Manifestations of Gluon Saturation in RHIC data

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Motivation:

• Very basics of gluon saturation
• Relativistic Heavy Ion Collider

Experimental signatures of saturation at RHIC

• Multiplicities in d-Au and Au-Au collisions
• Moving forward: d-Au
  - Particle spectra
  - Cronin enhancement / suppression
• Others

Summary and conclusions
@ High energy QCD: strong coherent gluon fields and gluon saturation. UNITARITY

Packing factor: $\rho_\perp \sigma^{gg} \sim O(1)$

$Q_s^2(x) \sim \frac{N_c \alpha_s(Q_s)}{\pi R_h^2} xG(x, Q_s)$

@ High density: Color Glass Condensate

- Large gluon occupation numbers at small-x
  \[
  \frac{dN^g}{dy \, d^2k} \sim \frac{1}{\alpha_s}, \quad k < Q_s
  \]
  \[
  Q_s \gg \Lambda_{QCD} \Rightarrow \alpha_s(Q_s) \ll 1
  \]
- Weak coupling methods
- Semiclassical approach:
  McLerran-Venugopalan model for nuclear gluon distributions
  +
  - Non-linear quantum evolution:
    Balitsky-Kovchegov, JIMWLK equations

@ Low density
- Dilute system: Incoherence
- Linear BFKL-DGLAP dynamics
Relativistic Heavy Ion Collider (BNL, Upton, NY)

<table>
<thead>
<tr>
<th>system</th>
<th>c.m.e. per nucleon (GeV)</th>
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<tbody>
<tr>
<td>Au-Au</td>
<td>19.6, 62.4, 130 and 200</td>
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<tr>
<td>Cu-Cu</td>
<td>62.4 and 200</td>
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<td>d-Au</td>
<td>200</td>
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<tr>
<td>p-p</td>
<td>62.4 and 200</td>
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Atomic numbers: 197, 64, 2

- **Nuclear enhancement**: large number of gluons in the nuclear wave function even at moderate energies:

\[
x G_A(x, Q^2) \sim A x G_N(x, Q^2)
\]

\[
Q_{sA}^2(x) \sim A^{1/3} Q_{sN}^2(x)
\]

\[
Q_{sAu}^2(\sqrt{s_{NN}} = 130 \text{ GeV}) |_{\eta=0} \sim 1 \text{ GeV}^2
\]
@ Au-Au collisions at high energies: Complicated dynamics

can we trace back initial saturation effects from detected particles??
@ d-Au collisions at high energies: (not so) complicated dynamics

Final state effects from QGP absent in d-Au collisions
Particle Multiplicities at RHIC

@ Incoherent independent scatterings: pQCD factorization, soft models (+ shadowing)

@ Saturation: Strong coherence effects. Reduced number of initial scattering centers:

- Final A-A multiplicity $\sim$ total number of gluons in nuclei wave function

$$\frac{dN_{AA}}{d\eta d^2b} \bigg|_{\eta=0} \sim \frac{1}{\alpha_s(Q_s)} Q_s^2 \sim xG(x, Q_s^2) \cdot A^{1/3}$$

Mostly small-$x$, transverse gluons
Particle Multiplicities at RHIC

@ Incoherent independent scatterings: pQCD factorization, soft models

@ Saturation: Strong coherence effects. Reduced number of initial scattering centers

PHOBOS Au-Au (200 GeV)

\[ \frac{dN_{AA}^{\text{ch}}}{d\eta} \mid \eta = 0 \]

Predictions prior to RHIC

Incoherent p+p superposition

CGC (McLerran, Venugopalan)

\[ \sqrt{S_{NN}} \]
Particle production in nuclear collisions:

@ \( k_t \)-factorization + saturation of unintegrated gluon distributions

\[
\frac{dN_{AB}^g}{d^2b\,d\eta} \sim \int \frac{d^2p}{p^2} \int d^2k \int d^2s \, \alpha_s \varphi_A(x_1, k; b) \varphi_B(x_2, p - k; b - s)
\]

\[\varphi(x, k_t)\]

\[x_0 < x_1 < x_2\]

\[h(x, k_t) = k^2 \nabla^2 \varphi\]

\[Q_s(x_1)\quad Q_s(x_2)\]

\[x_1(2) = \frac{p_t}{\sqrt{s}} e^{\pm \eta}\]

\( k_t < Q_s \) reduced respect linear dynamics BFKL

\[Q_s^2(x) \sim Q_0^2 \left( \frac{x_0}{x} \right)^\lambda \Rightarrow \frac{1}{N_{part}} \left. \frac{dN_{AB}^g}{d^2b\,d\eta} \right|_{\eta=0} = \begin{cases} \sqrt{s^\lambda} \ln \left( \sqrt{s^\lambda} N_{part} \right) ; \\
\sqrt{s^\lambda} N_{part}^{\frac{1-\delta}{3\delta}} ; \end{cases}\]

