QCD factorization for $\gamma^*\gamma^* \rightarrow \rho^0_L\rho^0_L$

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based on:

Diffractive production of two $\rho^0_L$ mesons in e+e- collisions

QCD factorizations in $\gamma^*\gamma^* \rightarrow \rho\rho$

BFKL resummation effects in $\gamma^*\gamma^* \rightarrow \rho\rho$

Double diffractive rho-production in $\gamma^*\gamma^*$ collisions
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1. Introduction: Exclusive processes at high energy QCD
   - Motivation
   - GDA and TDA for $\gamma^*\gamma^* \rightarrow \rho_L^0\rho_L^0$

2. Computation in the collinear factorization with DA at fixed $W^2$
   - Direct calculation
   - Interpretation in terms of QCD Factorization
     - GDA for transverse photon in the limit $\Lambda_{QCD} \ll W^2 \ll \text{Max}(Q_1^2, Q_2^2)$
     - TDA for longitudinal photon in the limit $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$)

3. Computation at large $W^2$: $k_T$ factorization approach
   - Motivation and aims
   - $k_T$ factorization
   - Non-forward Born order cross-section for $\gamma^*\gamma^* \rightarrow \gamma^*\gamma^*\rho_L^0\rho_L^0$
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   - Non-forward Born order cross-section for $e^+e^- \rightarrow e^+e^-\rho_L^0\rho_L^0$
     - Equivalent photon approximation
     - Kinematical cuts
     - ILC collider and LDC detector
     - Born result
     - LL BFKL enhancement
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Introduction: Exclusive processes at high energy QCD

Motivation

Since a decade, there have been much developpements in hard exclusive processes.

- form factors $\rightarrow$ Distribution Amplitudes
- DVCS $\rightarrow$ Generalized Parton Distributions,

... 

These tests are possible in **fixed target** experiments

- $e^\pm p$ HERA (HERMES), JLab, ...

as well as in **colliders**, mainly for fixed $s$

- $e^\pm p$ colliders: HERA (H1,ZEUS)
- $e^+e^-$ colliders: LEP, Belle, BaBar, BEPC

At the same time, the interest for phenomenological tests of hard Pomeron and related resummed approaches has become pretty wide:

- **inclusive** tests (total cross-section) and semi-inclusive tests (diffraction, forward jets, ...)
- **exclusive** tests (meson production)

These tests concern all type of collider experiments:

- $e^\pm p$ (HERA: H1, ZEUS)
- $p\bar{p}$ (TEVATRON: CDF, D0)
- $e^+e^-$ colliders (LEP, ILC)
We will focus on a specific exclusive process:

\[ \gamma^* \gamma^* \rightarrow \rho_0^0 \rho_0^0 \]

with both \( \gamma^* \) hard

It is a beautiful theoretical laboratory for investigating different dynamics (collinear, multiregge) and factorization properties of high energy QCD:

- at low energy (fixed \( s \)) it provides an (almost) full perturbative laboratory for extended GPDs: GDA and TDA
- at high energy (asymptotic \( s \)) it provides an (almost) full perturbative laboratory for BFKL and related resummed effects, at amplitude level.

The corresponding experimental process is

\[ e^+ e^- \rightarrow e^+ e^- \rho_0^0 \rho_0^0 \]

with double tagged outgoing leptons.
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GDA and TDA for $\gamma^*\gamma^* \rightarrow \rho^0_L\rho^0_L$: collinear factorization

Extensions from GPD

- **DIS**: inclusive process $\rightarrow$ forward amplitude ($t = 0$)

  \[
  \text{Structure Function} = \text{Coefficient Function} \otimes \text{Parton Distribution Function} \\
  \text{(hard)} \otimes \text{(soft)}
  \]

- **DVCS**: exclusive process $\rightarrow$ non forward amplitude ($-t \ll s = W^2$)

  \[
  \text{Amplitude} = \text{Coefficient Function} \otimes \text{Generalized Parton Distribution} \\
  \text{(hard)} \otimes \text{(soft)}
  \]
Extensions:

- **Meson production**: $\gamma$ replaced by $\rho, \pi, \cdots$

\[
\text{Amplitude} = \begin{array}{c}
\text{GPD} \\
\text{(soft)}
\end{array} \otimes \begin{array}{c}
\text{CF} \\
\text{(hard)}
\end{array} \otimes \begin{array}{c}
\text{Distribution Amplitude} \\
\text{(soft)}
\end{array}
\]

- **Crossed process**: $s \ll -t$

\[
\text{Amplitude} = \begin{array}{c}
\text{Coefficient Function} \\
\text{(hard)}
\end{array} \otimes \begin{array}{c}
\text{Generalized Distribution Amplitude} \\
\text{(soft)}
\end{array}
\]
starting from usual DVCS, one allows initial hadron $\neq$ final hadron example:

which can be further extended by replacing the outgoing $\gamma$ by any hadronic state

\[
\text{Amplitude} = \text{Transition Distribution Amplitude} \otimes \text{CF} \otimes \text{DA}
\]
Collinear factorization at $q\bar{q}\rho$ vertices

