Null Polygonal Wilson Loops in Full $\mathcal{N} = 4$ Superspace

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arxiv:1203.0525
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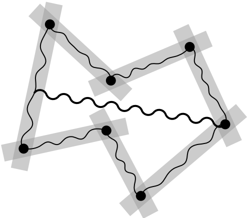

with
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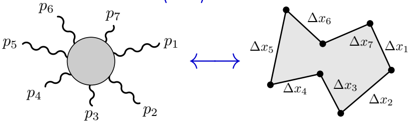
I. Introduction

Amplitudes vs. Null Polygonal Wilson Loops

Duality in Planar $\mathcal{N} = 4$ SYM: [Alday Maldacena] [Drummond Korchemsky Sokatchev] [Brandhuber Heslop Travaglini]

- Colour-ordered scattering amplitudes $\mathcal{A}$.
- Null polygonal Wilson loop expectation value $\langle \mathcal{W} \rangle$.

$$\mathcal{A}_{\mathrm{MHV}} = \mathcal{A}^{\mathrm{tree}}_{\mathrm{MHV}} \langle \mathcal{W}_{\mathrm{bosonic}} \rangle .$$



Identify:

- Null (on-shell) momenta $p \leftrightarrow$ null segments Δx.
- Momentum conservation $\leftrightarrow$ closure of Wilson loop.

Obey conformal + dual conf. = Yangian symmetry. [Drummond, Henn Korchemsky Sokatchev] [Drummond Henn Plefka]

What about Supersymmetry? [Mason Skinner] [Caron-Huot 1010.1167]

$$\mathcal{A} = \mathcal{A}^{\mathrm{tree}}_{\mathrm{MHV}} \langle \mathcal{W}_{\mathrm{chiral}} \rangle .$$

$\langle \mathcal{W}_{\mathrm{chiral}} \rangle$ has half of super (conformal) symmetry $\mathrm{Q}, \bar{\mathrm{S}}$, but not $\bar{\mathrm{Q}}, \mathrm{S}$.

Would like to use symmetries to construct $\mathcal{A}$, $\langle \mathcal{W} \rangle$ exactly.
Need to understand symmetries and anomalies.

Fully Supersymmetric Wilson Loops

Transformation of chiral WL under $\bar{\mathrm{Q}}$ computed. [Caron-Huot 1105.5606]
Use anomaly to reconstruct chiral WL expectation value. [Caron-Huot, He 1112.1060]

Need Wilson loop $\mathcal{W}$ in full superspace for full supersymmetry.

- How to define null polygons in full superspace?
- What is the SYM connection in full superspace?
- How to compute the expectation value?
- How is the full Wilson loop related to amplitudes?

This Talk: Want to compute WL expectation value in full superspace.

- Can compute from component fields. Better not!
  Mess: e.g. compare components and superspace for $A^{\mathrm{tree}}_{\mathrm{MHV}}$.
- No fully covariant off-shell superspace for $\mathcal{N} = 4$ SYM.
- Nevermind that. Use $\mathcal{N} = 4$ superspace anyway!

Outline

$\mathcal{N} = 4$ Super Yang–Mills in Full Superspace

- Define gauge theory on $\mathcal{N} = 4$ superspace. Constraints.
- Quantise and derive two-point functions.

Wilson Loops on Null Polygons in Superspace

- How to define null polygons?
- How to compute Wilson loop efficiently?

Twistors

- Simplify expressions using twistor variables.
- Take seriously: Wilson loop in ambitwistor space.

Regularisation

- UV divergences at cusps need to be regularised. How?

Superconformal and Yangian Anomalies

- Result is not invariant. Should it? How are symmetries broken?

II. $\mathcal{N} = 4$ SYM in Superspace

$\mathcal{N} = 4$ Super Yang–Mills Formulations

Various formulations of $\mathcal{N} = 4$ Super Yang–Mills theory:

- YM + matter,
- $\mathcal{N} = 1$ SYM + matter,
- $\mathcal{N} = 2$ SYM + matter in harmonic superspace,
- 10D YM + fermion reduced to 4D,
- 10D YM + fermion reduced to $4 - 2\epsilon$ dimensions,
- $\mathcal{N} = 4$ SYM on the light cone.