Kharzeev-Levin Nardi
Armesto-Salgado Wiedemann

Large energies:
Traveling waves
Energy dependence encoded in \( Q_s(x) \)
“geometric scaling”
• Geometric scaling found in DIS small-\(x\) data both in nuclear and proton reactions:

• (A good part of) RHIC phenomenology is based on this empirical information

\[
\sigma^{\gamma^*h}(x, Q^2) \rightarrow \sigma^{\gamma^*h}(\tau = Q^2/Q_s^2(x))
\]

- For proton: Golec-Biernat Wustoff
  \[
  Q_{sp}^2 = Q_0^2 \left( \frac{x_0}{x} \right)^{\lambda}
  \]
  \(\lambda \approx 0.28\)

- For nuclei: Armesto et al
  \[
  Q_{sA}^2 = Q_0^2 A^{1/3 \delta} \left( \frac{x_0}{x} \right)^{\lambda}
  \]
  \(\delta \approx 0.8\)
Saturation based models reproduce the collision energy, pseudorapidity and centrality dependence of multiplicity densities at RHIC

Kharzeev-Levin-Nardi

But: raw implementation of non-perturbative effects:
- Local parton-hadron duality
- Effective gluon mass
- Lack of impact parameter integration
• All main features of saturation models for gluon production remain after hydro simulation
  • Final state reflects early time properties of the system

@ CGC provides good initial conditions for hydrodynamics evolution

Further evolution (QGP, hadronic phase...): Hydrodynamics

Initial density of thermalized? gluons: CGC

- CGC+hydro
- PHOBOS

Hirano-Nara
Centrality dependence:

\[
\frac{1}{N_{\text{part}}} \frac{dN_{AB}^{g}}{d^{2}b d\eta} \bigg|_{\eta=0} \sim \begin{cases} 
\sqrt{s^{\lambda}} \ln \left( \sqrt{s^{\lambda}} N_{\text{part}} \right) \\
\sqrt{s^{\lambda}} N_{\text{part}}^{1-\delta/3\delta} 
\end{cases}
\]

- Factorization of energy and centrality dependence:
  
  collinear factorization: \( \frac{dN}{d\eta} \sim N_{\text{coll}} \)

- Ratio of \( dN/d\eta \) at \( \eta=0 \) relative to 200 GeV vs centrality

Armesto et al

PHOBOS
nucl-ex/0510042

C. Loizides

Cu+Cu
Au+Au

pp
preliminary (QM05)
Classical Gluodynamics:

- Includes factorization violating terms
- Numerical methods (lattice)
- Boost invariant: lack of rapidity dependence
  ... similar results to KLN approach

- Gluon fields in forward light cone given by classical YM EOM

\[ [D_{\mu}, F^{\mu\nu}] = J^{\nu} \]

\[ J^{\nu} = \delta^{\nu+} \rho_A(x_\perp) \delta(x^-) + \delta^{\nu-} \rho_B(x_\perp) \delta(x^+) \]

\[ \langle \rho^a(x_\perp) \rho^b(y_\perp) \rangle = g^2 \mu^2 \delta^{ab} \delta(x_\perp - y_\perp) \]
Moving forward in d-Au collisions

\[ Q_s^2(x) = Q_0^2 \left( \frac{x_0}{x} \right)^\lambda ; \quad x_1(2) = \frac{p_t}{\sqrt{s}} e^{\pm \eta} \]

- Scanning a large rapidity interval gives information about non-linear evolution:
  - Rapidity dependence of the saturation scale
  - Dynamical generation of geometric scaling and/or scaling violations
@ Nuclear modification factor in d-Au collisions:

\[ R_{dAu} = \frac{\frac{dN^{dAu}}{d\eta d^2bd^2p}}{N_{coll} \frac{dN^{pp}}{d\eta d^2bd^2p}} \]

- \( \eta \sim 0 \): multiple rescatterings (semiclassical: MV, Glauber-Mueller)

\[ p \lesssim Q_{sA} \Rightarrow R_{dAu} < 1 \]
\[ p \gtrsim Q_{sA} \Rightarrow R_{dAu} > 1 \]

Cronin enhancement

- \( \eta >> 0 \): non-linear evolution washes out the enhancement.

