Generic two-vertex strong field physics processes

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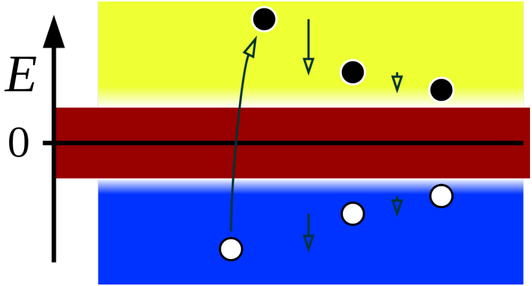
Synopsis

- Definition of a strong field
- Why is it important in collider physics?
- What strong field processes have been calculated/simulated?
- Furry picture/Volkov solutions
- Furry picture Feynman diagram components and rules
- **IPstrong:** a new event generator to produce strong field events

changing the vacuum state

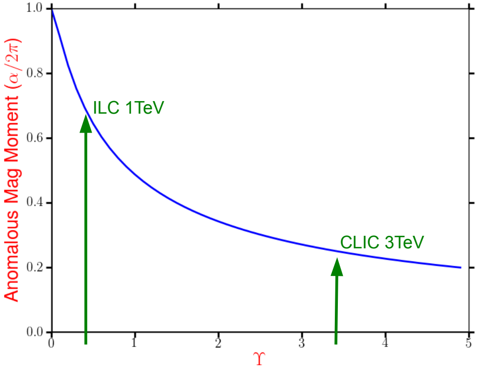


- Vacuum polarization at the Schwinger limit ($E_{\text{cr}} = 10^{18}$ V/m)
- In the incoming collider particles' rest frame, $E \to E_{\text{cr}}$
- Need to move beyond EPA and perturbation theory

Loop corrections in an external field

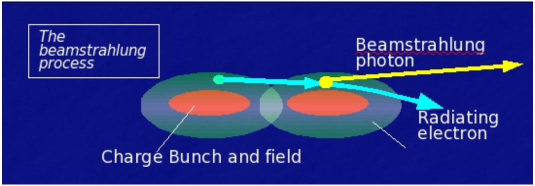
- anomalous magnetic moment (one-loop) in a charge bunch field

$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi} \int_0^\infty \frac{2\pi\, dx}{(1+x)^3} \left(\frac{x}{\Upsilon}\right)^{1/3} \mathrm{Gi}\left(\frac{x}{\Upsilon}\right)^{1/3}$$



- issues for spin tracking and... **ALL** loop corrections

Statement of the approach

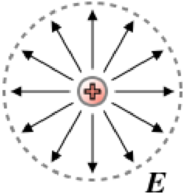


" Strong field processes are physics processes calculated simultaneously in the normal perturbation theory as well as exactly with respect to a strong electromagnetic field. "

" Such calculations are necessary when the external field seen by a particle approaches or exceeds E_{cr}. "

Strong fields at the collider IP

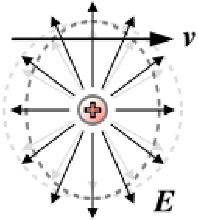
- moving charge has longitudinal length contraction



