Differential Cross Sections at NNLO

Alexander Mitov

DESY, Zeuthen

Alexander von Humboldt Fellow

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Quantum Chromodynamics: String Theory meets Collider Physics

In Short:

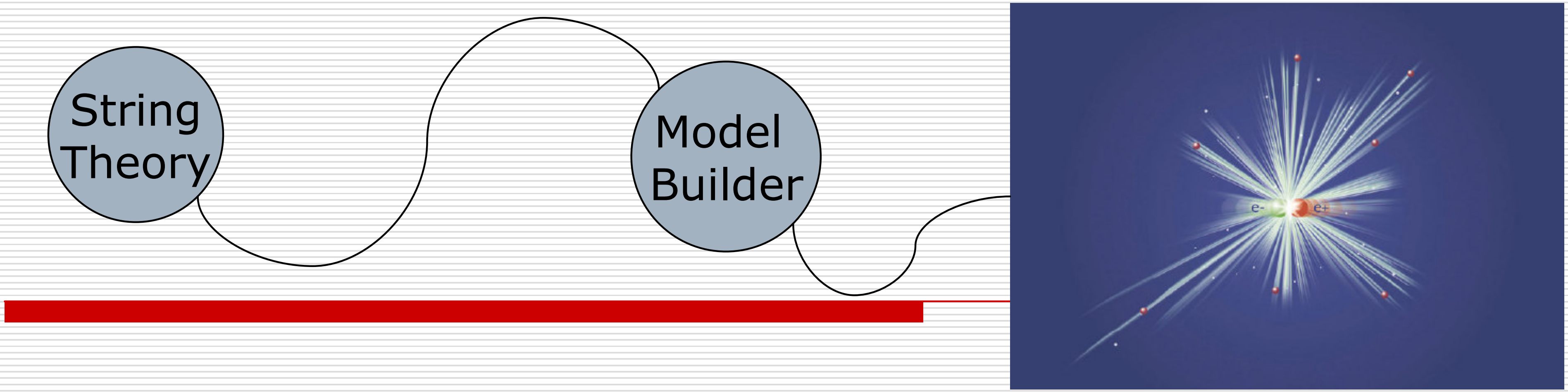
The subject is:

- ✓ technical ☺
- ✓ very relevant for the experiment,
- ✓ has seen a lot of progress in the last years,
- ✓ much more work remains to be done!

I'll introduce all of these on an example …

… from the viewpoint of a model builder.

Good model = ? Eureka!



What are the next steps?

- ✓ Try to connect to string theory
- ✓ Verify consistency with experiment

It is the second part I'll be concerned with in this talk

How can a two-loop spectrum be relevant here?

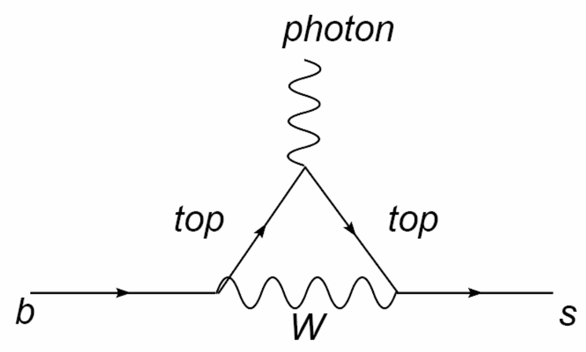
Here is an example:

- One thing one wants to make sure is that FCNC are under control.
- In the SM they are loop-induced and thus small.

A prime example: B → Xs + γ

- The decay of B-mesons to photons and “strange” particles

- ✓ High-quality data (Belle, BaBar)
- ✓ Very sensitive to new physics that might be running inside the loop
- ✓ Two-loop theoretical precision is mandatory



Completed just recently after ~ 10 years of hard work by ~ 30 people !