All formulations have:

- certain benefits,
- certain drawbacks,
- several followers,
- many enemies.

Eventually, all produce the same results.

Clearly: None displays full $\mathcal{N} = 4$ superspace $(x, \theta^a, \bar{\theta}_a)$ structure.
Inconvenient for computing a Wilson loop in full superspace.

$\mathcal{N} = 4$ Superspace

4 real bosonic coordinates $(-+++)$, 8 complex fermionic coordinates

$$X = (x^{\beta\dot{\alpha}}, \theta^{\beta a}, \bar{\theta}_b{}^{\dot{\alpha}}),$$

Our conventions:

- Use spinor notation: $x \sim \sigma_\mu x^\mu$.
  $x, \theta, \bar{\theta}$ are 2×2, 2×4, $\bar{4} \times 2$ matrices, respectively.
- Assume real Minkowski superspace: $x^\dagger = x$, $\theta^\dagger = \bar{\theta}$.
  No need for (2,2) signature or complex spacetime.

Superspace has translation-invariant vielbein $(e, d\theta, d\bar{\theta})$ with

$$e = dx - i d\theta \bar{\theta} - i\theta d\bar{\theta}.$$

Superspace has non-trivial torsion ($\{\mathrm{Q}, \bar{\mathrm{Q}}\} \sim \mathrm{P}$)

$$de = -2i d\theta d\bar{\theta}.$$

Yang–Mills Theory on $\mathcal{N}=4$ Superspace

Introduce a connection A on superspace [Sohnius]

$$A = \mathrm{Tr}(d\theta\, A_\theta) - \mathrm{Tr}(d\bar\theta\, A_{\bar\theta}) + \mathrm{Tr}(e\, A_x).$$

Connection A has (way) more component fields than $\mathcal{N}=4$ SYM.
Force certain components of field strength $F = dA + A^2$ to vanish:

$$\begin{aligned} F = &-\tfrac{1}{2}\,\mathrm{Tr}(d\theta^{\mathsf{T}}\varepsilon d\theta\,\bar\Phi) + \mathrm{Tr}(e^{\mathsf{T}}\varepsilon d\theta\,\bar\Psi) + \tfrac{1}{2}\,\mathrm{Tr}(e^{\mathsf{T}}\varepsilon e\,\bar\Gamma) \\ &-\tfrac{1}{2}\,\mathrm{Tr}(d\bar\theta\varepsilon d\bar\theta^{\mathsf{T}}\,\Phi) + \mathrm{Tr}(d\bar\theta\varepsilon e^{\mathsf{T}}\,\Psi) + \tfrac{1}{2}\,\mathrm{Tr}(e\varepsilon e^{\mathsf{T}}\,\Gamma), \end{aligned} \qquad \Phi^{ab} = \tfrac{1}{2}\varepsilon^{abcd}\bar\Phi_{cd}.$$

Remaining superfields Φ, Ψ, Γ contain scalar, spinor and field strength.

Jacobi Identity $dF + [A, F] = 0$ implies:

- Supersymmetry relations between the fields.
- Equations of motion of classical $\mathcal{N}=4$ SYM.

Inconvenient for full quantisation.

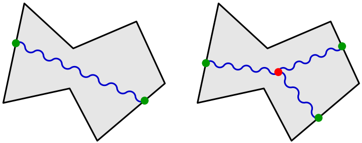
Wilson Line

Straight-forward to define Wilson line $\mathcal{W}$ on full superspace

$$\mathcal{W}_\gamma = \mathrm{P}\exp\int_\gamma A.$$

Want to compute $\langle\mathcal{W}_\gamma\rangle$, need to quantise, requires:

- Feynman propagator Δ_F,
- vertices (two loops and above),
- ghosts (two loops and above).



Start simple; content ourselves with one loop:

- Use canonical quantisation.
- Derive propagator $\Delta_\mathrm{F} = \langle 0|\mathrm{T}[A\,A']|0\rangle$
- Linearised/abelian theory sufficient.
- Can work on-shell.

