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Neutrino mass textures from String symmetries

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GREECE

based on:

I. Antoniadis & GKL, 1205.6930

and work in progress...

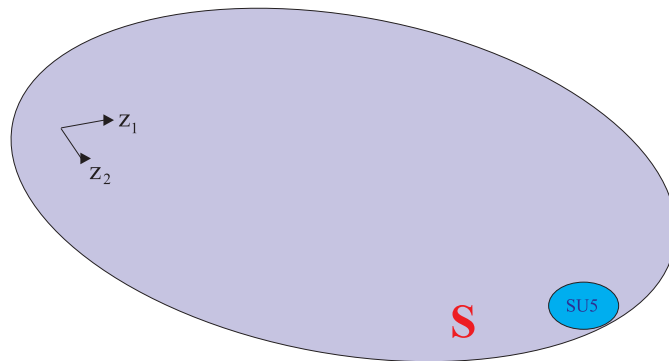
Outline of the Talk

- ▲ Basic ingredients of F-theory model building
- ▲ Mechanisms for fermion mass hierarchy
- ▲ Spectral cover description
- ▲ Classification of related **discrete** symmetries
- ▲ Examples of $SU(5)$ constructions
- ▲ Applications to neutrino physics

★ **F-theory** (*C. Vafa hep-th/9602022*)

- ▲ Defined on a background $\mathcal{R}^{3,1} \times \mathcal{X}$
- ▲ \mathcal{X} elliptically **fibered** **CY** 4-fold over B_3
- ▲ B_3 complex 3-fold base.

GUTs associated to **7-branes** wrapping certain class of ‘*internal*’
2-complex dim. surface S



F-theory: Model Building

▲ The precise gauge group is determined by the singular fibers over the surface \mathbf{S} .

▲ Elliptic Fibration: Highest singularity is E_8

▲ Gauge symmetry: any E_8 subgroup:

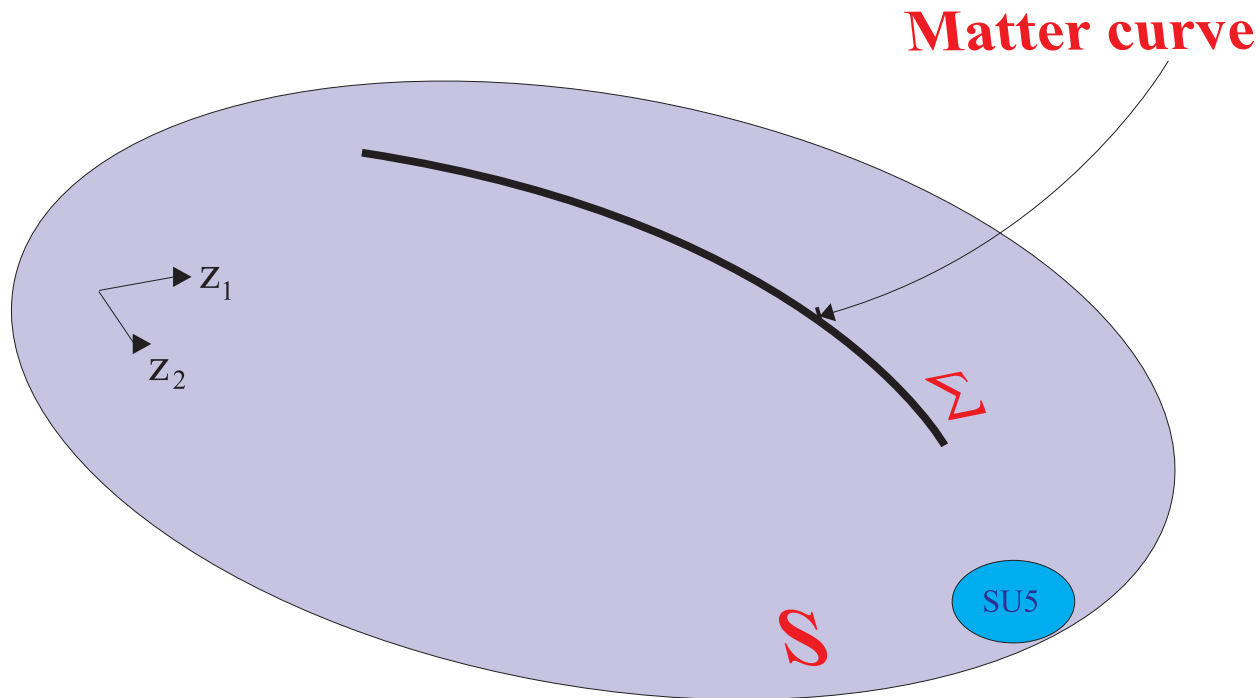
$$E_8 \rightarrow G_{GUT} \times C_{\text{spectral cover}}$$