Suppression:

\[ R_{dAu} < 1 \ \forall \ p \]
@ Nuclear modification factor in d-Au collisions:

- QCD non-linear evolution predicted the disappearance of the Cronin enhancement present in d-Au data at central rapidities at more forward rapidities:
Spectrum in d-Au collisions

\[ \frac{d\sigma}{d\eta d^2k} \sim \text{pdf}_{\text{proj}} \otimes \varphi_{\text{target}} \otimes D_{h/q} \]

\[ \varphi(x, k) = \int \frac{d^2r}{2\pi r^2} e^{ikr} N(x, r); \quad N(x, r) = 1 - e^{(r^2 Q_s^2(x))^\gamma} \]

\[ \gamma(r, Y) = \gamma_s + (1 - \gamma_s) \frac{\ln r Q_s}{\lambda Y + \ln r Q_s + d\sqrt{Y}}; \quad \gamma_s = 0.628... \]

Improved theoretical framework:

- **Large-x effects** in projectile: pdf’s + DGLAP evolution
- **Small-x solutions**: parametrizations based on HERA fits (GBW model)
- Fragmentation functions

**Dumitru et al**

- Strong scaling violations required to fit central rapidity data
- Approximate scaling in the forward region
Nuclear modification factor in d-Au collisions revisited

- Same theoretical framework + different parametrization of scaling violation
- Realistic parametrization of u.d.g. yield a much better quantitative comparison with experimental data. Leading-log evolution is too fast.

\[ \eta = 0 \]
\[ \eta = 1 \]
\[ \eta = 2.2 \]
\[ \eta = 3.2 \]
Limiting fragmentation

\[ \frac{dN_{ch}}{d\eta'}(\eta', s, b) \Rightarrow \frac{dN_{ch}}{d\eta'}(\eta', b) \]

\[ x_{1(2)} = \frac{p_t}{\sqrt{s}} e^{\pm \eta} \]

\[ \eta' = \eta - y_{beam} \]

\[ \frac{dN_{ch}}{d\eta'} \sim (x_q f_A^q(x_q) + x_g G_A(x_g)) \]
@ Quark production

- Net baryon transfer (valence quark production) (McLerran et al, JLA and Kovchegov)

- Sea quark production (Gelis et al, Kharzeev, Kovchegov, Tuchin)
Summary - Conclusions

- Manifestations of gluon saturation have been consistently identified in RHIC d-Au and Au-Au collisions at 130 and 200 GeV per nucleon.

  - Energy, pseudorapidity and centrality dependence of multiplicities in d-Au and Au-Au collisions
  - Suppression of Cronin enhancement in d-A collisions at forward rapidities
  - Limiting fragmentation, quark production.

- However:

  - RHIC is in the limit of applicability of high energy formalism. $Q_s \sim 1 \text{ GeV}$
  - Non-perturbative effects are not under control.
  - RHIC data does not have enough discriminating power to rule out other models/physical mechanisms
What’s next?

• More theoretical input:
  ⇒ Next-to leading order calculations in particle production
  ⇒ Evolution equations for dense-dense scattering
  ⇒ Thermalization/equilibration dynamics in A-A collisions

• More exclusive and ‘cleaner’ observables: correlations, photon and dilepton production

• Full high energy regime will be reached at the LHC:
  (Pb-Pb @ 5.5 TeV, d-Pb @ 7 TeV); \( Q_s^2 \sim 2\div3 \text{ GeV} \)

• Electron-ion collider (EIC):
  Ideal experimental ground to test the high density regime of QCD (eRHIC ??)
Back up slides
Back up slides
Ratio of dAu to pp: \( y = 3.2 \)

pQCD collinear factorization + LT shadowing
FIG. 2. Transverse momentum distributions of pions produced in a proton–gold collision computed by fragmenting gluons from conventional pQCD (dotted) and from CGC (solid) calculations. The topmost two curves are for $y = 0$ and the other pairs are for $y = 2$ and $y = 3$, respectively. The dashed line shows the suggested fit of the pion distribution.
• They are both linear equations ⇒ Dilute regime

• Only radiative/splitting processes taken into account:
  ⇒ Endless growth of gluon densities at large energy
  \[ xG^{BFKL}(x, Q^2) \sim x^{-4\alpha \ln 2} \sim x^{-0.5} \]

  ⇒ Unitarity violation: Scattering matrix: \( 0 \leq S \leq 1 \)
  Froissart bound: \( \sigma^{\text{hadron}} \lesssim \ln^2 s \)

• Coherence, non-linear effects are needed to restore unitarity: SATURATION

• Dual description of unitarization effects:

  multiple scattering ⇔ gluon recombination
\[ \kappa = \rho \sigma^{g g} \sim \frac{x G_h(x, Q_s^2)}{R_h^2} \frac{\alpha(Q_s^2)}{Q_s^2} \sim 1 \]