- relativistic charge bunch produces constant crossed plane wave field

$$A_\mu = a_{1\mu}(k \cdot x)$$

$$a_{1\mu} = (0, \vec{a})$$

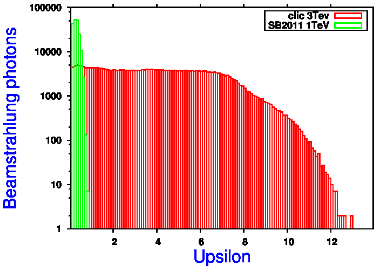


- particle p sees a field strength parameter Υ

$$\Upsilon = \frac{e|\vec{a}|}{m E_{\text{cr}}}(k \cdot p)$$

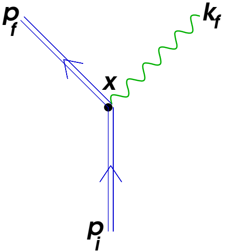
ILC and CLIC Strong field parameters

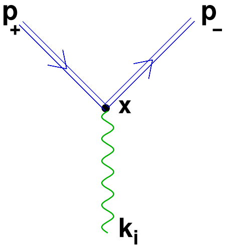
Parameter	ILC 1TeV	CLIC 3 TeV
$\mathcal{L}(\times 10^{34})$	4	3.6
N(incoh)	3.9e5	3.8e5
N(coh)	0	6.8e8
Υ (ave)	0.27	3.34
Υ (max)	0.94	10.9
δE_{bs}	10%	28%
$\langle \text{depol} \rangle_{\text{LW}}$	0.62%	3.5%



- CLIC far exceeds Schwinger critical field
- field strength varies from point to point through the beam collision
- depolarization due to spin precession (ILC) and spin flip (CLIC)
- # coherent pairs limited not by the field strength but by the beamstrahlung energy spread

Beamstrahlung, incoherent/coherent pair production

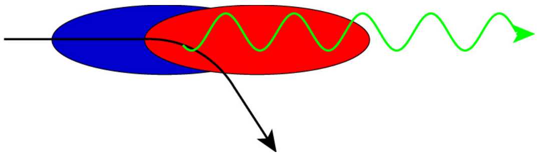




- simulated in IP simulators - CAIN, GP
- beamstrahlung & coherent pp calculated via quasi-classical approximation
- incoherent pairs calculated with beamstrahlung photon and EPA
- **more exactly** these are 1st and 2nd order Furry picture processes

bkgd pairs	current	proposed
coherent	quasi-classical	1 vertex Furry picture
incoherent	EPA	2 vertex Furry picture

Formation length



" distance travelled by a charged particle while a radiated photon moves one wavelength in front of it "

A bad argument: *" If the bunch is sufficiently short we dont need to worry about strong field effects"*

- classical argument that only applies to the beamstrahlung
- strong field propagator integrated over all length scales

(W.H.) Furry Picture

- Separate electromagnetic part into external and vacuum parts

$$\mathcal{L}^{\text{Furry}}_{\text{QED}} = \bar{\psi}^V (i\partial\!\!\!/ - eA\!\!\!/^e - m)\psi^V - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^V A\!\!\!/\, \psi^V$$

- require solutions ψ^V

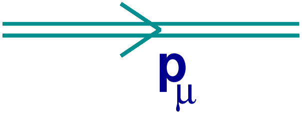
$$(i\partial\!\!\!/ - eA\!\!\!/^e - m)\psi^V = 0$$

- exact solutions known for
  - plane wave field
  - Coulomb field
  - collinear fields

- At E_{cr} vacuum becomes charged
- tadpole diagrams can contribute

lepton Volkov Solution

- Solution of the 2nd order Dirac equation with external potential



$$[D^2 + m^2 + \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}]\psi^{\mathsf{V}} = 0, \quad D_\mu = \partial_\mu + ieA^e_\mu$$

- Propose solution $\psi^{\mathsf{V}} = E_p\, e^{-ip\cdot x}\, u_p$

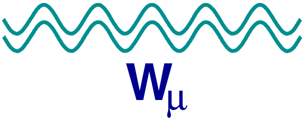
$$2i(kp)E_p' + [e^2 A^{e2} - 2e(A^e \cdot p) + ie\not{A}^{e\prime}\not{k}]E_p = 0$$

$$\therefore\ E_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(e\not{A}^e\not{k} + i2e(A^e\cdot p) - ie^2A^{e2}\right)\right]$$

- Fourier Transform $\psi^{\mathsf{V}} = \int dr\, \exp(-ip.x - irk\cdot x - \mathcal{FT}(E_p))\, u_p$