$Q^2_{1,2}$ : hard scales ⇒ collinear approximation at each $q\bar{q}\rho$ vertex

i.e. we neglect the transverse relative (anti-)quark momenta in the $\rho$ mesons:

$$\ell_1 \sim z_1 k_1 \quad \ell_2 \sim z_2 k_2$$

$$\bar{\ell}_1 \sim \bar{z}_1 k_1 \quad \bar{\ell}_2 \sim \bar{z}_2 k_2$$

We limit ourselves to longitudinally polarized mesons (to avoid potential end-point singularities due to higher twist contributions)

DA of the meson = matrix element of non local quarks fields correlator on the light cone

$$\langle 0 | \bar{q}(x) \gamma^\mu q(-x) | \rho_L(p) = \bar{q}q \rangle = f_\rho \ p^\mu \int_0^1 dz \ e^{i(2z-1)(px)} \phi(z)$$

with

$$\phi(z) = 6z(1-z) \left(1 + \sum_{n=1}^{\infty} a_{2n} C^{3/2}_{2n} (2z - 1) \right)$$

Note: $p_1, p_2$ are light-like Sudakov vectors along the meson momenta.
The moderate energy and the high energy factorizations

We will now consider two types of treatment for the hard part $M_H$

- at moderate $W^2 \gg \Lambda_{QCD}^2$, we perform the direct calculation.

We then show that it can be presented in a QCD factorized form involving

- either a GDA for $W^2 \ll \text{Max}(Q_1^2, Q_2^2)$

- or a TDA for $Q_1^2 \ll Q_2^2$ or $Q_1^2 \gg Q_2^2$

- at asymptotically large $W^2$, $k_T$ factorization involving impact factors
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The computation follows the line of the Brodsky, Lepage approach.
We consider the Born order, i.e. quark exchange.
We restrict ourselves to the forward case
We only consider longitudinally polarized mesons ⇒ leading twist

The amplitude can be expressed as the sum of two tensors:

\[ \mathcal{M} = T^{\mu \nu} \epsilon_\mu(q_1) \epsilon_\nu(q_2) \]

with

\[ T^{\mu \nu} = \frac{1}{2} g_T^{\mu \nu} T^\alpha \beta g_T \alpha \beta + \left( p_1^\mu + \frac{Q_1^2}{s} p_2^\mu \right) \left( p_2^\nu + \frac{Q_2^2}{s} p_1^\nu \right) \frac{4}{s^2} T^\alpha \beta p_2 \alpha p_1 \beta \]

\[ g_T^{\mu \nu} = g^{\mu \nu} - \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{p_1 \cdot p_2} \]
The photons polarization vectors read

\[ \epsilon_{\parallel}(q_1) = \frac{1}{Q_1} q_1 + \frac{2Q_1}{s} p_2 \quad \text{and} \quad \epsilon_{\parallel}(q_2) = \frac{1}{Q_2} q_2 + \frac{2Q_2}{s} p_1 . \]

- use QED gauge invariance
- remember that we only consider the forward kinematics

⇒ the number of diagrams reduces to 4

![Diagrams](image-url)
Longitudinally polarized photons

Result

\[
T_{\alpha \beta}^{\mu_2 \alpha} p_{\alpha}^{\mu_1} p_{\beta} = - \frac{s^2 f^2 C_F e^2 g^2 (Q_u^2 + Q_d^2)}{8N_c Q_1^2 Q_2^2} \int_0^1 dz_1 dz_2 \phi(z_1) \phi(z_2) \\
\times \left\{ \frac{(1 - Q_1^2/s)(1 - Q_2^2/s)}{(z_1 + \bar{z}_1 Q_2^2/s)(z_2 + \bar{z}_2 Q_1^2/s)} + \frac{(1 - Q_1^2/s)(1 - Q_2^2/s)}{z_1 + z_2 Q_2^2/s(z_2 + \bar{z}_2 Q_1^2/s)} + \frac{1}{z_2 \bar{z}_1} + \frac{1}{z_1 \bar{z}_2} \right\}
\]

with \( s = 2 p_1 \cdot p_2 \)

Note:

\( Q_1^2 \) and \( Q_2^2 \) are non-zero and DA vanishes at \( z_i = 0 \)

\( \Rightarrow \text{no end-point singularity in the } z_i \text{ integration} \)
Transversally polarized photons

Diagrams

In this case no simplification occurs. One needs to compute 12 diagrams.
Transversally polarized photons

