Non-linear QCD at high energies

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A mini-review with personal opinion and attitude which can be wrong
The state of the art

The problem of high density QCD

- has not been solved;
- is needed to be solved;

Non-linear QCD at high energies
Outline:

- Practical impact on the LHC physics;
- The BFKL Pomeron calculus: ups and downs;
- Statistical analogy approach: Langevin equation and generating functional;
- Summing Pomeron loops in the BFKL Pomeron calculus;
- The high energy asymptotic behaviour:
- Problems, ideas, solutions ... ≡ bright future;
Practical impact on the LHC physics

Survival Probability for diffractive Higgs production

Tel Aviv Group (1994 - present)
Durham Group (1998 - present)

\[ \langle |S^2| \rangle \approx 0.02 \]

\[ E \cdot L Levin 5 \]

\[ \langle |S^2| \rangle \approx 0.004 \]
W - Higgs boson correlations in inclusive Higgs production

CDF (1997)
\[ \sigma_{DP} = m \frac{\sigma(2 \text{ jets}) \sigma(2 \text{ jets})}{2 \sigma_{eff}} \]
\[ \sigma_{eff} = 14.5 \pm 1.7 \pm 2.3 \text{ mb} \]
\[ \frac{1}{\sigma_{eff}} = \frac{1}{2\pi R_H^2} \]
\[ R_H^2 = 5 \div 7 \text{ GeV}^{-2} \]

- Hannes Jung at GGI WS on high density QCD asked to estimate such correlation for the LHC energy
- J. Miller (preliminary)

\[ \sigma_{DP}(LHC) = \frac{\sigma(W) \sigma(H)}{2 \sigma_{eff}} \approx 3 \sigma_{DP}(CDF) \]
News:

- Everything that has been done during the past three years is nothing more than understanding of the BFKL Pomeron calculus (Kozlov, E.L. & Prygarin; Bondarenko);

- The feedback from the probabilistic interpretation: four Pomeron interactions (Lublinsky & E.L);

- The news: The Pomeron interaction generates a new state with the intercept large than intercept of two BFKL Pomeron (Hatta & Mueller; E.L, Miller & Prygarin);
\[ 1 + \gamma = \gamma_1 + \gamma_2 \]

\[ \omega(\gamma) = \omega(\gamma_1) + \omega(\gamma_2) \]

\[ A \propto \frac{1}{2\pi i} \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\omega}{\omega - \omega(\gamma)} \left( \frac{1}{\omega - \omega(\gamma_1) - \omega(\gamma_2)} \right) \]

\[ e^\omega Y \frac{1}{\omega - \omega(\gamma)} \]

\[ \frac{1}{\omega - \omega(\gamma_1) - \omega(\gamma_2)} \]

\[ \omega = \omega(\gamma) \]

\[ A \propto \frac{e^{\omega(\gamma)} Y}{\omega(\gamma) - \omega(\gamma_1) - \omega(\gamma_2)} \]

\[ \omega = \omega(\gamma_1) + \omega(\gamma_2) \]

\[ A \propto \frac{e^{(\omega(\gamma_1) + \omega(\gamma_2))} Y}{\omega(\gamma_1) + \omega(\gamma_2) - \omega(\gamma)} \]

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\[ \omega(2\gamma_0 - 1) = 2 \omega(\gamma_0) \]

\[ 2 \omega(\gamma_0) > \omega(\gamma = 1/2) \]

\[ A \propto Y e^{2 \omega(\gamma_0)} Y \]

The sad truth: we have to start from the very beginning not only in summing Pomeron loops but also in MFA ?!
We know from the old good days of Reggeon Field Theory (Pomeron calculus) that this theory is the field theory for directed percolation (Grassberger & Sudermeyer (1978), Obukhov (1980), Cardy & Sugar (1980)) but because of the advent of QCD we did not investigate this idea in the full strength.

