UHE Cosmic Rays above $10^{17}$ eV: The role of percolation

J. Alvarez-Muñiz\textsuperscript{1}, P. Brogueira\textsuperscript{2}, R. Conceição\textsuperscript{3}, J. Dias de Deus\textsuperscript{2,4}, M.C. Espírito Santo\textsuperscript{2,3}, M. Pimenta\textsuperscript{2,3}.

\textsuperscript{1} Dept. Física Partículas & IGFAE, Univ. Santiago de Compostela, 15782 Santiago, SPAIN,
\textsuperscript{2} Departamento de Física, IST, Av. Rovisco Pais, 1049-001 Lisboa, PORTUGAL,
\textsuperscript{3} LIP, Av. Elias Garcia, 14-1, 1000-149 Lisboa, PORTUGAL,
\textsuperscript{4} CENTRA, Av. Rovisco Pais, 1049-001 Lisboa, PORTUGAL.

Abstract

We show that the string percolation hadronic model provides a consistent interpretation of the experimentally measured energy dependence of the depth of shower maximum as well as of the muon content in ultra high energy cosmic ray showers above $10^{17}$ eV. We discuss the importance of the inelasticity and of the particle multiplicity in the most energetic shower interactions, as well as the crucial role played by the nature of the leading primary.

1 Introduction

The composition of ultra high energy cosmic rays (UHECRs) is only determined indirectly through the observation of two shower variables that are used as experimental handles on composition, namely, the atmospheric depth of shower maximum $X_{\text{max}}$, measured in fluorescence detectors, and the muon content of the shower on ground $N_{\mu}$, measured in surface array detectors.

Fly’s Eye/HiRes experimental data on the energy dependence of $X_{\text{max}}$ show an increase in the slope around $10^{17}$ eV. Concerning the muon component, at the AGASA array the lateral distribution function of muons was measured above $10^{17}$ eV and combined with the Akeno array data. It was observed that the slope of the density of muons at 600 m from the shower core $\rho_{\mu}(600)$ vs $E$ is flatter in data than predicted by hadronic models. Assuming $\rho_{\mu} \propto E^{\beta}$, data gives $\beta = 0.84 \pm 0.02$ [1, 2], while from simulation $\beta \sim 0.9$ ($\beta = 0.92$, 0.89 and 0.93 for protons with QGSJET-II.03, SIBYLL2.1 and EPOS 1.61 respectively). Both features have been interpreted as a change in composition, going from more Fe-like to more proton-like showers. An alternative interpretation is that these energy dependences are due to a change with energy of the features of the hadronic interactions. In this spirit, we show in this work that the string percolation model provides a consistent interpretation of the energy dependence of $X_{\text{max}}$ and $N_{\mu}$ in cosmic ray data above $10^{17}$ eV.

2 The two interpretations of UHE Cosmic Ray data

2.1 The “change in composition” interpretation

In order to make the argument simple, we use the original Heitler model of shower development [3]. The average location of $X_{\text{max}}$ in a shower initiated by a primary nucleus of energy $E$ and

\footnote{\textsuperscript{1} speaker}
mass $A$, is given by $X_{\text{max}} \simeq X_1 + X_0 \log(E/A)$, where $X_1$ is the average depth of the first collision and $X_0$ is the elongation rate. The muon content of the shower can be written as, $N_\mu \simeq A(E/A)\beta$. We then have,

$$\frac{dX_{\text{max}}}{d\log E} = X_0 \left[1 - \frac{d\log A}{d\log E}\right],$$  \hspace{1cm} (1)$$

and for the log $N_\mu$ dependence on $E$,

$$\frac{d\log N_\mu}{d\log E} = (1 - \beta) \frac{d\log A}{d\log E} + \beta.$$  \hspace{1cm} (2)$$

As experimentally, above $10^{17}$ eV $dX_{\text{max}}/d\log E$ is larger and $d\log N_\mu/d\log E$ is slightly smaller, in comparison with lower energies, the conclusion is:

$$\frac{d\log A}{d\log E} < 0,$$  \hspace{1cm} (3)$$
i.e., for $E \gtrsim 10^{17}$ eV the average mass number should, in this interpretation, decrease with $E$.

2.2 The “change in the features of hadronic interactions” interpretation

In this interpretation a key role is given to the variables characterising the first hadronic collisions. Among them the most relevant for shower development are the inelasticity $K$, defined as the fraction of energy distributed among the secondary particles except for the leading particle, the average (non-leading) multiplicity $\langle n \rangle$ at the collision energy and $P_0$, the probability of producing a leading $\pi^0$ in the collision.