★ **Spectral Cover** \Rightarrow useful local properties of G_{GUT}

▲ Sensible choice: $G_{GUT} = SU(5)$

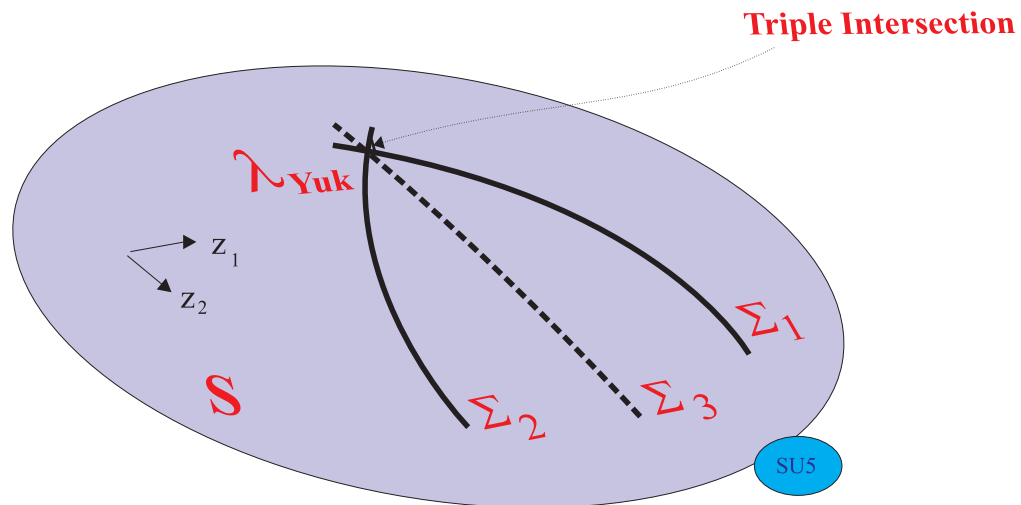
(a single condition $c_1(\mathcal{L}) \cdot c_1(\mathcal{L}) = -2$ ensures absence of exotics)

Matter is localised along intersections with other 7-branes...



Along a **matter curve** Σ gauge symmetry is **enhanced**...

Yukawa couplings are formed at triple intersections...