Linearised $\mathcal{N} = 4$ SYM and Prepotentials

There is a simple ansatz to solve linearised E.O.M.: [NB, He, Schwab, Vergu]

$$A_{\theta,a\beta} = \varepsilon_{\beta\delta} D_{a\gamma} B^{\gamma\delta}, \qquad A_{\bar\theta,\dot\alpha}{}^{b} = \varepsilon_{\dot\alpha\dot\delta} \bar D_{\dot\gamma}{}^{b} \bar B^{\dot\gamma\dot\delta}.$$

The prepotentials $B, \bar B$

- are (anti)-chiral: $\bar D B = 0$, $D \bar B = 0$,
- are harmonic: $D^2 B = 0$, $\bar D^2 \bar B = 0$,
- are self-conjugate: $D^2 \cdot B \sim \bar D^2 \cdot \bar B$,
- can be axial gauge fixed: $B^{\alpha\gamma} = \rho^\alpha \rho^\gamma B$, $\bar B^{\dot\alpha\dot\gamma} = \bar\rho^{\dot\alpha} \bar\rho^{\dot\gamma} \bar B$ ($\rho, \bar\rho$ fixed).

Solution in on-shell momentum space (solves chiral harmonic)

$$B(x^+, \theta) = \int d^2\lambda \, d^2\bar\lambda \, d^{0|4}\eta \, \exp\bigl(i\langle\lambda|x^+|\bar\lambda] + i\langle\lambda|\theta|\eta]\bigr) \frac{B(\lambda, \bar\lambda, \eta)}{\langle\lambda|\rho\rangle^2} \, .$$

Complete solution: $B(\lambda, \bar\lambda, \eta)$ has 16 components for each $p = \lambda\bar\lambda$.

Two-Point Correlators

Need to find $\langle 0|B(\lambda,\bar\lambda,\eta)\,B(\lambda',\bar\lambda',\eta')|0\rangle$. Canonical quantisation? [NB, He, Schwab, Vergu]
Action for $\mathcal{N}=4$ superspace not known/straight-forward. Construct:

$$\langle 0|B\,B'|0\rangle \sim \oint \frac{dz}{z}\,\delta^2(\lambda'+z^{-1}\lambda)\,\delta^2(\bar\lambda'-z\bar\lambda)\,\delta^{0|4}(\eta'-z\eta)\,\theta(E(\lambda,\bar\lambda)).$$

Mixed correlator via fermionic FT $\bar B(\bar\eta)\sim\int d^{0|4}\eta\exp(\eta\bar\eta)B(\eta)$:

$$\langle 0|B\,\bar B'|0\rangle \sim \oint \frac{dz}{z}\,\delta^2(\lambda'+z^{-1}\lambda)\,\delta^2(\bar\lambda'-z\bar\lambda)\,\exp(\eta\bar\eta')\,\theta(E(\lambda,\bar\lambda)).$$

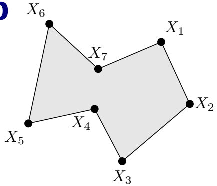
Transform to position space

$$\langle 0|B\,B'|0\rangle \sim \frac{\delta^{0|4}(\langle\theta|x^+|\bar\rho])}{\langle\rho|x^+|\bar\rho]^4(x^+)^2}+\ldots,\quad \langle 0|B\,\bar B'|0\rangle \sim \frac{x^2\log(x^2)}{\langle\rho|x|\bar\rho]^2}+\ldots.$$

Can now compute one-loop Wilson loop expectation values.

III. Wilson Loop

Null Polygonal Wilson Loop

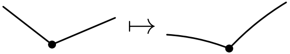


Consider a null polygon in full superspace.

- Sequence of vertices $X_j = (x_j, \theta_j, \bar{\theta}_j)$.
- Neighbours are null separated:

$$(x^{\pm}_{j,j+1})^2 = 0, \quad \langle x^{+}_{j,j+1}|\theta_{j,j+1} = 0, \quad \bar{\theta}_{j,j+1}|x^{-}_{j,j+1}] = 0.$$

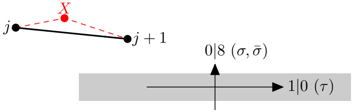
How to connect the vertices by a curve $X(\tau)$?



[NB Vergu]

- "Straight" line $X(\tau) = X_0 + \tau \Delta X$ not stable under superconformal.
- Demand that $X(\tau)$ is everywhere null separated from vertices.