**TIME HAS COME**

- **People:** Blaizot, Brunet, Derrida, Enberg, Golec-Biernat, Hatta, Iancu, Itakura, E.L., Lublinsky, McLerran, Marquet, Mueller, Munier, Peshanski, Shoshi, Soyez, Triantafyllopoulos + . . . . . nearly everybody
The **BEAUTY**: we can model our high energy amplitude by creating some statistical system (glass???) and make the real experiment, may be, just on the desk.

Langevin equation:

\[
\frac{\partial \Phi}{\partial Y} = \frac{\bar{\alpha}_S}{2\pi} \mathbf{K} \otimes \Phi - \frac{2\pi \bar{\alpha}_S^2}{N_c} \Phi^2 + \zeta
\]

\[
< |\zeta| > = 0; \quad < |\zeta \zeta| > \neq 0
\]

Langevin equation for Einstein diffusion:

\[
\frac{d\vec{v}}{dt} = -\lambda \vec{v} + \zeta
\]
The main prediction: 
( Iancu, Mueller & Munier (2004))

- violation of the geometrical scaling behaviour;
- appearance of new saturation scale;

\[ A(z, Y) = A\left(\frac{\ln(r^2 Q_{new,s}^2)}{\sigma}\right) = \frac{1}{\sigma \sqrt{2\pi}} \int dz T(z) e^{-\frac{(z-<z>)^2}{2\sigma^2}} \]

- \( \sigma^2 \propto Y \);
- \( z = \ln(r^2 Q_s^2) \) where \( r \) is the dipole size;
- \( <z> = \ln(r^2 Q_{new,s}^2) \);
- \( T(z) = \) solution in the MFA;
- \( Q_{new,s} = \) new saturation (diffusion) scale.
The BFKL Pomeron calculus leads to (Kozlov, E.L. & Prygarin)

- $< \zeta(x_1, x_2; Y) \zeta(x'_1, x'_2; Y') > = B \delta(Y - Y') \delta(x_1 - x'_1) \delta(x_2 - x'_2)$

with

- $B \equiv 2 \frac{2\pi \bar{\alpha}_S}{N_c} \left( \frac{1}{p_1^2 p_2^2 \Phi^+(x_1, x_2; Y)} \right)^2$

\[ \times \int \frac{d^2 x_3}{x_{12}^2 x_{13}^2 x_{23}^2} (L_{12}\Phi(x_1, x_2; Y))\Phi^+(x_1, x_3; Y)\Phi^+(x_3, x_2; Y) \]

Price for simplification:

1. assumption: $L_{12} \Phi(x_1, x_2; Y) = \Phi(x_1, x_2; Y)$;
2. simplification: using the momentum representaion and looosing the connection to correct D.O.F.;
3. assumption: $b \gg \{x_{12}; x_{13}; x_{32}\}$ and, therefore, we are looosing a possibly to calculate the Pomeron loops;
Finally

\[ < \zeta(k, b; Y) \zeta(k', b'; Y) > = \]
\[ \frac{4\pi \bar{\alpha}_S^2}{N_c} \Phi(k, b; Y) \delta^{(2)}(\vec{b} - \vec{b}') \delta^{(2)}(\vec{k} - \vec{k}') \delta(Y - Y') \]

Questions and Surprises:

- The Langevin equation for the amplitude looks as follows:

\[ \frac{\partial N}{\partial Y} = \bar{\alpha}_S \left( N - N^2 + \bar{\alpha}_S \sqrt{2N} \zeta \right) \]

Only at \( N < \bar{\alpha}_S^2 \) the third term is essential. On the other hand it sums the enhanced diagrams, how is it possible?
- In the toy-model equation does not lead to decreasing amplitude at high energy (Munier (talk at GGI WS); Naftali (private communication)) in contradiction with the exact solution?! 

- For $z < 0$ the solution of the BFKL equation leads to the geometrical scaling behaviour (Iancu, Itakura & McLerran (2002)). How to match this solution with the scaling violating one?! 

- At first sight the $b$ correlation depend on future. What is wrong?
For up-looking fan diagrams:

\[
\frac{\partial N}{\partial Y} = \tilde{\alpha}_S \left\{ K \otimes N + \sqrt{\alpha_S^2 N} \right\}
\]

(i) what is the initial condition to get MFA?