$B \to X_s + \gamma$

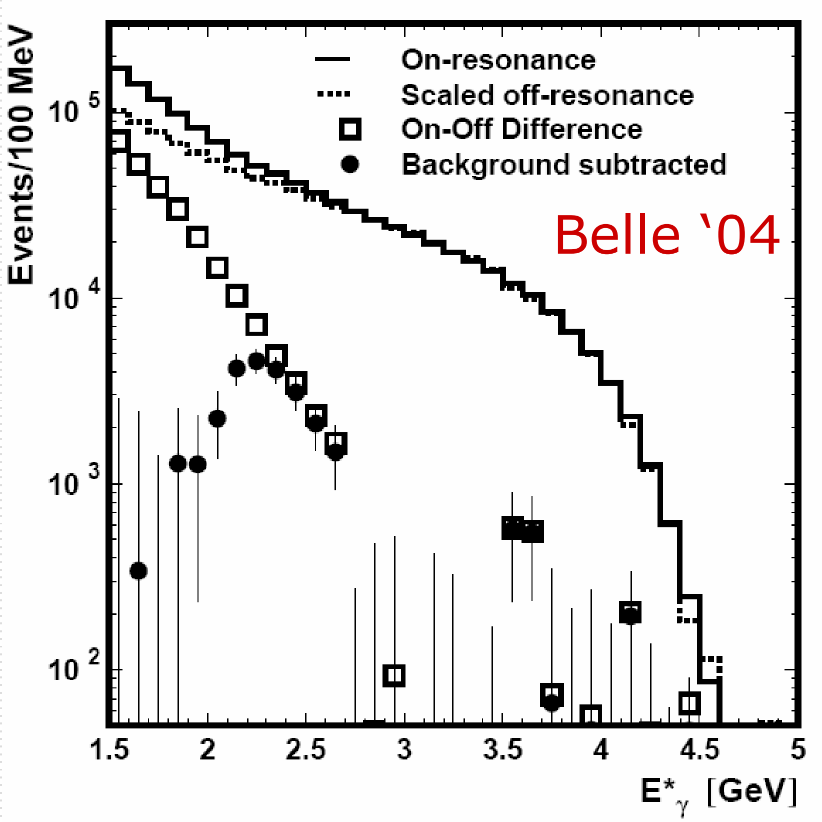
❖ This is a simple counting experiment:

The total decay rate is reconstructed from the "integrated" spectrum.

❖ Let us look at the fine print:

The emitted photons can have energy in the interval 0 ~2.5 GeV (follows from simple kinematics)

❖ However, experimentally photons with energy less than 1.8GeV cannot be seen (effectively do not exist)