Results

\[ T^{\alpha \beta} g_{\alpha \beta} = -\frac{e^2 (Q_u^2 + Q_d^2) g^2 C_F f_R^2}{4 N_c \hat{s}} \int_0^1 dz_1 dz_2 \phi(z_1) \phi(z_2) \]

\[ \times \left\{ 2 \left( 1 - \frac{Q_2^2}{s} \right) \left( 1 - \frac{Q_1^2}{s} \right) \left[ \frac{1}{(z_2 + \bar{z}_2 \frac{Q_1^2}{s})^2(z_1 + \bar{z}_1 \frac{Q_2^2}{s})^2} + \frac{1}{(\bar{z}_2 + z_2 \frac{Q_1^2}{s})^2(\bar{z}_1 + z_1 \frac{Q_2^2}{s})^2} \right] + \left( \frac{1}{\bar{z}_2 z_1} - \frac{1}{\bar{z}_1 z_2} \right) \left[ \frac{1}{1 - \frac{Q_2^2}{s}} \left( \frac{1}{\bar{z}_2 + z_2 \frac{Q_1^2}{s}} - \frac{1}{z_2 + \bar{z}_2 \frac{Q_1^2}{s}} \right) - \frac{1}{1 - \frac{Q_1^2}{s}} \left( \frac{1}{\bar{z}_1 + z_1 \frac{Q_2^2}{s}} - \frac{1}{z_1 + \bar{z}_1 \frac{Q_2^2}{s}} \right) \right] \right\} \]

Same remark:

\( Q_1^2 \) and \( Q_2^2 \) are non-zero and DA vanishes at \( z_i = 0 \)

\[ \Rightarrow \text{no end-point singularity in the } z_i \text{ integration} \]
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Interpretation in terms of QCD Factorization

GDA for transverse photon in the limit $\lambda_{QCD} \ll W^2 \ll \text{Max}(Q_1^2, Q_2^2)$

When $W^2$ is smaller then the highest photon virtuality

For example $\frac{W^2}{Q_1^2} = \frac{s}{Q_1^2} \left( 1 - \frac{Q_1^2}{s} \right) \left( 1 - \frac{Q_2^2}{s} \right) \approx 1 - \frac{Q_1^2}{s} \ll 1$

the result obtained from direct calculation simplifies into

$$T^{\alpha \beta} g_{T^{\alpha \beta}} \approx \frac{e^2 (Q_u^2 + Q_d^2) g^2 C_F f_{\rho}^2}{4 N_c W^2}$$

$$\times \int_0^1 dz_1 \, dz_2 \left( \frac{1}{z_1 + z_1 \frac{Q_2^2}{s}} - \frac{1}{z_1 + \bar{z}_1 \frac{Q_2^2}{s}} \right) \left( \frac{1}{\bar{z}_2 z_1} - \frac{1}{\bar{z}_1 z_2} \right) \phi(z_1) \phi(z_2)$$

which can be interpreted as $(p \sim p_1, n \sim p_2)$

![Diagram](image-url)
Interpretation in terms of QCD Factorization

GDA for transverse photon in the limit $\Lambda_{QCD} \ll W^2 \ll \text{Max}(Q_1^2, Q_2^2)$: PROOF

**GDA computation**

At leading twist, the GDA is calculated in the Born order of perturbation theory

$$
\langle \rho^0_L(k_1) \rho^0_L(k_2) | \bar{q}(\alpha n/2) \text{exp} \left[ ig \int dy n_y A^y(y) \right] q(\alpha n/2) | 0 \rangle
$$

$$
= \int_0^1 dz e^{-i(2z-1)\alpha(np)/2} \Phi \rho^0_L \rho^0_L(z, \zeta, W^2)
$$

($P \sim p_1$ and $n \sim p_2$ for $Q_1 > Q_2$)

Since $W^2$ is hard, the GDA can be factorized:

$$
\text{GDA} = \text{GDA}_n + \text{DA}
$$

In our kinematics, the QCD Wilson line vanishes:
Interpretation in terms of QCD Factorization

GDA for transverse photon in the limit $W^2 \ll \text{Max}(Q_1^2, Q_2^2)$: PROOF

**Hard Part** computation at Born order

\[
T_H(q_1, q_2) = \left( \frac{1}{z + \bar{z}} + \frac{1}{z + \bar{z}} \right) \left( 4 e^2 N_c Q^2_q \right)
\]

In the case of one flavored quark, it equals:

\[
T_H(z) = -4 e^2 N_c Q^2_q \left( \frac{1}{z + \bar{z} \frac{Q^2_q}{s}} - \frac{1}{z + \bar{z} \frac{Q^2_q}{s}} \right)
\]
Interpretation in terms of QCD Factorization

GDA for transverse photon in the limit $W^2 \ll \text{Max}(Q_1^2, Q_2^2)$: SUMMARY

We have thus shown that $T^{\alpha \beta} g_{T \alpha \beta}$ factorizes into Hard part $\otimes$ GDA:

$$T^{\alpha \beta} g_{T \alpha \beta} = \frac{e^2}{2} \left( Q_u^2 + Q_d^2 \right) \int_0^1 \frac{dz}{z} \left( \frac{1}{\tilde{z} + z \frac{Q_u^2}{s}} - \frac{1}{z + \tilde{z} \frac{Q_d^2}{s}} \right) \Phi^{PLPL}(z, \zeta \approx 1, W^2)$$

with the GDA which itself factorizes into Hard part $\otimes$ DA DA:

$$\Phi^{PLPL}(z, \zeta \approx 1, W^2) = -\frac{f_{PL}^2 g^2 C_F}{2 N_c W^2} \int_0^1 dz_2 \phi(z) \phi(z_2) \left[ \frac{1}{z_2 \bar{z}_2} - \frac{1}{\bar{z}_2 z_2} \right]$$