W boson Volkov Solution

- Equation of motion for the W boson



$$(D^2 + m_W^2)W_\nu + i2eF^{\mu}{}_{\nu}W_\mu = 0, \quad D^\mu W_\mu = 0$$

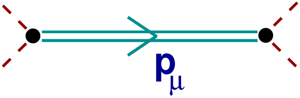
- with solution $W_\mu = E_p^W \, e^{-ip\cdot x} \, w_p$ where

$$E_p^W = \left(g_{\mu\nu} + \frac{e}{k\cdot p}\int F_{\mu\nu} - \frac{e^2}{2(k\cdot p)^2} A^{e2} k_\mu k_\nu \right)$$
$$. \exp\left[-\frac{i}{2(k\cdot p)} \left(2e(A^e \cdot p) - e^2 A^{e2} \right) \right]$$

- similar solutions can be found for other particles that couple to A^e eg. charginos

Strong field propagator

- Look for Green's function solution



$$(\not{D}(x) - m) G^{\mathsf{V}}(x, x') = \delta(x - x')$$

- ψ^V are orthogonal and complete (Ritus, Ann Phys **69** 555, 1970)

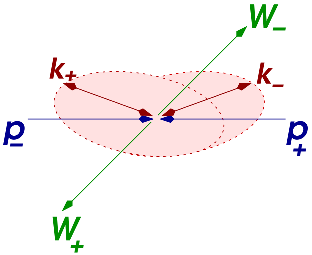
- Solution is the fermion propagator flanked by Volkov E_p functions

$$G^{\mathsf{V}}(x, x') = \int d^4p \; E_p(x) \frac{\not{p} + m}{p^2 - m^2 + i\epsilon} \bar{E}_p(x') \, e^{ip\cdot(x'-x)}$$

$$\psi^{\mathsf{V}} \equiv E_p(x) e^{-ip\cdot x} \, u_p$$

- photon propagator remains unchanged

Volkov solutions in two external fields



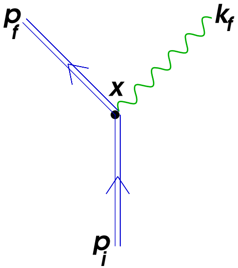
- both incoming bunches contribute external fields
- external field wavevectors are generally anti-collinear
- Need new Volkov-type solution

- Propose solution $\psi^{\mathsf{V}} = E_p(\phi_-, \phi_+)\, e^{-ip\cdot x}\, u_p \quad ; \quad \phi_\pm = (k_\pm \cdot p)$

$$2(k_-{\cdot}k_+)\frac{\partial^2 E}{\partial\phi_-\partial\phi_+} + B\frac{\partial E}{\partial\phi_-} + C\frac{\partial E}{\partial\phi_+} + DE = A^2(\phi_-, \phi_+)$$

- solution possible by bunch symmetry, $\vec{a_-} \approx -\vec{a_+} \quad ; \quad \vec{k_-} \approx -\vec{k_+}$

Modified Feynman Rules



- double fermion lines are Volkov solutions
- conservation of momentum allows 1 vertex diagrams
- Volkov E_p functions as adjacent to the vertex

$$\gamma^e_\mu = \int d^4x \bar{E}_{p_f}(x)\gamma_\mu E_{p_i}(x)\, e^{i(p_f - p_i + k_f)\cdot x}$$

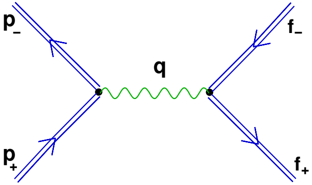
- momentum space vertex has contribution rk from external field

$$\gamma^e_\mu = (2\pi)^4 \int dr\ \bar{E}_{p_f}(r)\gamma_\mu E_{p_i}(r)\delta^4(p_f + k_f - p_i - rk)$$

Generic two vertex Furry picture S channel

$$M_{fi} = g_1 g_2 \int dr_1 dr_2 \; \bar{v}_{p_+} \gamma^{e\mu}_{p_- p_+} u_{p_-} \bar{\epsilon}_{f_+} \gamma^{e}_{f_- f_+ \mu} \epsilon_{f_-} \; \frac{\delta(F - I - (r_1 + r_2)(k_- + k_+))}{(I + (r_1 + r_2)(k_- + k_+))^2}$$

