NLO jet production in $k_T$-factorisation

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Motivation

- We want to understand the strong force (QCD)
  - per se as one of the fundamental forces in nature
  - as background at collider experiments
- soft energy scale $\sim$ confinement $\sim$ no free quarks/ gluons observable, but jets of hadronized particles
- hard energy scale $\sim$ asymptotic freedom $\sim$ access via perturbative QCD
- factorization to disentangle soft from hard physics
Motivation - need for BFKL resummation

perturbative QCD = expansion in coupling $\alpha_s$

- large but ordered scales (e.g. $s \gg |t| \gg \Lambda_{QCD}^2$) $\sim$ large logs ($\log s/t$) for each additional emission in multi Regge kinematics $\sim$ compensating smallness of $\alpha_s$

- need to resum terms $\sim (\alpha_s \log s/t)^n$
  $\sim$ LO Balitsky-Fadin-Kuraev-Lipatov equation ['75-'78]

- resummation of terms $\sim \alpha_s(\alpha_s \log s/t)^n$
  $\sim$ NLO BFKL equation ['98]
Outline

1 Motivation and Introduction ✓
2 Jet production vertex at central rapidity
   - Jet production at LO
     $\gamma^* \gamma^*$, pp, unintegrated gluon density
   - Jet production at NLO
     $\gamma^* \gamma^*$, pp, unintegrated gluon density
3 Summary
Total cross section at LO BFKL
Total cross section at LO BFKL

\[ \sigma(s) = \int \frac{d^2 k_a}{2\pi k_a^2} \int \frac{d^2 k_b}{2\pi k_b^2} \Phi_A(k_a) \Phi_B(k_b) \]
\[ \times \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{s_0} \right)\omega f_\omega(k_a, k_b). \]

- with impact factors \( \Phi \)
- Green’s function \( f_\omega \) obeys BFKL equation

\[ \omega f_\omega(k_a, k_b) = \delta^{(2)}(k_a - k_b) \]
\[ + \int d^2 k \mathcal{K}(k_a, k) f_\omega(k, k_b) \]
Jet production at LO BFKL

\[
\frac{d\sigma}{d^2k_{Jet} dy_{Jet}} = \int \frac{d^2k_a}{2\pi k_a^2} \int \frac{d^2k_b}{2\pi k_b^2} \Phi_A(k_a) \Phi_B(k_b) \\
\times \int d^2q_a \int d^2q_b \int \frac{d\omega}{2\pi i} \left( \frac{s_{AJ}}{s_0} \right)^\omega f_\omega(k_a, q_a) \\
\times \mathcal{V}(q_a, q_b; k_{Jet}, y_{Jet}) \\
\times \int \frac{d\omega'}{2\pi i} \left( \frac{s_{BJ}}{s'_0} \right)^{\omega'} f_{\omega'}(-q_b, -k_b)
\]

with the LO emission vertex

\[
\mathcal{V} = \mathcal{K}_{real}^{(LO)}(q_a, -q_b) \delta^{(2)}(q_a + q_b - k_{Jet}).
\]
\( \gamma^* \gamma^* \) scattering (LO)

- impact factors and jet provide hard scale as well
- symmetric situation, choose \( s_0 \) as
  \[
  s_0 = |k_a| |k_{Jet}|, \quad s'_0 = |k_{Jet}| |k_b|
  \]
- natural language of rapidities:
  \[
  \left( \frac{s_{AJ}}{s_0} \right)^\omega = e^{\omega(y_A - y_{Jet})}
  \]
**pp scattering (LO)**

- only jet provides hard scale
- asymmetric situation, choose $s_0$ as
  
  $$s_0 = k^2_{Jet}, \quad s'_0 = k^2_{Jet}$$

- natural language of longitudinal momentum fractions

  $$\left( \frac{s_{AJ}}{s_0} \right) ^\omega = \left( \frac{1}{x_1} \right) ^\omega$$
Jet production at LO BFKL