- This is a limiting case of the original equation obtained by D. Müller et al (2000)
- It extends the studies of $\gamma^* \gamma^* \rightarrow \pi \pi$ by M. Diehl et al (2000)
- We limited ourselves to the case of $t = t_{\text{min}}$
Interpretation in terms of QCD Factorization

TDA for longitudinal photon in the limit $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$)

The direct calculation of the amplitude $M = T_{\alpha \beta}^\alpha p_2 \alpha p_1 \beta$ can be interpreted, in the limiting case $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$), as

$$M = TDA \otimes CF \otimes DA$$

TDA kinematics = GPD kinematics

$$n_1 = (1 + \xi)p_1 \text{ and } n_2 = \frac{p_2}{1 + \xi}$$

$x, \xi$ are momentum fraction along $n_2 = \frac{p_2}{1 + \xi}$

More precisely, we prove that $M$ factorizes as

$$T_{\alpha \beta}^\alpha p_2 \alpha p_1 \beta$$

$$= -i f_{\rho} e^2 (Q_u^2 + Q_d^2) g_s^2 \frac{C_F}{8 N_c} \int_{-1}^{1} dx \int_{0}^{1} dz_1 \left[ \frac{1}{z_1 (x - \xi)} + \frac{1}{z_1 (x + \xi)} \right] \phi(z_1)$$

$$\times N_c \left[ \Theta(1 \geq x \geq \xi) \phi\left(\frac{x - \xi}{1 - \xi}\right) - \Theta(-\xi \geq x \geq -1) \phi\left(\frac{1 + x}{1 - \xi}\right) \right]$$
Interpretation in terms of QCD Factorization

TDA for longitudinal photon in the limit $Q_2^2 \gg Q_1^2$ (or $Q_1^2 \ll Q_2^2$): PROOF

**TDA computation at Born order**

The TDA $\gamma^* \to \rho_L^0$ is defined through $(n \sim n_1)$

$$
\int \frac{dz^-}{2\pi} e^{ix(P,z)} \langle \rho_L^q(k_2) | \bar{q}(-z/2) | \gamma^* (q_2) \rangle
$$

$$
= e^{Qf} \frac{2}{P^+} \frac{Q^2}{Q_2^2} \epsilon_{\nu}(q_2) \left( (1 + \xi)n_2^\nu + \frac{Q_2^2}{s(1+\xi)n_1^\nu} \right) T(x, \xi, t_{min}),
$$

where the QED Wilson line is explicitly indicated (QCD Wilson line gives no contribution)

Since $Q_2^2$ is hard, the TDA can be factorized:

Explicit computation gives

$$
T(x, \xi, t_{min}) \equiv N_c \left[ \Theta(1 \geq x \geq \xi) \phi \left( \frac{x - \xi}{1 - \xi} \right) - \Theta(-\xi \geq x \geq -1) \phi \left( \frac{1 + x}{1 - \xi} \right) \right]
$$
Interpretation in terms of QCD Factorization

TDA for longitudinal photon in the limit \( Q_1^2 \gg Q_2^2 \) (or \( Q_1^2 \ll Q_2^2 \)): PROOF

**Hard computation at Born order**

\[
T_H(z_1, x) = -if_\rho g^2 e Q_q \frac{C_F}{2 N_c} \frac{\phi(z_1)}{Q_1^2} \epsilon^\mu(q_1) \left( 2 \xi n_{2 \mu} + \frac{1}{1 + \xi n_{1 \mu}} \right) \\
\times \left[ \frac{1}{z_1(x + \xi - i\epsilon)} + \frac{1}{\bar{z}_1(x - \xi + i\epsilon)} \right],
\]
Interpretation in terms of QCD Factorization

TDA for longitudinal photon in the limit $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$):

**SUMMARY**

We have shown, at Born order, that $T^{\alpha \beta} p_2 \alpha p_1 \beta$ factorizes into TDA $\otimes$ Hard part $\otimes$ DA:

$$T^{\alpha \beta} p_2 \alpha p_1 \beta = -i f_\rho^2 e^2 (Q_u^2 + Q_d^2) g^2 \frac{C_F}{8N_c} \int_{-1}^{1} dx \int_{0}^{1} dz_1 T(x, \xi, t_{min}) \left[ \frac{1}{z_1 (x - \xi)} + \frac{1}{z_1 (x + \xi)} \right] \phi(z_1)$$

with the TDA which itself factorizes into Hard part $\otimes$ DA:

$$T(x, \xi, t_{min}) \equiv N_c \left[ \Theta(1 \geq x \geq \xi) \phi \left( \frac{x - \xi}{1 - \xi} \right) - \Theta(-\xi \geq x \geq -1) \phi \left( \frac{1 + x}{1 - \xi} \right) \right]$$

Note:

Only the DGLAP part of the TDA contributes because of support properties of the $\rho$ meson DA

\[
\text{DGLAP(1)} \quad -1 \leq x \leq -\xi \\
\text{ERBL} \quad -\xi \leq x \leq \xi \\
\text{DGLAP(2)} \quad \xi \leq x \leq 1
\]
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Computation at large $W^2 : k_T$ factorization approach

Theoretical motivations

QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in $t$ channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominates with respect to Born order at large relative rapidity.
Computation at large $W^2 : k_T$ factorization approach

Theoretical motivations

\[ e^+ e^- \rightarrow e^+ e^- \rho_L^0 \rho_L^0 \] is a good observable in order to test this limit:

- **IR-safe probes:** double tagging of the final leptons and cut-off over soft photons \( \Rightarrow \) the hard virtual photons give the hard scales on both sides of the \( t \)-channel exchanged state \( \Rightarrow \) fully perturbative process (except for DAs of \( \rho \)).

- Observable dominated by the "soft" (but still perturbative) dynamics of QCD (BFKL and extensions) and not by its collinear dynamics (DGLAP, ERBL):
  - we impose \( Q_1^2 \sim Q_2^2 \)
  - gives access to the interplay between collinear and soft dynamics by getting away from \( Q_1^2 \sim Q_2^2 \) domain and by playing with the relative rapidity
  - one can control the spread in \( k_T \) of the partons: transition from linear to non-linear (saturated regime), when increasing \( s_{\gamma*\gamma*} \) for given values \( Q_1^2 \) and \( Q_2^2 \).
  - Experimentally feasible by increasing \( s_{e^+ e^-} \)
  - it gives access to non-forward dynamics
    - can reveal Pomeron structure apart from the forward limit
    - for saturation studies, it is important to get a full impact parameter picture of the process (Froissart bound is for each impact parameter)
    - Note that for \( t = 0 \), the simplest model for non-linearity is the Balitskii Kovchegov equation

- Cross-section are expected to be peaked in the forward limit
  - \( \Rightarrow \) the total cross-section gives the general trends
Computation at large $W^2$ : $k_T$ factorization approach

**Aims**

- Compute the scattering amplitude for $\gamma^* \gamma^* \rightarrow \gamma^* \gamma^* \rho_0^L \rho_0^L$ with gluon exchange, in the range $s_{\gamma^* \gamma^*} \ll -t, Q_1^2, Q_2^2$ for every photons polarizations and check dominance with respect to quarks exchange at ILC energies.

- We focus on $Q_1^2 \sim Q_2^2 \Rightarrow$ no DGLAP evolution (this is practically imposed by the small range in both $Q_i^2$ due to the lower perturbative cut-off and by the fast decreasing amplitude as powers of $Q_i^2$).

- We prove the experimental feasability at ILC, with LDC detector project.

- Study linear and non linear dynamical effects, and the expected enhancement at large rapidity.
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- Motivation and aims
- $k_T$ factorization
  - Non-forward Born order cross-section for $\gamma^*\gamma^* \rightarrow \gamma^*\gamma^*\rho^0_L\rho^0_L$
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Computation at large $W^2$: $k_T$ factorization approach

$k_T$ factorization

- Use Sudakov decomposition $k = \alpha p_1 + \beta p_2 + k_\perp$
- write

$$d^4k = \frac{s}{2} d\alpha d\beta d^2k_\perp$$

and rearrange integrations in the large $s$ limit:

\[\Rightarrow \text{set } \alpha_k = 0 \text{ and } \int d\beta\]

\[\Rightarrow \text{set } \beta_k = 0 \text{ and } \int d\alpha\]
impact representation (written here for the whole process) \( \text{note: } \vec{k} = \text{Eucl.} \iff k_{\perp} = \text{Mink.} \)

\[
\mathcal{M} = is \int \frac{d^2 k}{(2\pi)^4 k^2 (r - k)^2} \mathcal{J}_{\gamma^*_\nu, T}(q_1) \rightarrow \rho^0_L(k_1)(k, r - k) \mathcal{J}_{\gamma^*_\nu, T}(q_2) \rightarrow \rho^0_L(k_2) (-k, -r + k)
\]

For longitudinally polarized photons the impact factor reads

\[
\mathcal{J}_{\gamma^*_\nu, L}(q_i) \rightarrow \rho_L(k_i)(k, r - k) = 8\pi^2 \alpha_s \frac{e}{\sqrt{2}} \delta^{ab} f_\rho \alpha(k_i) \int_0^1 dz_i z_i \bar{z}_i \phi(z_i) \mathcal{P}(z, k, r, \mu_i)
\]

where

\[
\mathcal{P}(z, k, r, \mu_i) = \frac{1}{z_i r^2 + \mu_i^2} + \frac{1}{\bar{z}_i r^2 + \mu_i^2} - \frac{1}{(z_i r - k)^2 + \mu_i^2} - \frac{1}{(\bar{z}_i r - k)^2 + \mu_i^2} \propto \mathcal{J}_{\gamma^*_\nu, L}(q_i) \rightarrow q \bar{q}
\]