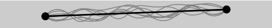
$$x = x_j + \tau\lambda_j\bar{\lambda}_j + i\lambda_j\sigma\bar{\theta}_j - i\theta_j\bar{\sigma}\bar{\lambda}_j,$$
$$\theta = \theta_j + \lambda_j\sigma, \qquad \bar{\theta} = \bar{\theta}_j + \bar{\sigma}\bar{\lambda}_j.$$



Solution $X(\tau,\sigma,\bar{\sigma})$ depends on 8 free fermionic curve parameters $\sigma,\bar{\sigma}$.

Null segment is fat. Which curve $X(\tau)$ to pick?

- Connection A flat on $X(\tau,\sigma,\bar{\sigma})$: $F|_{\text{segment}} = 0$.



- All curves $X(\tau)$ on $X(\tau,\sigma,\bar{\sigma})$ physically equivalent! [Witten]

Segment Potential Shift

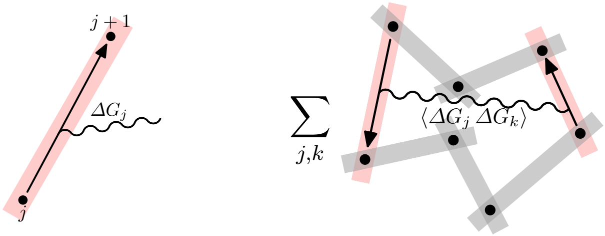
Can compute WL expectation value by brute force.

Better: Integrate connection $A = dG_j + d\bar{G}_j$ on fat segment j [NB, He, Schwab, Vergu]

$$G_j(x^+, \theta) \sim \int d^2\lambda\, d^2\bar{\lambda}\, d^{0|4}\eta\, e^{i\langle\lambda|x^+|\bar{\lambda}]+i\langle\lambda|\theta|\eta]}\, \frac{\langle\lambda_j|\rho\rangle\, B(\lambda, \bar{\lambda}, \eta)}{\langle\lambda|\rho\rangle\langle\lambda|\lambda_j\rangle}\,.$$

(Abelian) Wilson loop $\mathcal{W}$ is sum over potential shifts across segments

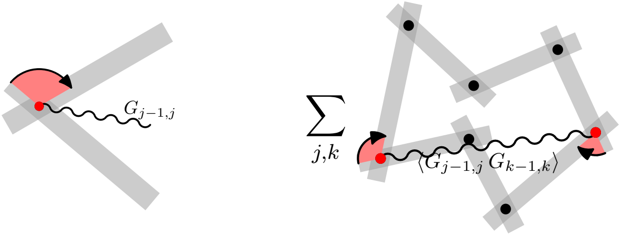
$$\log \mathcal{W} = \oint A = \sum_{j=1}^{n} \Delta G_j, \quad \Delta G_j = G_j(X_{j+1}) - G_j(X_j).$$



Vertex Potential Shift

Can rearrange Wilson loop as sum over vertices [Mason Skinner] [NB, He Schwab Vergu]

$$\oint A = \sum_{j=1}^{n} G_{j-1,j}, \quad G_{j-1,j} = G_{j-1}(X_j) - G_j(X_j),$$



Vertex potential shift reads

$$G_{j-1,j} \sim \int d^2\lambda\, d^2\bar{\lambda}\, d^{0|4}\eta\, e^{i\langle\lambda|x_j^+|\bar{\lambda}] + i\langle\lambda|\theta_j|\eta]}\, \frac{\langle\lambda_{j-1}|\lambda_j\rangle\, B(\lambda, \bar{\lambda}, \eta)}{\langle\lambda|\lambda_{j-1}\rangle\langle\lambda|\lambda_j\rangle}\,.$$

Localised at point X_j. Gauge dependence on ρ drops out!