Changing $A$ has a similar effect on the behavior of $X_{\text{max}}$ and $N_\mu$ with energy as changing $K$. The fastest particle in the collision, carrying an energy $(1 - K)E$, will originate the shower branches that go deeper in the atmosphere. We can then write, $X_{\text{max}} \simeq X_1 + X_0 \log[(1 - K)E_0]$, where $E_0$ is a low energy threshold. Regarding the muon content of the shower, a possible assumption is that while energy flows in the $(1 - K)$ direction, the number of muons flows in the $K$ direction and $N_\mu \sim N_\pi^\pm \propto KE$. We thus have,

$$\frac{dX_{\text{max}}}{d\log E} = X_0 \left[\frac{d\log(1 - K)}{d\log E} + 1\right],$$  \hspace{1cm} (4)$$

and

$$\frac{d\log N_\mu}{d\log E} = \frac{d\log K}{d\log E} + 1.$$  \hspace{1cm} (5)$$

Since above $10^{17}$ eV, $dX_{\text{max}}/d\log E$ is larger and $d\log N_\mu/d\log E$ smaller than at lower $E$, we infer that:

$$\frac{d\log K}{d\log E} < 0,$$  \hspace{1cm} (6)$$
i.e., some of the features in CR data can be alternatively explained by an inelasticity $K$ decreasing with $E$. 
Moreover, the nature of the leading particle, plays a fundamental role in the interpretation of cosmic ray data, affecting the relation \( N_\mu \propto K \). In fact, when the leading particle is a proton carrying an energy \((1 - K)E\), the number of muons decreases with \( K \) due to the combination of two effects: on one hand the production of secondary \( \pi^0 \)'s, which decay into \( \gamma \)'s, carrying an energy \( \sim 1/3 KE \) that is lost for muon production, and on the other hand, the larger the \( K \) the smaller the energy carried by the leading particle, and hence the smaller the number of new charged pions and muons reaching ground it will produce. However, if \( \pi^0 \)'s are themselves leading particles - \( P_0 > 0 \), muons can only be produced in the \( K \) direction and \( N_\mu \) is larger the larger the inelasticity. This can be seen in a very simplistic model in which the number of muons is proportional to the energy available for muon production in the first interaction \( N_\mu \sim (1 - P_0)(1 - K)E + (2/3)KE \), where the factors \((1 - P_0)\) and \(2/3\) account for the energy lost into the electromagnetic branch. As a consequence,

\[
\frac{dN_\mu}{dK} \sim (P_0 - \frac{1}{3})E, \tag{7}
\]

showing that the behavior of \( N_\mu \) with \( K \) depends on \( P_0 \). The fact that it inects at the particular value \( P_0 = 1/3 \) comes from the very simplistic treatment and should not be taken seriously. This effect has been confirmed by a numerical implementation of \( P_0 \) in a full hybrid Monte Carlo simulation of shower development [4]. The crucial role played by \( P_0 \) on the dependence of \( N_\mu \) with \( K \) is apparent in the left panel of Fig. 1.

Fig. 1: Left: Hybrid Monte Carlo simulation prediction for the number of muons at ground as a function of inelasticity for different values of the probability \( P_0 \) of having a leading \( \pi^0 \) in a proton-induced shower of \( E = 10^{18} \text{ eV} \). Right: Energy dependence of the average inelasticity \( K \). The dotted curve is the SIBYLL simulation; the dashed-dotted curve corresponds to QGSJET simulations; and the dashed line is a model of [8]; the full line is our percolation model.
Fig. 2: Left: $N_p$ vs $E$. The results of the string percolation model (solid line) are compared to the Akeno/AGASA data (crosses). The dotted line corresponds to $N_p \propto E^{0.9}$ as predicted by most of the standard hadronic models. Right: $X_{\text{max}}$ vs $E$ in the percolation model (solid line) along with data (symbols) and the predictions of SIBYLL and QGSJET.