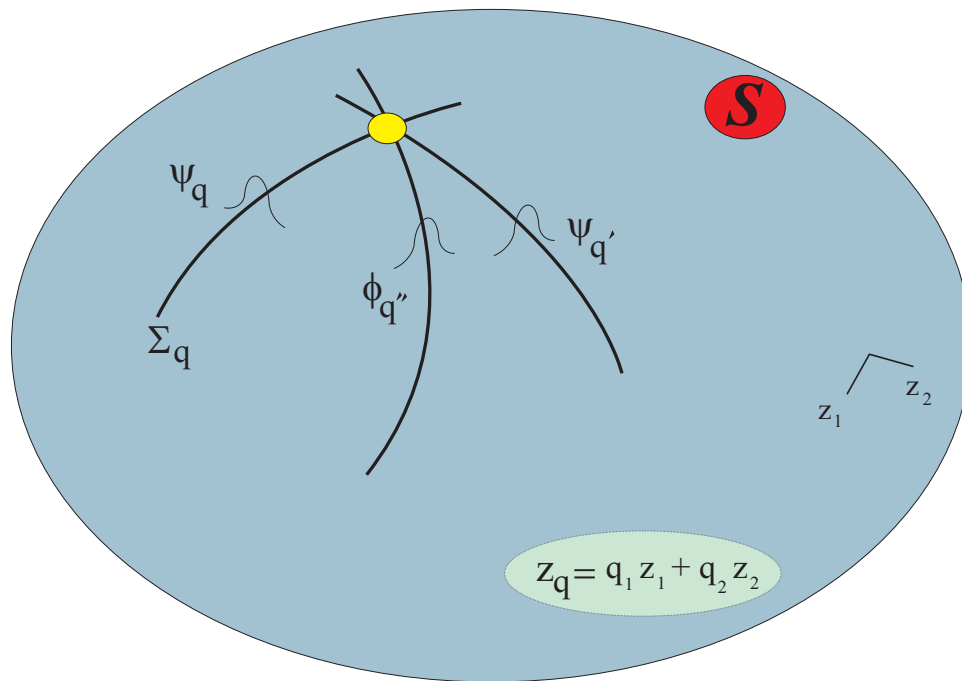


where gauge symmetry is further enhanced:

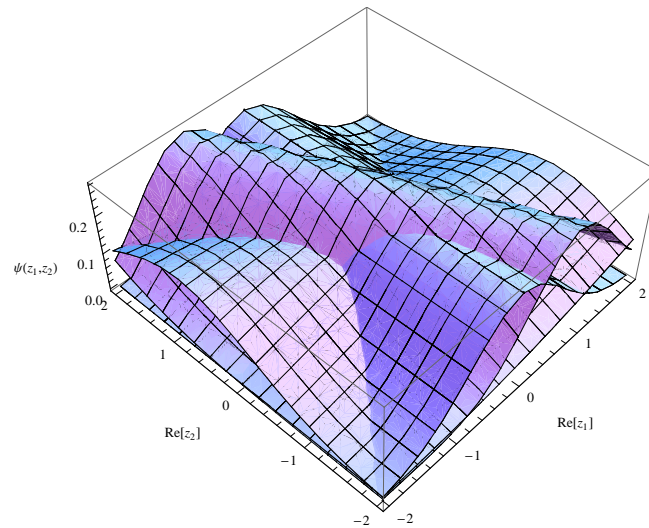
$$\lambda_b 10 \cdot \bar{5} \cdot \bar{5} \in SO(12)$$

$$\lambda_t 10 \cdot 10 \cdot 5 \in E_6$$

▲ Matter fields are represented by wavefunctions ψ_i, ϕ on the intersections of 7-branes with **S**.



Matter Wavefunctions fall off rapidly away from the curves



Yukawa coupling \propto integral of overlapping wavefunctions
at the **intersection**

$$\lambda_{ij} \sim \int_S \psi_U^j \psi_Q^i \psi_H$$

Integral's main dependence is on local details near the intersection

\Rightarrow **reliable λ_{ij} -estimation without knowing global geometry!**

Mechanisms for Fermion mass hierarchy

▼ If all three families are on the same matter curve, masses to lighter families can be generated by:

i) **non-commutative fluxes** *Cecotti et al, 0910.0477*

ii) **non-perturbative effects**, *Aparicio et al, 1104.2609*

▼ If families are distributed on different matter curves:

Implementation of **Froggatt-Nielsen mechanism**,

Dudas and Palti, 0912.0853

GKL and G.G. Ross, 1009.6000

▲▲ **Combined mechanism:**

Only two families on the same matter curve

★ **Origin and Nature of Family Symmetries** ★

In F-theory all matter descends from the E_8 -adjoint decomposition

We already assumed

$$E_8 \rightarrow SU(5)_{GUT} \times SU(5)_{\perp}$$

therefore

$$248 = (24, 1) + (1, 24_{\perp}) + (10, 5_{\perp}) + (\bar{5}, 10_{\perp}) + (5, \bar{10}_{\perp}) + (\bar{10}, \bar{5})_{\perp}$$

Interpretation from geometric point of view:

$SU(5)_{GUT}$ fields reside on **matter curves**:

$$\Sigma_{10_{t_i}} : n_{10} \times 10_{t_i} + \bar{n}_{\bar{10}} \times \bar{10}_{-t_i} \quad (1)$$

$$\Sigma_{5_{t_i+t_j}} : n_5 \times \bar{5}_{t_i+t_j} + \bar{n}_{\bar{5}} \times 5_{-t_i-t_j} \quad (2)$$

Families on different curves distinguished by roots $t_i, t_j \in SU(5)_{\perp}$

★ **Monodromies** reduce $SU(5)_\perp$ symmetry ★

Geometric equivalent description useful to local F-theory:

Spectral Cover Description:

★ *local patch around GUT singularity described by*

$$\mathcal{C}_5 = \prod_{i=1}^5 (s - t_i) = b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 = 0, \quad b_1 = 0$$

coefficients b_k ($\in \mathcal{F}$) carry information of geometry...