For transversally polarized photons, one obtains

\[
\mathcal{J}_{\gamma^*_\tau, T}(q_i) \rightarrow \rho_L(k_i)(k, r - k) = 4\pi^2 \alpha_s \frac{e}{\sqrt{2}} \delta^{ab} f_\rho \alpha(k_i) \int_0^1 dz_i (z_i - \bar{z}_i) \phi(z_i) \varepsilon \cdot Q(z_i, k, r, \mu_i)
\]

where

\[
Q(z_i, k, r, \mu_i) = \frac{z_i r}{z_i^2 r^2 + \mu_i^2} - \frac{\bar{z}_i r}{\bar{z}_i^2 r^2 + \mu_i^2} + \frac{k - z_i r}{(z_i r - k)^2 + \mu_i^2} - \frac{k - \bar{z}_i r}{(\bar{z}_i r - k)^2 + \mu_i^2} \propto \mathcal{J}_{\gamma^*_\tau, T}(q_i) \rightarrow q \bar{q}
\]

we denote \( \mu_i^2 = Q_i^2 z_i \bar{z}_i + m^2 \), where \( m \) is the quark mass (set to zero in practice)

due to QCD gauge invariance (probes are colorless), both impact factor vanishes when \( k \rightarrow 0 \) or \( r - k \rightarrow 0 \)
Outline

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Non-forward Born order cross-section for $\gamma^* \gamma^* \rightarrow \gamma^* \gamma^* \rho_L^0 \rho_L^0$

Analytical dimensional integration through conformal transformations: principle

- All the 2-d integrations with respect to $k$ are treated analytically.
- The method relies on conformal transformation in the transverse momentum plane (method inspired by Vassiliev in 2-d coordinate space).
- The idea is to reduce the number of propagators, in order to be able to perform standard Feynman parameter integration.
- The whole computation involves integrals with up to 4 propagators (2 massive, with different masses) which we would have been enable to compute without this method.
The integral \((\bar{a} \equiv 1 - a)\)

\[
J_{3\mu}(a) = \int \frac{d^2k}{k^2(k-r)^2} \left[ \frac{1}{(k-ra)^2 + \mu^2} - \frac{1}{a^2r^2 + \mu^2} + (a \leftrightarrow \bar{a}) \right]
\]

has 3 propagators (1 massive)

- perform the inversion on integration variable and parameters:
  \[
k \rightarrow \frac{K}{K^2}, \quad r \rightarrow \frac{R}{R^2}, \quad m \rightarrow \frac{1}{M}
\]

- perform a shift of variable: \(K = R + k'\)
- perform another inversion
- one then obtains an integral with 2 propagators (1 massive)

\[
J_{3m} = \frac{1}{r^2} \int \frac{d^2k}{k^2} \left[ \frac{(r+k)^2}{(r^2a^2 + m^2)\left(k - r\frac{r^2a^2+2m}{r^2a^2+m^2}\right)^2 + m^2r^4}\right] - \frac{1}{a^2r^2 + m^2} + (a \leftrightarrow \bar{a})
\]

which is easily computed.
Non-forward Born order cross-section for $\gamma^*\gamma^* \rightarrow \gamma^*\gamma^* \rho_L^0 \rho_L^0$

Results

- The integration over momentum fractions $z_1$ and $z_2$ are performed numerically
- we use $Q_1 Q_2$ as a scale for $\alpha_S$ (3 loops)

Differential cross-sections for $\gamma_i^* \gamma_j^* \rightarrow \gamma_i^* \gamma_j^* \rho_L^0 \rho_L^0$
Non-forward Born order cross-section for $\gamma^*\gamma^* \rightarrow \gamma^*\gamma^* \rho_L^0 \rho_L^0$

Results

- The cross-sections strongly decrease with $Q^2$ (as $1/Q^4$ for LL).
- Any cross-section with at least one transverse photon vanishes at $t = 0$ (due to $s$-channel helicity conservation): remember that $\rho$ is transverse.
- At large $t$, $\gamma_T^* \gamma_T^*$ dominates (photon are then almost on-shell with respect to $t$).
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Equivalent photon approximation

$$\gamma^* \gamma^* \rightarrow \rho_L^0 \rho_L^0 \quad \rightarrow \quad e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0$$

using equivalent photon approximation

$$\frac{d\sigma(e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0)}{dy_1 dy_2 dQ_1^2 dQ_2^2} = \frac{1}{y_1 y_2 Q_1^2 Q_2^2} \left( \frac{\alpha}{\pi} \right)^2 \left[ l(y_1) l(y_2) \sigma(\gamma_L^* \gamma_L^* \rightarrow \rho_L^0 \rho_L^0) + t(y_1) l(y_2) \sigma(\gamma_T^* \gamma_L^* \rightarrow \rho_L^0 \rho_L^0) 
+ l(y_1) t(y_2) \sigma(\gamma_L^* \gamma_T^* \rightarrow \rho_L^0 \rho_L^0) + t(y_1) t(y_2) \sigma(\gamma_T^* \gamma_T^* \rightarrow \rho_L^0 \rho_L^0) \right]$$

with the usual flux factors given by

$$t(y_i) = \frac{1 + (1 - y_i)^2}{2}, \quad l(y_i) = 1 - y_i,$$

$y_i$ ($i = 1, 2$) are the longitudinal momentum fractions of the bremsstrahlung photons with respect to the incoming leptons