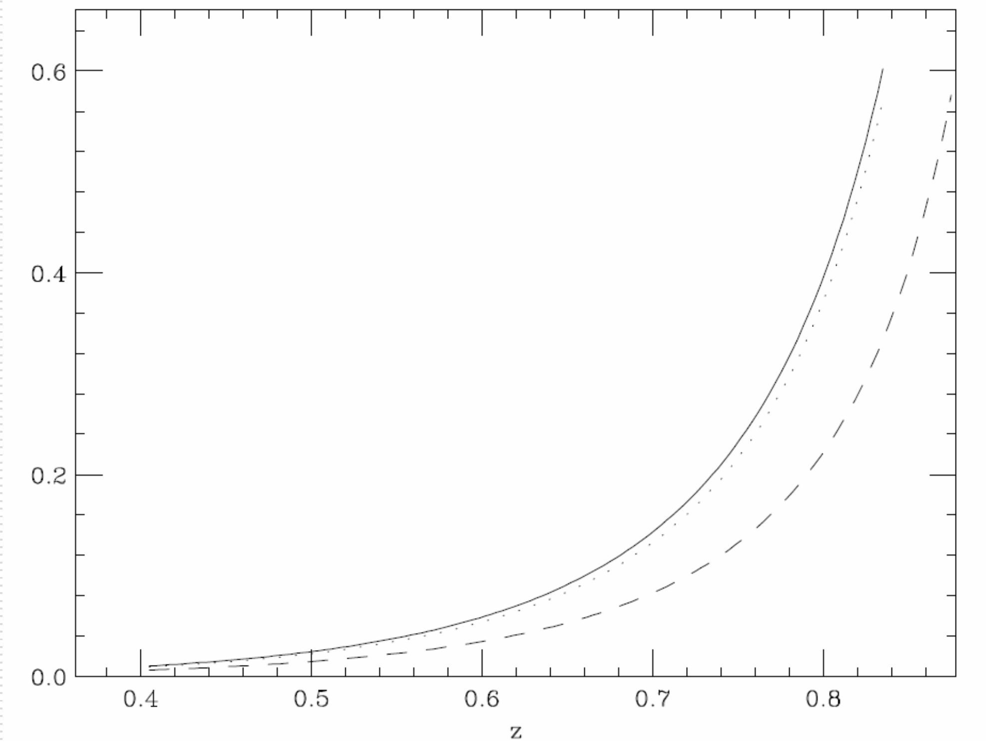
$B \to X_s + \gamma$

- To account for the unobserved low energy photons one has to know the shape of the photon spectrum with two-loop (NNLO) precision

A.M. , Melnikov '05;
Asatrian, Ewerth, Ferroglia, Gambino, Greub '06
Blokland, Czarnecki, Misiak, Slusarczyk, Tkachov '05

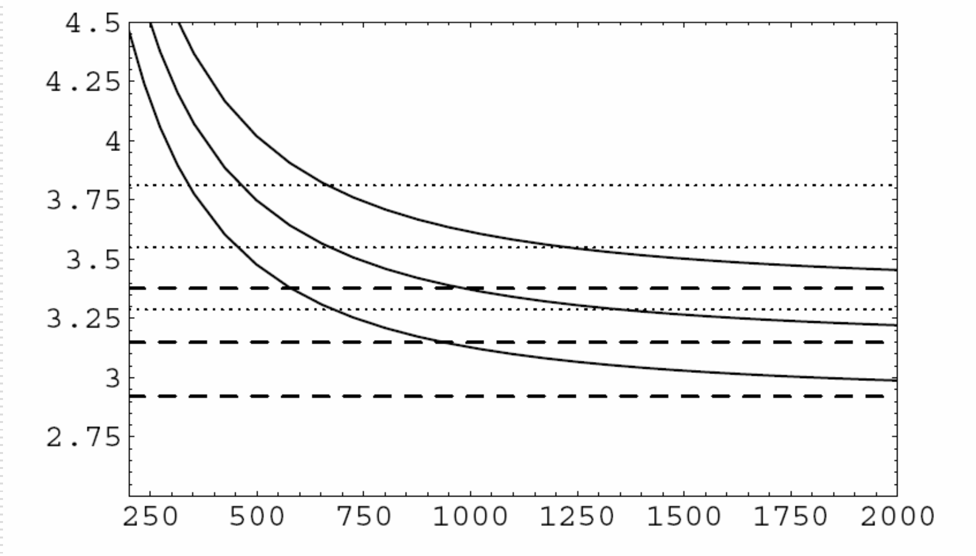
NNLO corrections important to reduce the uncertainty.

A.M., Melnikov '05



The first complete NNLO SM prediction for this process:

M. Misiak et al. '06



Generally, NNLO differential distributions deliver:

- ✓ More adequate match of the experimental setup (w/r to inclusive ones),
- ✓ Improved precision in the theoretical prediction,
- ✓ Quantitative estimate of the theoretical error.

Three basic approaches for evaluation of distributions:

- ❖ Fixed order calculations:
  - ❖ Fully analytic,
  - ❖ Purely numerical,
- ❖ All order resummations.

Each one has its pluses and minuses and range of applicability.

Analytical evaluation of differential distributions at NNLO

- ✓ Appropriate for not-very-differential distributions,
- ✓ Appropriate for singular parton level distributions (in presence of kinematic end-point singularities),
- ✓ Supplies analytical results that can be used elsewhere.

Methods

Evaluation of phase-space integrals with (possibly) additional constraints.

Very tedious! Requires more work than a typical loop integral.

Several single-scale NNLO results were calculated this way:

Drell-Yan, e+e-,Higgs

Hamberg, van Neerven, Matsuura `91;
Rijken, van Neerven `96;
Ravindran, Smith, van Neerven, `03

Analytical evaluation (cont.)