Vertex Correlators: Purely Chiral

Compute correlator of vertex potential shifts $\langle 0|G_{j-1,j}\, G_{k-1,k}|0\rangle$.
Chiral-chiral: Integrals over delta-functions

$$\sim \int d^2\lambda\, d^2\bar\lambda\, \theta(E)\, e^{i\langle\lambda|x^+_{j,k}|\bar\lambda]}\, \frac{\delta^{0|4}(\langle\lambda|\theta_{j,k})\langle j-1|j\rangle\langle k-1|k\rangle}{\langle\lambda|j-1\rangle\langle\lambda|j\rangle\langle\lambda|k-1\rangle\langle\lambda|k\rangle}\,.$$

Perform remaining Fourier integral (differential operators)

$$\sim \frac{\langle j-1|j\rangle\,\langle k-1|k\rangle\,\delta^{0|4}(\theta_{j,k}|x^+_{j,k}|\bar\rho])}{\langle j-1|x^+_{j,k}|\bar\rho]\,\langle j|x^+_{j,k}|\bar\rho]\,\langle k-1|x^+_{j,k}|\bar\rho]\,\langle k|x^+_{j,k}|\bar\rho]\,(x^+_{j,k})^2}\,.$$

- Well-known vertex correlator: R-invariant. [Drummond Henn]
- Yields tree-level NMHV/MHV upon summation.
- Reference spinor $\bar\rho$ drops out in sum.
- Purely anti-chiral correlator is complex conjugate.

Vertex Correlators: Mixed Chiral

Compute mixed chirality correlator $\langle 0|G_{j-1,j}\,\bar{G}_{k-1,k}|0\rangle$.

$$\sim \int d^2\lambda\, d^2\bar{\lambda}\, \theta(E)\, e^{i\langle\lambda|x^{+-}_{j,k}|\bar{\lambda}]}\, \frac{\langle j-1|j\rangle[k-1|k]}{\langle\lambda|j-1\rangle\langle\lambda|j\rangle[\bar{\lambda}|k-1][\bar{\lambda}|k]}\,.$$

Perform remaining Fourier integral (differential operators)

$$\sim \tfrac{1}{2}\log\frac{\langle j-1|x^{+-}_{j,k}|k]\,\langle j|x^{+-}_{j,k}|k-1]}{\langle j-1|x^{+-}_{j,k}|k-1]\,\langle j|x^{+-}_{j,k}|k]}\log\Big(\langle j-1|x^{+-}_{j,k}|k-1]\,\langle j|x^{+-}_{j,k}|k]\Big)$$
$$-\operatorname{Li}_2\frac{\langle j-1|x^{+-}_{j,k}|k]\,\langle j|x^{+-}_{j,k}|k-1]}{\langle j-1|x^{+-}_{j,k}|k-1]\,\langle j|x^{+-}_{j,k}|k]}\,.$$

- Expected one-loop form: $\operatorname{Li}_2 + \log^2$, superconformal cross ratios.
- Singular for $|k-j| \leq 2$; regularisation needed.
- Fermionic contribution only in $x^{+-}_{jk} := x^-_k - x^+_j + 2i\theta_j\bar{\theta}_k$.
- Used same correlator as for purely chiral case up to fermionic FT.

VEV vs. Time-Ordered Expectation Value

In fact, computed $\langle 0|W|0\rangle$ instead of $\langle W\rangle = \langle 0|\mathrm{T}[W]|0\rangle$.
Used $\mathit{\Delta} = \langle 0|\mathit{\Phi}\mathit{\Phi}'|0\rangle$ instead of $i\mathit{\Delta}_{\mathrm{F}} = \langle \mathit{\Phi}\mathit{\Phi}'\rangle$.

- $\mathit{\Delta} \sim \theta(E)\delta(p^2)$ is on-shell, $i\mathit{\Delta}_{\mathrm{F}} \sim 1/p^2$ is off-shell.
- E.O.M. required for exploiting constraints: flatness on segments.

Note that difference is in imaginary part (scalar field)

$$i\mathit{\Delta}_{\mathrm{F}} - \mathit{\Delta} \sim i\theta(\pm t)\delta(x^2).$$

Difference $\langle W\rangle - \langle 0|W|0\rangle$ is small: lower transcendentality.

- Leading transcendentality in $\langle 0|W|0\rangle$ correct! Disregard lower ones.
- Compute cut $\operatorname{disc}\langle W\rangle$, $\operatorname{disc} i\mathit{\Delta}_{\mathrm{F}} \sim \delta(p^2)$: on-shell!
- Brute force calculation in position space: $i\mathit{\Delta}_{\mathrm{F}} = \mathrm{T}[\mathit{\Delta}]$. Okay!