- usual coupling constants and spinors/polarisation
- modified (Furry) vertices γ^e
- r_1, r_2 momentum contribution from (two) external fields



$$\frac{|M_{fi}|^2}{VT} = (g_1 g_2)^2 \int dr_1 dr_2 \;\; \mathrm{Tr}[..r_1..r_2..] \frac{d\vec{f_-} d\vec{f_+}}{4\omega_{f_-}\omega_{f_+}} \frac{\delta(F - I - (r_1 + r_2)(k_- + k_+))}{(I + (r_1 + r_2)(k_- + k_+))^2}$$

Furry picture phase integrals

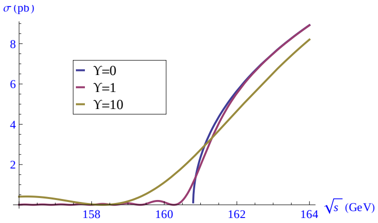
- usually, 6 integrations and 4 delta functions leaves $d\Omega$

$$\int dr_1 dr_2 \;\mathrm{Tr}(r_1, r_2) \frac{d\vec{f}_- d\vec{f}_+}{4\omega_{f_-}\omega_{f_+}} \frac{\delta(F - I - (r_1 + r_2)(k_- + k_+))}{(I + (r_1 + r_2)(k_- + k_+))^2}$$

- two extra integrations mean that process threshold is smeared

$$\int \frac{dr\, d\Omega}{8} \left[1 - \frac{m_f^2}{(E - r\,((\Upsilon_+ + \Upsilon_-)/E)^{1/3})^2}\right]^{1/2} \int dr_2 \;\mathrm{Tr}(r - r_2, r_2)$$

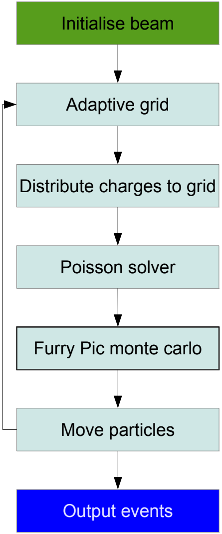
- eg. W pair production



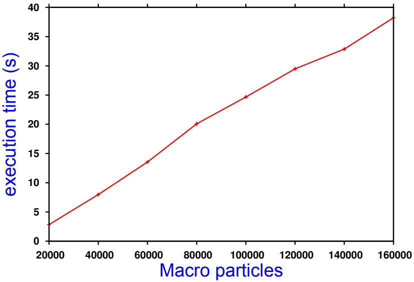
Requirements for a strong field event generator

REQUIREMENTS:	SOLUTION:
To simulate a charge bunch collision and calculate the field strength at each point of production →	A PIC code using an efficient method for modeling the electrodynamics – crosscheck with CAIN/GP
To have a finely scaled simulation in order to accurately model disruption, hour glass effect etc. →	(same as above)
To perform a relatively complex cross-section calculation at each point of production →	MPI using openMPI or GPU programming
To have full spin tracking →	T-BMT with higher order corrections to AMM, Sokolov-Ternov and higher order helicity amplitudes
To be flexible enough to include new higher order processes →	Allow new processes to be loaded externally

IPstrong - towards a strong field generator



- Fortran 2003 with openMPI (Fortran 2008 has inbuilt gpu)
- 3D electrostatic poisson solver (MPI)
- Furry picture processes replace all other processes
- output in multiple formats (stdhep, lcio)
- cross-checks with existing programs



Summary

- ILC/CLIC bunch fields are ”strong” as regards the Schwinger critical field
- only ”nuisance” pair backgrounds, have so far been (approximately) considered
- pair backgrounds can (should) be calculated with 1st and 2nd order Furry picture processes
- Such analysis can (should) be applied to all collider physics processes - particular precision spin processes
- further theoretical development needed - such as new Volkov-type solns
- threshold energies are smeared, cross-sections and loop corrections will vary
- A new EM solver/generic event generator, **IPstrong** is being developed