★ $SU(5)_\perp$ breaking corresponds to any of the possible splittings of the *Spectral Cover*:

$$\mathcal{C}_5 \rightarrow \mathcal{C}_4 \times \mathcal{C}_1$$

$$\mathcal{C}_5 \rightarrow \mathcal{C}_3 \times \mathcal{C}_2$$

...

★ Examples ★

▲ **Application:** The $\mathcal{C}_4 \times \mathcal{C}_1$ case

▲ **Motivation:** The neutrino sector

▲ $\mathcal{C}_4 \times \mathcal{C}_1$ implies the splitting of the polynomial in two factors

$$\sum b_k s^{5-k} = \underbrace{(a_1 + a_2 s + a_3 s^2 + a_4 s^3 + a_5 s^4)}_{\mathcal{C}_4} \underbrace{(a_6 + a_7 s)}_{\mathcal{C}_1}$$

Topological properties of a_i are fixed in terms of those of b_k , by equating coefficients of same powers of s

$$b_0 = a_5 a_7, \quad b_5 = a_1 a_6, \quad \text{etc...}$$

Moreover:

▲ \mathcal{C}_1 : associated to a $\mathcal{U}(1)$

▲ \mathcal{C}_4 : reduction to (i) continuous $SU(4)$ subgroup, or

(ii) to Galois group $\in S_4$ (see also Marsano et al 0906.4672)

Properties and Residual Spectral Cover Symmetry

▲ If $\mathcal{H} \in S_4$ the **Galois** group, final symmetry of the model is:

$$SU(5)_{GUT} \times \underbrace{\mathcal{H} \times U(1)}_{\text{family symmetry}}$$

▲ The final subgroup $\mathcal{H} \in S_4$ is linked to specific **topological** properties of the polynomial coefficients a_i .

▲ a_i coefficients determine useful properties of the model, such as

i) **Geometric** symmetries \rightarrow \mathcal{R} -symmetries

ii) **Flux** restrictions on the **matter curves**

▲ **Fluxes** determine useful properties on the **matter curves**

including :

Multiplicities and **Chirality** of matter/Higgs **representations**

Determining the **Galois** group in \mathcal{C}_4 -spectral cover

In order to find out which is the **Galois** group, we examine *partially symmetric* functions of roots t_i (*Lagrange method*)

1.) The Discriminant Δ

$$\Delta = \delta^2 \quad \text{where} \quad \delta = \prod_{i < j} (t_i - t_j)$$

δ is invariant under S_4 -**even** permutations $\Rightarrow A_4$

Δ symmetric \rightarrow can be expressed in terms of coefficients $a_i \in \mathcal{F}$

$$\Delta(t_i) \rightarrow \Delta(a_i)$$

If $\Delta = \delta^2$, such that $\delta(a_i) \in \mathcal{F}$, then

$$\mathcal{H} \subseteq A_4 \text{ or } V_4 \quad (= \text{Klein group})$$

If $\Delta \neq \delta^2$, (i.e. $\delta(a_i) \notin \mathcal{F}$), then

$$\mathcal{H} \subseteq S_4 \text{ or } \mathcal{D}_4$$

2.) To study possible reductions of S_4, A_4 to their subgroups, another partially symmetric function should be examined:

$$f(x) = (x - x_1)(x - x_2)(x - x_3)$$

$$x_1 = t_1t_2 + t_3t_4, \quad x_2 = t_1t_3 + t_2t_4, \quad x_3 = t_2t_3 + t_1t_4$$

$x_{1,2,3}$ are invariant under the three Dihedral groups $D_4 \in S_4$.