IV. Twistors

Fat Null Lines and Twistors Variables

Recall fat null line:

$$x = x_j + \tau\lambda_j\bar{\lambda}_j + i\lambda_j\sigma\bar{\theta}_j - i\theta_j\bar{\sigma}\bar{\lambda}_j, \quad \theta = \theta_j + \lambda_j\sigma, \quad \bar{\theta} = \bar{\theta}_j + \bar{\sigma}\bar{\lambda}_j.$$

Equivalent to solution of twistor equations

$$\langle\lambda_j|x^+ = \langle\lambda_j|x_j^+ =: \mu_j, \qquad x^-|\bar{\lambda}_j] = x_j^-|\bar{\lambda}_j] =: \bar{\mu}_j,$$
$$\langle\lambda_j|\theta = \langle\lambda_j|\theta_j =: \chi_j, \qquad \bar{\theta}|\bar{\lambda}_j] = \bar{\theta}_j|\bar{\lambda}_j] =: \bar{\chi}_j.$$

The $4|4$ projective vectors $W_j := (\lambda_j, \mu_j, \chi_j)$ are (momentum) twistors.
$\bar{W}_j := (\bar{\mu}_j, \bar{\lambda}_j, \bar{\chi}_j)$ are conjugate twistors; $(W_j, \bar{W}_j)$ is real ambitwistor.

Twistor product is superconformal invariant

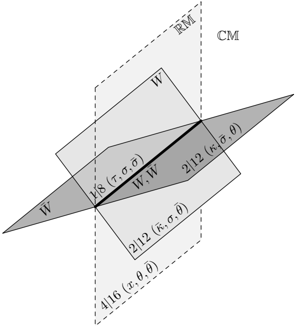
$$\langle j, k] := W_j \cdot \bar{W}_k \sim \langle\lambda_j|x_{jk}^{+-}|\bar{\lambda}_k].$$

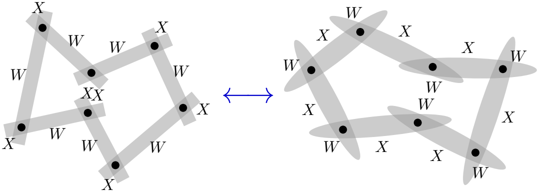
Note: $\langle j, k] = 0$ for $k = j, j \pm 1$.

Ambitwistors and Twistor Space Polygon

Twistors as subspaces of spacetime:

- Twistor W describes $(2|12)$D null subspace of $(4|16)$D complex space.
- Conjugate twistor $\bar{W}$ describes opposite chirality $(2|12)$D null subspace.
- When $\langle W, \bar{W}] = 0$ twistor and conjugate twistor intersect.
- Intersection is $(1|8)$D subspace of real superspace: ambitwistor $(W, \bar{W})$.
- Dual polygon in twistor space: W is point, X is $\mathbb{CP}^1$





Twistor Fields

Can perform calculations in twistor space. On-shell twistor fields [Witten]. . .

$$B(\lambda,\mu,\chi) = \int d^2\bar\lambda\, d^{0|4}\eta\, \exp\bigl(i[\mu|\bar\lambda] + i\chi\eta\bigr) B(\lambda,\bar\lambda,\eta).$$

Vertex potential shift as integral over $\mathbb{CP}^1$ (twistor dual of vertex)

$$G_{j-1,j} = \int \frac{ds}{s}\, B(W_{j-1} + sW_j), \quad \bar G_{j-1,j} = \int \frac{ds}{s}\, \bar B(\bar W_{j-1} + s\bar W_j).$$

Purely chiral and mixed chiral correlator (reference twistors $W_\star, \bar W_\star$)

$$\langle BB'\rangle \sim \int \frac{ds}{s}\,\frac{dt}{t}\,\delta^{4|4}(sW + tW' + \langle W,\bar W_\star] W_\star),$$
$$\langle B\bar B'\rangle \sim \int \frac{ds}{s}\,\frac{dt}{t}\,\exp\bigl(s\langle W,\bar W'] + t\langle W,\bar W_\star]\langle W_\star,\bar W']\bigr).$$

Problem: leave real spacetime; integration contours obscured.