(ii) why there is no scaling violation?
Summing Pomeron loops in the BFKL Pomeron calculus

(E.L., Miller & Prygarin)

\[ 1 \approx \bar{\alpha}_s Y \leq \ln 1/\bar{\alpha}_s^2 \leq \bar{\alpha}_s Y \leq \bar{\alpha}_s Y \leq 1/\bar{\alpha}_s \]

- \[ 1 \approx \bar{\alpha}_s Y \leq \ln 1/\bar{\alpha}_s^2 \rightarrow \text{LO BFKL Pomeron} \]
- \[ \ln 1/\bar{\alpha}_s^2 \leq \bar{\alpha}_s Y \leq 1/\bar{\alpha}_s \rightarrow \text{BFKL Pomeron calculus} \]
- \[ 1/\bar{\alpha}_s \leq \bar{\alpha}_s Y \rightarrow \text{NLO BFKL Pomeron and non-linear QCD} \]
\[ A \propto \int \frac{d\omega}{2\pi i} \frac{1}{\omega - \omega(\gamma)} \frac{1}{\omega - \omega(\gamma_1) - \omega(\gamma_2)} \frac{1}{\omega - \omega(\gamma')} \]

- \[ \omega = \omega(\gamma) = \omega(\gamma') \]

\[ A \propto Y \frac{e^{\omega(\gamma)} Y}{\omega(\gamma) - \omega(\gamma_1) - \omega(\gamma_2)} \]

- \[ \omega = \omega(\gamma_1) + \omega(\gamma_2) \]

\[ A \propto \frac{Y e^{(\omega(\gamma) + \omega(\gamma_2)) Y}}{(\omega(\gamma) + \omega(\gamma_2 - \omega(\gamma)))^2} \]
Overlapping singularity:

\[ \omega = \omega(2\gamma_0 - 1) = 2\omega(\gamma_0) \]

\[ A \propto Y^2 e^{2\omega(\gamma_0)Y} > e^{2\omega_{BFKL}(\gamma=1/2)Y} \]

Scenario:

\[ \gamma_0 > \gamma_{cr} \text{ therefore, two Pomerons } (\gamma_1 \text{ and } \gamma_2) \text{ are inside the saturation region;} \]

Inside the saturation region \( \omega_{sat}(\gamma) = \frac{\omega(\gamma_{cr})}{1-\gamma_{cr}} (1 - \gamma) \) (Bartels & E.L. (1992))

\[ 2\omega_{sat}(\gamma_0) = \omega_{pert}(2\gamma_0 - 1) \text{ has no solution;} \]
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The scenario:

- We can neglect the overlapping singularities;
- We are dealing with the system of the non-interacting BFKL Pomerons;
- For summing Pomeron loops we can use the Iancu-Mueller-Patel -Salam approximation, improved by the renormalization of the scattering amplitude at low energies;
An example:

\[ Y - Y' + - \]

\[ \gamma_R = \gamma + \gamma^2 \]

\[ \Delta_R = \Delta + \alpha_s^3 \]
\[ A = \quad \gamma^{BA} \quad \text{low energy amplitude} \quad \gamma_R \rightarrow \gamma^{BA} \]

**Solution:**

**For model BFKL kernel**

\[ \omega(\gamma) = \tilde{\alpha}_s \left\{ \begin{array}{cl} \frac{1}{\gamma} & \text{for } r^2 Q_s^2 \ll 1 \quad \text{summing} \quad (\tilde{\alpha}_s \ln(1/(r^2 Q_s^2)))^n; \\ \frac{1}{1 - \gamma} & \text{for } r^2 Q_s^2 \gg 1 \quad \text{summing} \quad (\tilde{\alpha}_s \ln(r^2 Q_s^2))^n; \end{array} \right. \]
we obtain:

- geometrical scaling behaviour;
- rather slow approaching the asymptotic value, namely

\[ 1 - N \propto \exp(-z) \quad \text{where} \]

\[ z = \ln(r^2Q_s^2); \]
Conclusions

“Once you eliminate the impossible what remains is the solution - no matter how improbable it may seem”