$s_{\gamma^* \gamma^*} \sim y_1 y_2 s_{e^+e^-}$

$$\Rightarrow \sigma^{e^+e^- \rightarrow e^+e^- \rho_L \rho_L} \text{ is peaked in the low } y \text{ and } Q^2 \text{ region}$$
Non-forward Born order cross-section for $e^+ e^- \rightarrow e^+ e^- \rho_0^0 \rho_0^0$

Kinematical cuts

- **photon momentum fractions:** (in the laboratory frame = center of mass system (cms) for an $e^+ e^-$ collider)

$$y_i = \frac{E - E'_i \cos^2(\theta_i/2)}{E}$$

- **virtualities:**

$$Q^2_i = 4EE'_i \sin^2(\theta_i/2)$$

- cross-section peaked at small $Q^2_i$ and $y_i$

$\Rightarrow$ one needs to get access to the (very) forward region

- **kinematical constraints:**
  - minimal detection angle (detector constraint)
  - conditions on the energies of outgoing leptons (detector constraint)
  - Regge condition

$$y_{i \text{ max}} = 1 - \frac{E_{\text{min}}}{E}$$

$$y_{1 \text{ min}} = \max \left(f(Q_1), 1 - \frac{E_{\text{max}}}{E}\right)$$

$$y_{2 \text{ min}} = \max \left(f(Q_2), 1 - \frac{E_{\text{max}}}{E}, \frac{c Q_1 Q_2}{s y_1}\right)$$

with $f(Q_i) = 1 - \frac{Q^2_i}{s \tan^2(\theta_{\text{min}}/2)}$
Non-forward Born order cross-section for $e^+e^- \rightarrow e^+e^- \rho^0_L \rho^0_L$

ILC collider and detectors

Reference Design Report for International Linear Collider

- $\sqrt{s_{e^+e^-}} = 2E_{\text{lepton}}$: nominal value of 500 GeV
- High luminosity, with 125 fb$^{-1}$ per year within 4 years of running at 500 GeV
- Possible scan in energy between 200 GeV and 500 GeV.
- Upgrade at 1 TeV, with a luminosity of 1 ab$^{-1}$ within 3 to 4 years
- Two interaction regions are highly desirable: one which could be at low crossing-angle, and one compatible with $e\gamma$ and $\gamma\gamma$ physics (through single or double laser Compton backscattering)
  - At the moment, 3 options are considered: 2 mrad, 14 mrad and 20 mrad

- Note that in $e\gamma$ and $\gamma\gamma$ modes, for which $\alpha_c > 25$ mrad, no BeamCal can be placed around the beampipe, at least at $\alpha < 12$ mrad (angular size of the disrupted outgoing beam after laser Compton backscattering)

- It thus means that if a single detector would be used at the same interaction point (in order to reduce the budget devoted to $\gamma\gamma$ mode, this solution without displacement of the detector has been suggested: Telnov), no forward calorimeter like BeamCal could be installed
Non-forward Born order cross-section for $e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0$

ILC collider and detectors

In the case of $e^+e^-$ mode

- Each design of detector for ILC project involves a very forward electromagnetic calorimeter for luminosity measurement, with tagging angle for outgoing leptons down to 5 mrad (design 10 years ago were considering 20 mrad as almost impossible!)
- This is an ideal tool for diffractive physics: cross-section are sharply peaked in the very forward region
- Luminosity is enough to give high statistics, even with exclusive events
- There are 4 concepts of detectors at the moment:
  - GLD
  - Large Detector Concept (LDC)
  - Silicon Design Detector Study (Sid)
  - 4th
Non-forward Born order cross-section for $e^+ e^- \rightarrow e^+ e^- \rho_L^0 \rho_L^0$

We focus specifically on the LDC project

- The BeamCal is an electromagnetic calorimeter devoted to luminosity measurement, located at 3.65 m from the vertex

- It can be used for diffractive physics
- The main background is due to beamstrahlung photons, which leads to energy deposit in cells close from the beampipe
  \[ \Rightarrow \text{in practice we cut-off the cells for lepton tagging with} \]
  \[ E_{\text{min}} = 100 \text{ GeV} \]
  \[ \theta_{\text{min}} = 4 \text{ mrad} \]

\[
\frac{d\sigma}{dt} \frac{e^+ e^- \rightarrow e^+ e^- \rho_L \rho_L}{dQ_1^2 \ dQ_2^2} = \int_{Q_{1\text{min}}}^{Q_{1\text{max}}} \ dQ_1^2 \int_{Q_{2\text{min}}}^{Q_{2\text{max}}} \ dQ_2^2 \int_\epsilon^{\gamma_{\text{max}}} \ dy_1 \int_\gamma_{\text{max}}^{\gamma_{\text{max}}} \ dy_2 \int_{Q_{1\text{min}} Q_{2\text{min}}}^{Q_{1\text{max}} Q_{2\text{max}}} \ \frac{d\sigma}{dt \ dy_1 \ dy_2 \ dQ_1^2 \ dQ_2^2}.\]
Non-forward Born order cross-section for $e^+ e^- \rightarrow e^+ e^- \rho_L^0 \rho_L^0$