Important analytic result: the Higgs cross-section at NNLO (single scale)

Harlander, Kilgore '01

The breakthrough after the work on Higgs by Anastasiou, Melnikov '02

They realized one can apply IBP identities to integrals involving not only propagators but also delta-functions:

$$\delta(k^2) \sim \frac{1}{k^2 - i\varepsilon} - \frac{1}{k^2 + i\varepsilon}$$

What are IBP identities?

$$\int d^d k \frac{d}{dk^\mu} \left(\frac{p^\mu}{[(p_1-k)^2]^{n_1} \ldots [(p_N-k)^2]^{n_N}} \right) = 0$$

Analytical evaluation (cont.)

IBP identities lead to algebraic relations between the following integrals:

$$I(n_1,\ldots,n_N) \;=\; \int d^dk \frac{1}{[(p_1-k)^2]^{n_1}\ldots[(p_N-k)^2]^{n_N}}$$

It turns out these span a finite dimensional space

Therefore, each integral can be mapped to a finite basis of Masters:

$$I(n_1,\ldots,n_N) = \sum_{\alpha} C_{\alpha}(n_1,\ldots,n_N)\, M_{\alpha}$$

For n_1 , … , n_N fixed positive or negative integers the coefficients C_α are just rational functions.

The most practical method for IBP solving: Gauss elimination Laporta '00

Other methods proposed by Baikov, Tarasov, Smirnov

Analytical evaluation (cont.)

✓ Application of IBP's allow much more efficient evaluation.

Processes calculated with this method:

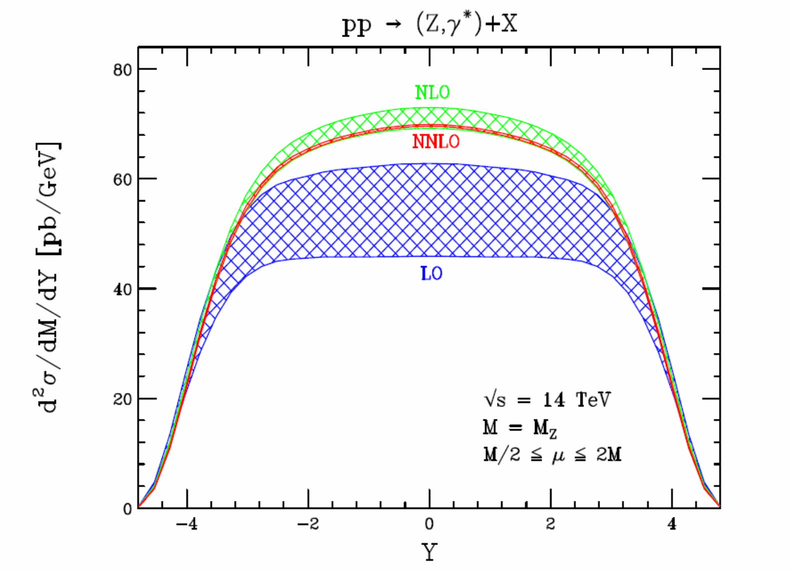
❖ Higgs and pseudo-scalar Higgs cross-sections

Anastasiou, Melnikov '02

❖ (Drell-Yan) vector boson rapidity distribution

Anastasiou, Dixon, Melnikov, Petrielo '03

hep-ph/0312266



Note:

✓ reduced scale dependence at NNLO

✓ significant analytical complexity with 2 variables

Analytical evaluation (cont.)

- Perturbative Fragmentation Function of a heavy quark at NNLO

  Melnikov, A. M.; A. M. '04

Allows computations of spectra of massive particless from massless calculations and resummation of ln(m)-terms to all orders in the coupling

Mele, Nason '91

From x to N space: calculation in Mellin space

A.M. '05

N-space calculations applied previously to DIS; different in nature.

- ✓ Very efficient method
- ✓ No complications from endpoint singularities
- ✓ Difference instead of differential equations

- Application: e^+e^- massless coefficient functions at NNLO

  Moch, A.M. '06

Complete evaluation required only 7 integrals to be evaluated by hand.

Analytical evaluation (cont.)