• Packing factor:

• In nuclear collisions the saturation regime is reached at lower energies:

\[ x G_A(x, Q^2) \sim A x G_N(x, Q^2); \quad R_A^2 \sim A^{2/3} R_p^2 \quad \Rightarrow \quad Q_{sA}^2(x) \sim A^{1/3} Q_{sN}^2(x) \]
The Balitsky-Kovchegov equation

The BK equation describes the energy evolution of the forward scattering amplitude of a quark-antiquark dipole on a hadronic target at leading logarithmic accuracy:

\[ \frac{\partial S(x, y; Y)}{\partial Y} = \int d^2 z \ K(x, y, z) \left[ S(x, z; Y)S(z, y; Y) - S(x, y; Y) \right] \]

The kernel: probability of small-x gluon emission at leading \( \alpha_s \ln(1/x) \):

\[ K(x, y, z) = \frac{\alpha_s N_c}{2\pi^2} \frac{(x - y)^2}{(x - z)^2(z - y)^2} \]

+ virtual corrections:
BK is the large-$N_c$ limit of the B-JIMWLK hierarchy of coupled evolution equations:

$$\frac{\partial N(\bar{x}, \bar{y})}{\partial Y} = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 z}{2\pi} \frac{(\bar{x} - \bar{y})^2}{(\bar{x} - \bar{z})^2 (\bar{y} - \bar{z})^2} \left[ N(\bar{x}, \bar{z}) + N(\bar{y}, \bar{z}) - N(\bar{x}, \bar{y}) - N(\bar{x}, \bar{z}) N(\bar{y}, \bar{z}) \right]$$

$$\langle N(\bar{x}, \bar{z}) N(\bar{y}, \bar{z}) \rangle = \langle N(\bar{x}, \bar{z}) \rangle \langle N(\bar{y}, \bar{z}) \rangle + \mathcal{O}(1/N_c^2)$$

Despite its apparent simplicity, its analytical solutions are not known: Numerical methods.

Analytical approaches: Saddle-point BFKL + boundary(ies), Mueller et al.

Connection to reaction-diffusion processes in statistical physics
@ NUMERICAL SETUP:

- No impact parameter dependence: Translational invariant app.

\[
\mathcal{N}^{GBW}(Y, r) = 1 - \exp \left\{ -\frac{r^2 Q_s^2(Y)}{4} \right\}
\]

\[
\mathcal{N}^{MV}(Y, r) = 1 - \exp \left\{ -\frac{r^2 Q_s^2(Y)}{4} \ln \frac{1}{r\Lambda} \right\}
\]

\[
\text{NEXT-TO-LEADING LOG EFFECTS: Ansatz for the running of the coupling:}
\]

- Coordinate space: \( \alpha_s \rightarrow \alpha_s(\mathbf{r}) \), “parent dipole running”

- Momentum space: \( \alpha_s \rightarrow \alpha_s(\max\{k, q, k-q\}) \)
Numerical Solutions

\[ \phi(k) \quad \phi(k) \quad h(k) = k^2 \nabla_k^2 \phi(k) \]

**Fixed**
- BFKL
- BK

**Running**
- BFKL
- BK

$k$
The solutions of the evolution at large rapidity exhibit the property of geometric scaling:

\[ \phi(k, Y) \rightarrow \phi(\tau), \quad \tau = \frac{k}{Q_s(Y)} \]

However, the scaling functions are different for fixed and running coupling:

**Fix:** \[ \phi(\tau) \sim \tau^{-2(1-\lambda)} \ln(b\tau) \]

**Run:** \[ \phi(\tau) \sim \tau^{-2(1-\lambda)} \ln(b\tau) \]

\[ \lambda_{fix} \sim 0.37 \]

\[ \lambda_{run} \sim 0.15 \]

\[ \tau > 1 \]
- BFKL evolution clearly violates unitarity: \( N > 1 \)

- Running coupling effects considerably slow down the evolution w.r.t. the fixed coupling case (emission of small dipoles is suppressed)
• Geometric scaling: $\mathcal{N}(Y, r) \rightarrow \mathcal{N}(\tau = r Q_s(Y))$

- $Q_s^2(Y): \mathcal{N}(Y, 1/Q_s) = 0.5$

- Scaling fully realized at extremely large rapidities: $Y \sim 80$.

- Fixed and running coupling scaling solutions are different.

\begin{align*}
Q_{fix,s}^2(Y) & \sim \exp \{4.8\alpha_s Y\} \\
Q_{run,s}^2(Y) & \sim \exp \left\{\sqrt{Y} \right\} \sim \exp \{0.3Y\}
\end{align*}