Correlators using Twistors

Purely chiral vertex correlator using twistors $W = (\omega, \chi)$

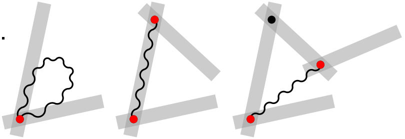
$$\frac{\delta^{0|4}\big(\chi_\star \varepsilon_{ABCD}\omega^A_{j-1}\omega^B_j\omega^C_{k-1}\omega^D_k + 4\text{ perm.}\big)}{\varepsilon_{ABCD}\omega^A_{j-1}\omega^B_j\omega^C_{k-1}\omega^D_k \times 4\text{ perm.}}$$

- Superconformal invariant. Reminiscent of superdeterminant.
- Reference twistor $W_\star$ disappears in sum.

Mixed chiral vertex correlator using twistors

$$\tfrac{1}{2}\log\frac{\langle j-1,k]\langle j,k-1]}{\langle j-1,k-1]\langle j,k]}\log\left(\langle j-1,k-1]\langle j,k]\right) - \mathrm{Li}_2\,\frac{\langle j-1,k]\langle j,k-1]}{\langle j-1,k-1]\langle j,k]}\,.$$

- Only superconformal twistor brackets.
- Weight in W_j and $\bar{W}_j$ vanishes in sum.
- Singular when one $\langle j,k] = 0$.
  Regularise three terms:

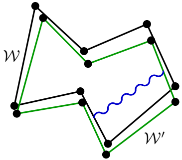


V. Regularisations

Framing

"Point-splitting": Correlator of two nearby Wilson loops $\mathcal{W}, \mathcal{W}'$ [Bullimore Skinner]

$$\langle \mathcal{W} \rangle_{\text{frame}} = \left(\frac{\langle \mathcal{W}\mathcal{W}' \rangle}{\langle \mathcal{W} \rangle \langle \mathcal{W}' \rangle} \right)^{1/2}$$



Shift second polygon using twistor variables (weights preserved) [NB, He Schwab Vergu]

$$W'_j = W_j + \epsilon \, \frac{\langle W_j, \bar{W}_\star]}{\langle W_\star, \bar{W}_\star]} \, W_\star + \mathcal{O}(\epsilon^2).$$

Twistor products regularised: $\mathcal{O}(\epsilon)$ instead of 0 for $|j-k| \leq 1$

$$\langle j, k'] = \langle j, k] + \epsilon \, \frac{\langle j, \bar{\star}] \langle \star, k]}{\langle \star, \bar{\star}]} + \mathcal{O}(\epsilon^2).$$

Result depends on reference twistor $W_\star, \bar{W}_\star$: Axial symmetry breaking.
Result diverges for $\epsilon \to 0$ as it should (UV).

Supersymmetric Extrapolation

Superspace formalism tied to 4D. Cannot do dimensional regularisation.

Construct a supersymmetric expression: [NB, He, Schwab, Vergu]

- Start with known bosonic result.
- Express through $\langle j|k\rangle$, $[j|k]$, $\langle j|x_{jk}|k]$.
- Adjust proper weights in spinors $\langle j|$ and $|k]$.
- Lift $\langle j|x_{jk}|k]$ to $\langle j|x_{jk}^{+-}|k]$ and thus $\langle j,k]$.
- Obtain an expression in $\langle j|k\rangle$, $[j|k]$ and $\langle j,k]$.

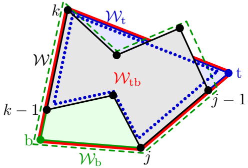
Features:

- $\langle j,k]$ is superconformal invariant.
- $\langle j|k\rangle$, $[j|k]$ break (super)conformal boosts.
- Expression has correct collinear behaviour.
- First principles derivation desirable.