We obtain, at $\sqrt{s_{e^+e^-}} = 500$ GeV (and $c = 1$)

$$
\begin{align*}
\sigma^{LL} &= 32.4 \text{ fb} \\
\sigma^{LT} &= 1.5 \text{ fb} \\
\sigma^{TT} &= 0.2 \text{ fb} \\
\sigma^{tot} &= 34.1 \text{ fb}
\end{align*}
$$

which leads to $4.3 \times 10^3$ events per year with foreseen luminosity
Non-forward Born order cross-section for \( e^+ e^- \rightarrow e^+ e^- \rho^0_L \rho^0_L \)

Born results

- The background (dominated by \( \gamma \) which would be misidentified in BeamCal as \( e^+ \) or \( e^- \)) is completely negligible at \( \sqrt{s_{e^+e^-}} = 500 \) GeV.
- Quarks contribution are indeed negligible. This is related to \( c \) through \( s_{\gamma^*\gamma^*} > c Q_1 Q_2 \).
- More drastic Regge constraint by performing \( c = 1 \rightarrow c = 10 \) reduces the cross-section by 40% \( \Rightarrow \) still statistically measurable.
- Changing order of loop for \( \alpha_S \) only has a few % effect.

```
\[
\frac{d\sigma_{\text{min}}}{dt} (fb/GeV^2)
\]
```

- Red curve: \( c = 1 \)
- Green curve: \( c = 2 \)
- Yellow curve: \( c = 3 \)

From up to down:
- Gluon exchange
- Quark-exchange with \( \gamma_L^* \)
- Quark-exchange with \( \gamma_T^* \)
Enhancement is enormous but it is well known that NLL BFKL is below LL BFKL and almost always above Born. This latter issue is true except at small rapidity in certain peculiar scale fixing scheme: see Ivanov Papa.

At the level of $\gamma^* \gamma^*$ this has been studied earlier

- resummed BFKL à la Khoze, Martin, Ryskin, Stirling: Enberg, Pire, Szymanowski, S.W with LL impact factor and BLM scale fixing
- NLL BFKL with NLL impact factor: Ivanov Papa.
- Work to implement this at $e^+ e^-$ level is in progress.

due to detector constraint, the expected increase of the cross-section with $\sqrt{s_{e^+e^-}}$ is washed-out for $\sqrt{s_{e^+e^-}} > 500$ GeV: flattish curve to be compared with sharked curve at Born level.
\( \gamma^* \gamma^* \rightarrow \rho_0^0 \rho_0^0 \) is a very nice process for studying QCD dynamics in its perturbative regime, with a minimal onset of non-perturbative physics.

At low energy, it is dominated by quark exchange:

- Its perturbative analysis in the Born approximation, in the forward case, leads to two different types of QCD factorization.
- We have shown that the polarization states of the photons dictate either the factorization involving a GDA or involving a TDA.
- Usually these two types of factorizations are applied to two different kinematical regimes.
- The arbitrariness in choosing values of photon virtualities \( Q_i^2 \) shows that there may exist an intersection region where both types of factorization are simultaneously valid.
- The obtained TDA contains a perturbative part which could give a hint for modelling in non-perturbative cases.

Further generalizations:

- non-forward kinematics (rather easy)
- transverse photon (hard: higher twist contributes)
- charged meson pair (hard: non-trivial QED gauge invariance)

The measure could be done at Babar, Belle, BEPC-II,...,ILC.
At high energy, it is dominated by gluon exchange

- we gave a precise estimation of the Born order cross-section for arbitrary photon polarization
- we have demonstrated the feasibility of the measurement at the level of $e^+e^- \rightarrow e^+e^- \rho_0^0 \rho_0^0$ with double tagged outgoing leptons, within ILC collider and LDC detector with a forward electromagnetic calorimeter
- this evaluation can be considered as the background for any resummation à la BFKL
- we have made a first estimate of BFKL evolution at LL.
- our previous estimate of resummed BFKL evolution for $\gamma^* \gamma^* \rightarrow \rho_0^0 \rho_0^0$ should now be implemented at $e^+e^- \rightarrow e^+e^- \rho_0^0 \rho_0^0$ level
- there is a potential very interesting possibility of entering smoothly into the non-linear saturation regime when the machine would be upgraded up to 1 TeV:
  - at $\sqrt{s_{e^+e^-}} = 500$ GeV, $Q_{sat} \sim 1.1$ GeV, saturation is at the border, almost negligible
  - at $\sqrt{s_{e^+e^-}} = 1$ TeV, $Q_{sat} \sim 1.4$ GeV, saturation effects should start to be rather important (but still in the almost linear regime)