3 The role of the string percolation model

In string models of hadronic interactions, multiparticle production is described in terms of colour strings stretched between partons of the projectile and target particles. Colour strings might be viewed as small discs in the transverse space of radius $r_0 \sim 0.2$ fm, filled with the colour field. These strings fragment into new strings through $q - \bar{q}$ production and subsequently hadronize into particles. At low energy, valence strings are formed, forward and backward in the center of mass frame, containing most of the collision energy. As the energy increases additional sea strings, central in rapidity, are created carrying part of the energy of the valence strings. Softer secondaries are produced and the inelasticity increases with energy. With growing energy and/or atomic number of colliding particles, the number of strings increases further, and they begin to overlap forming clusters. A cumulative effect then occurs, the length in rapidity of fused strings is larger than that of individual strings and fast particles are produced. As a consequence, the inelasticity $K$ starts to decrease with energy as can be seen in Fig. 1 [5]. The onset of this effect has been estimated in [5] to occur above $\sim 10^{17}$ eV for proton-proton collisions when the area occupied by the overlapping strings is $\sim 1.13$ times the area of the interaction region. Percolation also affects the multiplicity. Due to the fusion of strings less softer secondaries are produced and the multiplicity is reduced with respect to models in which strings decay independently. Moreover, and since fused strings are sea strings of the $q - \bar{q}$ type, the fast leading secondaries that are produced can be $\pi^0$s with a probability $P_0$ that tends to 0.3.

These predictions are in contrast with those of string models with no string fusion, in which the inelasticity and multiplicity increases monotonically with energy, and the probability...
of $\pi^0$ being the leading particle is $P_0 \sim 0.1$ at most.

In order to test the effects of these predictions on the air shower observables, $X_{\text{max}}$ and $N_\mu$, we have implemented them into a hybrid and fast Monte Carlo simulation of shower development in the atmosphere described in [4]. In particular we have implemented an inelasticity $K$ decreasing with energy [6] as shown in Fig. 1, a reduction of the multiplicity realized through a color summation factor [5], and a probability $P_0 = 0.3$ of the leading particle being a $\pi^0$, in all hadronic interactions above $10^{17}$ eV. In the left panel of Fig. 2 we show the prediction of percolation on the behavior of $N_\mu$ with $E$. A fit to our results yields $\beta = 0.83$ compatible with the experimental value measured by the Akeno/AGASA array $\beta = 0.84 \pm 0.02$. It is important to notice that in this study protons were used as primaries and the result in Fig. 2 corresponds to shifting the proton result to an intermediate and constant composition $A \sim 20$ using that $N_\mu \sim A(E/A)^\beta$. It is interesting to note that when $E > 10^{19}$ eV, the Lorentz boost factor of $\pi^0$s is large enough so that they begin to interact in the atmosphere instead of decaying, increasing the production of muons, steepening the slope of $N_\mu$ vs $E$ line. A hint of this effect is clearly seen in the left panel of Fig. 2. This behaviour is consistent with observations of the total number of muons at the ground in inclined hybrid events collected at the Pierre Auger Observatory above $6.3 \times 10^{18}$ eV [7].

In the right panel of Fig. 2 we show the prediction of the string percolation model on the behaviour of $X_{\text{max}}$ with $E$ along with the predictions of other models based on QCD taken from [8]. The prediction is reasonably consistent with data, following the tendency of the slope around $10^{17}$ eV. A slight overshoot of $X_{\text{max}}$ can be seen at $E > 10^{18.5}$ eV which we believe might be due to the constant value of $P_0 = 0.3$ used in all interactions instead of a smoothly increasing function of the energy. The solid line in Fig. 2 corresponds to the prediction for proton primaries scaled by a constant composition of $A \sim 15-20$, assuming $X_{\text{max}} \sim \log(E/A)$. It is important to remark that the value of $A$ is the same with which the $N_\mu$ vs $E$ prediction was scaled.

4 Conclusions, discussion and outlook

We have shown that, in the interpretation of the energy dependence of the depth of the shower maximum and of the muon content in high energy cosmic ray showers ($E \geq 10^{17}$ eV), other variables besides the composition may play an important role, in particular those characterising the first (high energy) hadronic collisions. The role of the inelasticity $K$, the probability $P_0$ of the leading particle being a $\pi^0$, and the particle multiplicity $<n>$ were discussed. Special emphasis has been put on the crucial role played by $P_0$ on the dependence of the number of muons with $K$.

For the first time the predictions of the behaviour of these variables in a string percolation model were included systematically and used to derive predictions on the behaviour of the depth of the shower maximum $X_{\text{max}}$ and on the number of muons in the shower $N_\mu$ as a function of the energy. The model reasonably describes the trends seen in data.

The next step will be to create a full Monte Carlo simulation including percolation, adapted to the low and high energy regions and including proton-air and nuclei-air collisions where more complex effects may arise. Also we should account for the fact that the clusters of percolating
strings have a large colour content, and an increased production of baryons is likely and cannot be neglected since it affects the number of muons in the shower [9].

References