Jet production at NLO BFKL

Summary

\textbf{pp scattering (LO)}

- only jet provides hard scale
- asymmetric situation, choose $s_0$ as
  \[ s_0 = k_{Jet}^2, \quad s'_0 = k_{Jet}^2 \]

natural language of longitudinal momentum fractions

\[ \left( \frac{s_{AJ}}{s_0} \right) = \left( \frac{1}{x_1} \right) \]
Unintegrated gluon density

define the unintegrated gluon density

\[ g(x, k) = \int \frac{d^2 q}{2\pi q^2} \Phi_P(q) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} x^{-\omega} f_\omega(q, k) \]

which obeys the evolution equation

\[ \frac{\partial g(x, q_a)}{\partial \ln 1/x} = \int d^2 q \ K(q_a, q) g(x, q) \]

Then cross section can be written in \( k_T \) factorization

\[ \frac{d\sigma}{d^2 k_{Jet} dy_{Jet}} = \int d^2 q_a \int d^2 q_b \ g(x_1, q_a) g(x_2, q_b) \mathcal{V}(q_a, q_b; k_{Jet}, y_{Jet}) \]
Q: Can we just replace the LO expressions for impact factors, kernel and Green’s function by their NLO counterparts?
Q: Can we just replace the LO expressions for impact factors, kernel and Green's function by their NLO counterparts?
A: No!

- real Kernel $\mathcal{K}_{\text{real}}$ contains at NLO two particle production
  - jet algorithm
  - separation MRK ↔ QMRK ~ scale $s_\Lambda$
- energy scale $s_0$ is now a relevant parameter
Jet definition

- remember: at LO $\mathcal{K}_{\text{real}} \sim \leftarrow \sim \nu$

- at NLO $\mathcal{K}_{\text{real}} \sim \uparrow + \int \downarrow$

- for $\downarrow$ two possibilities:
  - both together form a jet
  - one forms the jet, other one unresolved

- define distance in rapidity-azimuthal angle space

$$R_{12} = \sqrt{(y_1 - y_2)^2 + (\phi_1 - \phi_2)^2}$$

- $\theta(R_0 - R_{12}) : \downarrow$

- $\theta(R_{12} - R_0) : \downarrow^x$

- open integration to extract jet

$$\nu \sim \uparrow + \int \downarrow + \int \downarrow^x$$
Subtraction term

- real and virtual parts with different \(x_{1,2}\) configurations \(\sim\) different \(g(x_1, q_a)g(x_2, q_b)\) \(\sim\) cancellation of divergences?

\[
\nu = \begin{pmatrix}
\nu^R \\
\int \nu^L \\
\int \nu^L^x
\end{pmatrix} + \int \nu^L + \int \nu^L^x
\]
Subtraction term

- real and virtual parts with different $x_{1,2}$ configurations $\sim$ different $g(x_1, q_a)g(x_2, q_b)$ $\sim$ cancellation of divergences?

$$\nu = \left( \int \mathcal{K} \right) + \int \left( \mathcal{K} - \mathcal{K}^x \right) + \int \left( \mathcal{K}^x - \mathcal{K}^x \right)$$

- add singular part of 2 particle production (in $x$ configuration of virtual part) times $0 = 1 - \theta(R_0 - R_{12}) - \theta(R_{12} - R_0)$
- first bracket: analytical cancellation of divergences
- second and third bracket: numerical cancellation of divergences
\(\gamma^*\gamma^*\) scattering (NLO)

- NLO calculation of the kernel was performed in framework with hard scale impact factors.
- \(\sim\) can keep (in principle) LO formula with NLO impact factors, Green’s functions, jet vertex.
pp scattering (NLO)

proton: soft scale   jet: hard scale

- in asymmetric situation: necessity of scale change
  \[ s_0 = |k_a| |k_{Jet}| \rightarrow s_0 = k_{Jet}^2 \]

- symmetric change
  \[ s_0 = |k_a| |k_{Jet}| \rightarrow s_0 = f_1(|k_a|) f_2(|k_{Jet}|) \]
  could be compensated by change in only the impact factors and the vertex

- asymmetric change effects complete evolution; now from a soft scale to a hard scale
pp scattering - consequences of scale change