Massive particles (much less studied so far):

- Energy spectrum of heavy quarks (mesons) in e^+e^- at NNLO

In progress: Cacciari, A.M., Moch, Vogt

Obtained in the small mass limit as convolution of:

$$\frac{d\sigma_Q}{dz}(z,Q,m) = \sum_a \int_z^1 \frac{dx}{x}\frac{d\hat{\sigma}_a}{dx}(x,Q,\mu) D_{a/Q}\left(\frac{z}{x},\frac{\mu}{m}\right)$$

- Small angle $e^+e^- \to e^+e^-$ Bhabha scattering in NNLO QED

Important process: needed for precise luminosity monitoring at the ILC.

Special process: small mass limit; only soft radiation allowed.

Therefore: the bottleneck are the two-loop (virtual) corrections. Predicted!

Penin '05	Effective theory methods,
Becher, Melnikov '07	Factorization method $M^{(m)} = Z \cdot M^{(0)}$

Truly complicated problem; direct calculation not yet available! Moch, A.M. '06

The massless NNLO result known for some time now Bern, Dixon, Ghinculov `00

Numerical evaluation of differential distributions at NNLO

- ✓ Appropriate for completely differential distributions,
- ✓ Invaluable for implementing arbitrary cuts that exactly reflect the experimental setup.

Methods

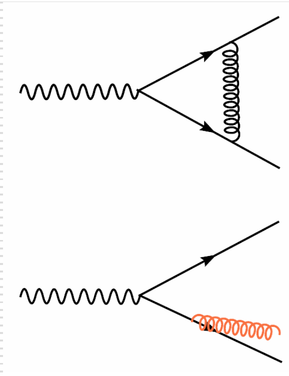
Essentially two approaches available and worked out:

- ❖ Subtraction method,
- ❖ Sector decomposition method.

Numerical evaluation at NNLO: subtraction method

An observable represents a sum of

- ✓ virtual corrections
  (typically evaluated exactly)
- ✓ unresolved real emissions
  (typically evaluated numerically)



$$\mathrm{d}\sigma_{NNLO} = \int_{\mathrm{d}\Phi_5} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) + \int_{\mathrm{d}\Phi_4} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) + \int_{\mathrm{d}\Phi_5} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_4} \mathrm{d}\sigma^{VS,1}_{NNLO} + \int_{\mathrm{d}\Phi_3} \mathrm{d}\sigma^{V,2}_{NNLO}$$

Idea: add and subtract something which is computable and has the same singularities as the exact result. Each line is numerically integrable.

0707.1285[hep-ph]

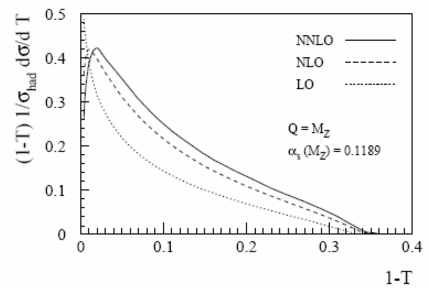


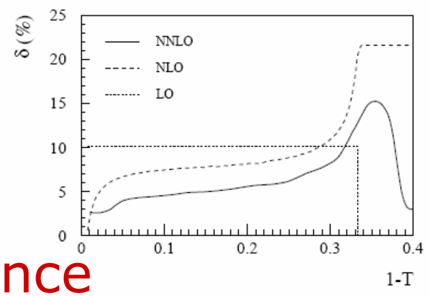
FIG. 1: Thrust distribution at $Q = M_Z$.

Specific method used here:
antenna subtraction at NNLO

Gehrmann-De Ridder, Gehrmann, Glover `05

First application:
thrust distribution in $e^+e^- \to 3$ jets at NNLO:
most accurate extraction of α_S

Gehrmann-De Ridder, Gehrmann, Glover, Heinrich `07



Perturbative convergence and reduction of scale dependence

Numerical evaluation at NNLO: sector decomposition

Old and well known method.

Turns out it is suitable for evaluation of integrals

Binoth, Heinrih `00
Anastasiou, Melnikov, Petrielo `04

Idea of the method:

Step 1: $$\int d^d k \frac{1}{(p-k)^2 \dots} = \text{Feynman parametrization} = \int_0^1 [dx_1 \dots dx_n] \frac{P^a(\varepsilon, x)}{Q^b(\varepsilon, x)}$$

Step 2: start a long but totally algorithmic procedure to re-map [x]→ [x'].
The RHS becomes a sum of integrals over the unit hypercube which contain no singularities!