Boxed Wilson Loop

Compute a UV-finite quantity instead. Boxed Wilson Loop: [Alday, Gaiotto, Maldacena, Sever, Vieira]

$$C_{\text{box}} = \frac{\langle \mathcal{W} \rangle \langle \mathcal{W}_{\text{tb}} \rangle}{\langle \mathcal{W}_{\text{t}} \rangle \langle \mathcal{W}_{\text{b}} \rangle} \,.$$



Features:

- Result is finite and independent of regularisation.
- Needs only minor regularisation for special vertices.
- Only twistor brackets $\langle j, k]$: Manifestly superconformal! [NB, He, Schwab, Vergu]
- Framed and guessed results reduce to this.
  Test of collinear behaviour. Consistent!

VI. Superconformal and Yangian Symmetry

Framing and Guessing

Act with generators on $\langle\mathcal{W}\rangle$ and observe: "Anomaly". [NB, He Schwab Vergu]

Superconformal Anomaly:

- Framing: Reference twistors $W_\star, \bar{W}_\star$ break superconformal invariance.
- Guessing: Spinor brackets $\langle j|k\rangle$, $[j|k]$ leave super-Poincaré invariance.
- Anomaly expression depends on regularisation.

Yangian Anomaly:

- Anomaly expression depends on regularisation.
- Superconformal anomaly reverberates.
- Additional anomaly intrinsic to Yangian?

Divergent Quantities:

- Futile discussion: Quantities are divergent; depend on regularisation.

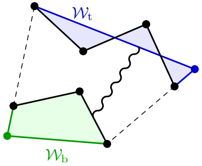
Boxing

Finite Observable:

- Boxed Wilson loop C_{box} is UV-finite.
- No superconformal anomaly.
- Yangian anomaly remains; even worse.

Yangian Anomalous?

- Boxed Wilson loop not a regularisation but finite observable.
- Can be computed consistently in any regularisation.
- Introduces additional data, new dependent vertices t, b.
- One-loop: correlator of two Wilson loops.



Yangian invariance expected only for disc amplitude.

Yangian Anomalous?

Is there a Yangian-Invariant WL Observable?

E.g. renormalised Wilson loop:

$$\tilde{\mathcal{W}} = \frac{\mathcal{W}}{\langle \mathcal{W} \rangle}.$$

- Fully Yangian invariant expectation value $\langle \tilde{\mathcal{W}} \rangle$.
- Expectation value boring: 1.
  Even boring results can be correct...
- Correlators $\langle \tilde{\mathcal{W}}_1 \tilde{\mathcal{W}}_2 \ldots \rangle$ non-trivial. Finite?

And even if Yangian Symmetry is Anomalous:

- Yangian not an exact symmetry for local operators.
- Commutes only with spin chain Hamiltonian up to boundary terms.
- Still integrability applies and can be exploited.
- Can try to reconstruct exact result from anomaly.

VII. Conclusions

Conclusions

$\mathcal{N} = 4$ Super Yang–Mills in Full Superspace

- Can indeed be used in calculations.

Null Polygonal Wilson Loops in Full Superspace

- Null polygons are fat.
- Computed one-loop expectation value efficiently.
- Contains NMHV tree and fully supersymmetric MHV loop.
- Dual description in ambitwistor space: Null lines.

Regularisation and Symmetries

- Can regularise in several ways.
- Superconformal and Yangian symmetry broken in different ways.
- Can compute finite quantities with superconformal symmetry.
  Not Yangian invariant. Yangian anomalous? Yes and no.

Wilson Loop Dual to Amplitudes?

Rather Not:

- Many more components (η, $\bar{\eta}$) than particles (η).
- Consider supersymmetric intervals $x_{j,j+1}$ vs. bosonic ones $x_{j+1} - x_j$:

$$x_{j,j+1}^2 = 0, \qquad \textstyle\sum_j x_{j,j+1} \neq 0,$$
$$(x_{j+1} - x_j)^2 \neq 0, \qquad \textstyle\sum_j (x_{j+1} - x_j) = 0.$$

Extended amplitude either on-shell or momentum conservation.

Full Wilson Loop Contains Amplitude:

- Can set $\bar{\theta} = 0$ to restrict to chiral superspace.
- Unless. . .

MHV Prefactor: Wilson loops lack $A_{\text{MHV}}^{\text{tree}}$.

- Wilson loops require $\bar{\theta}$-dependence for full superconformal symmetry.
- Amplitudes are superconformal on their own: $A_{\text{MHV}}^{\text{tree}}$ compensates.