- modified Kernel for evolution of Green’s function

\[ \tilde{\mathcal{K}}(q_1, q_2) = \mathcal{K}(q_1, q_2) - \frac{1}{2} \int d^2 q \, \mathcal{K}^{(LO)}(q_1, q) \mathcal{K}^{(LO)}(q, q_2) \ln \frac{q^2}{q_2^2} \]

\[ \omega \tilde{f}_\omega(k_a, q_a) = \delta^{(2)}(k_a - q_a) + \int d^2 q \, \tilde{\mathcal{K}}(k_a, q) \tilde{f}_\omega(q, q_a) \]
Jet production at LO BFKL

Jet production at NLO BFKL

Summary

`pp` scattering - consequences of scale change

- modified Kernel for evolution of Green’s function

\[
\tilde{\mathcal{K}}(q_1, q_2) = \mathcal{K}(q_1, q_2) - \frac{1}{2} \int d^2 q \mathcal{K}^{(LO)}(q_1, q) \mathcal{K}^{(LO)}(q, q_2) \ln \frac{q^2}{q_2^2}
\]

\[
\omega \tilde{f}_\omega(k_a, q_a) = \delta^{(2)} (k_a - q_a) + \int d^2 q \tilde{\mathcal{K}}(k_a, q) \tilde{f}_\omega(q, q_a)
\]

- modified proton impact factor

\[
\tilde{\Phi}(k_a) = \Phi(k_a) - \frac{1}{2} k_a^2 \int d^2 q \frac{\Phi^{(LO)}(q)}{q^2} \mathcal{K}^{(LO)}(q, k_a) \ln \frac{q^2}{k_a^2}.
\]
Jet production at LO BFKL

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Summary

$pp$ scattering - consequences of scale change

- modified Kernel for evolution of Green’s function

$$\tilde{\mathcal{K}}(q_1, q_2) = \mathcal{K}(q_1, q_2) - \frac{1}{2} \int d^2 q \mathcal{K}^{(LO)}(q_1, q) \mathcal{K}^{(LO)}(q, q_2) \ln \frac{q^2}{q_2^2}$$

$$\omega \tilde{f}_\omega(k_a, q_a) = \delta^{(2)}(k_a - q_a) + \int d^2 q \tilde{\mathcal{K}}(k_a, q) \tilde{f}_\omega(q, q_a)$$

- modified proton impact factor

$$\tilde{\Phi}(k_a) = \Phi(k_a) - \frac{1}{2} k_a^2 \int d^2 q \frac{\Phi^{(LO)}(q)}{q^2} \mathcal{K}^{(LO)}(q, k_a) \ln \frac{q^2}{k_a^2}.$$

$$\Rightarrow$$ new NLO unintegrated gluon distribution

$$g(x, k) = \int d^2 q \frac{\tilde{\Phi}_P(q)}{2\pi q^2} \int \frac{d\omega}{2\pi i} \tilde{f}_\omega(k, q) x^{-\omega}$$

$$\frac{\partial g(x, q_a)}{\partial \ln 1/x} = \int d^2 q \tilde{\mathcal{K}}(q_a, q) g(x, q).$$
pp scattering - consequences of scale change

⇒ new NLO unintegrated gluon distribution

\[
\frac{\partial g(x, q_a)}{\partial \ln 1/x} = \int d^2q \tilde{K}(q_a, q) g(x, q).
\]
pp scattering - consequences of scale change

- new NLO unintegrated gluon distribution

\[
\frac{\partial g(x, q_a)}{\partial \ln 1/x} = \int d^2 q \tilde{\kappa}(q_a, q) g(x, q).
\]

- modified vertex

\[
\tilde{\mathcal{V}}(q_a, q_b) = \mathcal{V}(q_a, q_b) - \frac{1}{2} \int d^2 q \mathcal{K}^{(LO)}(q_a, q) \mathcal{V}^{(LO)}(q, q_b) \ln \frac{q^2}{(q - q_b)^2}
- \frac{1}{2} \int d^2 q \mathcal{V}^{(LO)}(q_a, q) \mathcal{K}^{(LO)}(q, q_b) \ln \frac{q^2}{(q_a - q)^2}.
\]
Summary

We constructed a jet vertex
- in NLO $k_T$ factorization
- explicitly free of divergences
- implications for definition of uPDFs at NLO
- kept track of dependence on all scales involved