Step 3: expand in ε to get:

$$\int d^d k \frac{1}{(p-k)^2 \dots} = \sum_n \varepsilon^n \int d\,[\text{unit hypercube}(x)] \times f_n(x)$$

All ε dependence is now explicit! The functions $f_n(x)$ – integrable!

Numerical evaluation at NNLO: sector decomposition

Example: hep-ph/0311311

Start with the integral:

$$I = \int_0^1 \mathrm{d}x\,\mathrm{d}y\,x^{-1-\epsilon}y^{-1-\epsilon}\,(x+y)^{-\epsilon}$$

Cannot be expanded in ε

Split into two:

$$I_1 = \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y\,x^{-1-\epsilon}y^{-1-\epsilon}\,(x+y)^{-\epsilon}$$

$$I_2 = \int_0^1 \mathrm{d}y \int_0^y \mathrm{d}x\,x^{-1-\epsilon}y^{-1-\epsilon}\,(x+y)^{-\epsilon}$$

New variables:

y` = y/x     x` = x/y

$$I_1 = \int_0^1 \mathrm{d}x\,\mathrm{d}y\,x^{-1-3\epsilon}y^{-1-\epsilon}\,(1+y)^{-\epsilon}$$

$$I_2 = \int_0^1 \mathrm{d}x\,\mathrm{d}y\,y^{-1-3\epsilon}x^{-1-\epsilon}\,(1+x)^{-\epsilon}$$

Expand in ε using the identity:

$$\lambda^{-1+\epsilon} = \frac{1}{\epsilon}\delta(\lambda) + \sum_{n=0}^{\infty}\frac{\epsilon^n}{n!}\left[\frac{\ln^n(\lambda)}{\lambda}\right]_+$$

Numerical evaluation at NNLO: sector decomposition

In some cases the functions $f_n(x)$ can be evaluated analytically.

$$\int d^d k \frac{1}{(p-k)^2 \ldots} = \sum_n \varepsilon^n \int d\,[\text{unit hypercube}(x)] \times f_n(x)$$

➔ A very nice alternative for calculating Feynman integrals

However: The real advantage of the method is that any single integral can be integrated numerically.

Note: Sector decomposition is a particular powerful tool for numerical checks of analytical results!

But most of all: can be applied to phase-space integrals

$$\int d^d k \frac{1}{(p-k)^2 \ldots} \times \delta(k) \times J(k) = \sum_n \varepsilon^n \int d\,[\text{unit hypercube}(x)] \times f_n(x) J_n(x)$$

$\delta(k)$ – a set of phase-space delta functions,

$J(k)$ – arbitrary observation function,

$J_n(x)$ – integrable functions that result from $J(k)$ after the re-mapping.

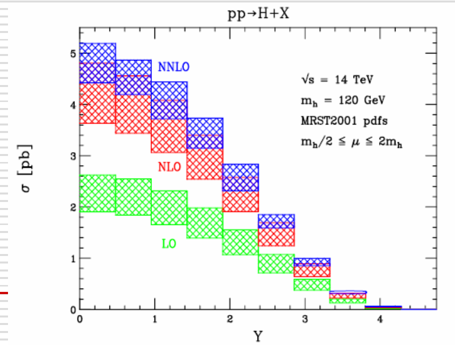
Numerical evaluation at NNLO: sector decomposition

Several important results have been obtained this way.

Differential Higgs production:

Anastasiou, Melnikov, Petriello `04

hep-ph/0409088



Fully differential Higgs production (arbitrary experimental cuts allowed)

Anastasiou, Melnikov, Petriello `04 Decay $H \to \gamma\gamma$ included

Electron energy spectrum in muon decay

Anastasiou, Melnikov, Petriello `04

What are the limitations of the method?

☺ Applicable to any problem

☹ Restricted by the proliferation of number of integrals. Each one is computable but the total takes time. Room for optimization!

All order resummations

- ✓ Typical applicability: the same as the analytical calculations,
- ✓ Important corrections close to kinematic thresholds
- ✓ Easy(-er) access to important information at higher perturbative orders.

Note: this is not an exhaustive review of this vast subject!
I'll cover only few selectively chosen features/results.

Main idea:

At threshold, real radiation is not possible due to kinematics.

➔ very close to threshold only soft emissions ($E \to 0$) are allowed.

- ❖ One can exploit the universality of soft emissions to derive the relevant contributions working directly in the eikonal approximation.

All order resummations

In QED and QCD it is known that

- ✓ in the soft limit the (eikonal) cross-section exponentiates,
- ✓ can be computed by:
  - ✓ considering Wilson lines
  - ✓ or simply matching few anomalous dimensions to known FO results.

Example: B-> s + γ

$$\frac{1}{\Gamma_{\rm total}^{O_7,{\rm PT}}}\frac{d\Gamma^{O_7,{\rm PT}}(x)}{dx} = \int_{\mathcal{C}} \frac{dN}{2\pi i}\, x^{-N}\, \bar{M}_N^{{\rm PT},O_7}$$

Andersen, Gardi `05

$$\widetilde{\rm Sud}(m,N) = \exp\left\{\int_0^1 dx \frac{x^{N-1}-1}{1-x}\left[\int_{(1-x)^2m^2}^{(1-x)m^2}\frac{d\mu^2}{\mu^2}\mathcal{A}\Big(\alpha_s(\mu^2)\Big) + \mathcal{B}\Big(\alpha_s((1-x)m^2)\Big) - \mathcal{D}\Big(\alpha_s((1-x)^2m^2)\Big)\right]\right\}$$

The functions A,B,D have perturbative expansion in the coupling

B,D – known at NNLO from DIS and heavy quark fragmentation.

Important role of the cusp anomalous dimensions A:
They control soft and collinear radiation (and → are process independent).

All order resummations

Example – Higgs production:

Moch, Vogt `05

hep-ph/0508265

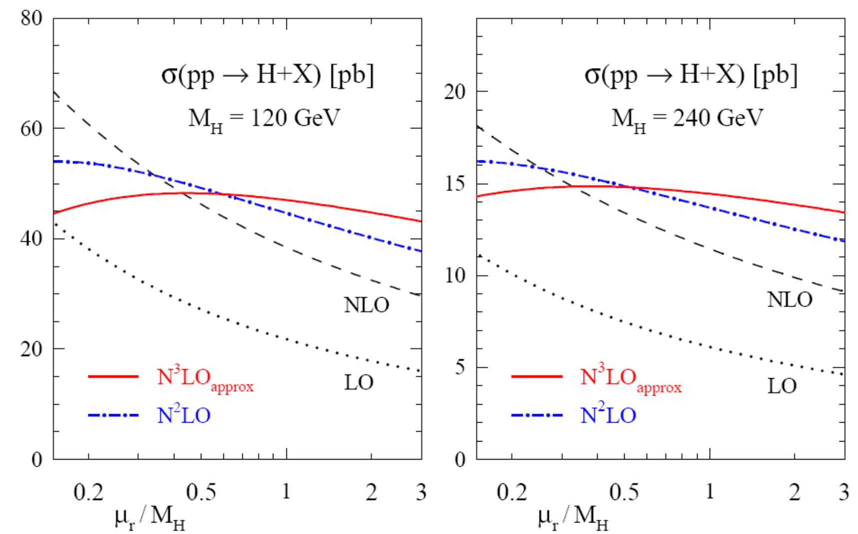


Figure 2: The dependence of the fixed-order predictions for the LHC cross section on the renormalization scale μ_r at $\mu_f = M_H$ for two representative values of the Higgs boson mass M_H.

The higher order effects indicate perrturbative stability !

Summary

- NNLO differential distributions – vital for precision collider physics
- Will be of utmost importance for the LHC and future ILC programs.
- Discovery and undesrtanding of New Physics will depend on it.

Status

- Many new technical developments in the last several years
- Complementary calculational approaches exist and produce results
- There is also plenty of room for further contributions

Expectations for the LHC

- Many things have been done at NNLO: Higgs, Drell-Yan
- Many things remain: Jets, Top-production.
- Work is underway!