Combined results of Δ and $f(x)$:

	$\Delta \neq \delta^2$	$\Delta = \delta^2$
$f(x)$ irreducible	S_4	A_4
$f(x)$ reducible	D_4, Z_4	V_4

The induced restrictions on the coefficients a_i

1. Tracelessness condition $b_1 = 0$ demands

$$a_4 = a_0 a_6, \quad a_5 = -a_0 a_7$$

2. The requirement that the discriminant is a square $\Delta = \delta^2$ imposes the following relations among a_i :

$$a_1 = e_1^2, \quad a_0 a_7 = e_2^2, \quad a_2 a_6 \rightarrow \lambda a_1 a_7$$

3. Reducibility of the function $f(x)$ is achieved if

$$f(0) = 4a_5 a_3 a_1 - a_1 a_4^2 - a_5 a_2^2 = 0$$

▲ 1st Example : A_4

Gauge Symmetry: $SU(5)_{\text{GUT}} \times A_4 \times U(1)$

$F = (10, 3)$	t_a	$F = (Q, u^c, e^c)$
$F_x = (10, 1)$	t_s	$F_x = (Q, u^c, e^c)$
$\bar{F}_y = (\bar{10}, 1)_{-t_5}$	$-t_5$	$\bar{F} = (\bar{Q}, \bar{u}^c, \bar{e}^c)$
$H = (\bar{5}, 3)$	$t_s \pm t_a$	h_u
$\bar{f} = (\bar{5}, 3)_{+t_5}$	$\frac{1}{4}(t_s - t_a) + t_5$	$\bar{f}_i = (\ell, d^c)_i$
$\bar{H} = (\bar{5}, 1)_{+t_5}$	$\frac{1}{4}(t_s + 3t_a) + t_5$	\bar{h}_d
$\theta_a = (1, 3)$	0	θ_{ij}
$\theta_b = (1, 3)$	t_a	θ_{i4}
$\theta_c = (1, 3)_{-t_5}$	$\frac{1}{4}(t_s - t_a) - t_5$	θ_{i5}
$\theta' = (1, 1')_{-t_5}$		θ_{45}
$\theta'' = (1, 1'')_{+t_5}$		θ_{54}

Yukawa terms

1st choice: $3 \times (Q, u^c, e^c) \in F = (10, 3) \rightarrow$ tree-level coupling:

$$\mathcal{W}_u \supset (10, 3)_{t_i} (10, 3)_{t_i} (5, 3)_{-2t_i}$$

\rightarrow **Wrong** quark mass relations!

2nd choice: $3 \times (Q, u^c, e^c) \in F_x = (10, 1) \rightarrow$ fourth-order coupling:

$$\frac{1}{\Lambda} (10, 1) (10, 1) (5, 3) (1, 3) \leftrightarrow \lambda_t F_x F_x H \theta_b$$

In this case, lighter generations receive masses from non-commutative fluxes and/or non-perturbative effects

Neutrinos

$$\mathcal{W}_\nu \supset \frac{1}{\Lambda^3} (\bar{5}, 3)_{t_i+t_5} (\bar{5}, 3)_{t_i+t_5} (5, 3)_{-2t_i} (5, 3)_{-2t_i} \theta_{i5} \theta_{i5}$$

$\mathcal{F} - \mathcal{A}_4$ has a rich neutrino sector

Example

Take the vevs: $\langle \theta_{(1,3)} \rangle \sim a_i$, $\langle h_{(5,3)} \rangle \sim v_i$

$$\{a_1 \rightarrow 1, a_2 \rightarrow 0, a_3 \rightarrow 0, v_1 \rightarrow 0, v_3 \rightarrow v_2\}$$

$$m_\nu \propto \begin{pmatrix} 2 & 1c & 1c \\ 1c & 13 & -4c \\ 1c & -4c & 13 \end{pmatrix}$$

with c accounting for corrections (charged leptons, etc).

For $c = 2$ we get the right mixing, and the mass ratio

$\Delta m_{23}^2 / \Delta m_{13}^2 \sim 10$ close to the expected value.

▲ 2nd Example : $SU(5)_{GUT} \times Z_2 \times Z_2 \times U(1)$
 Spectral cover equation and field content:

$$C_5(s) = (a_3 s^2 + a_2 s + a_1) (a_6 s^2 + a_5 s + a_4) (a_7 + a_8 s)$$

$SU(5)$	$U(1)_Y$ -flux	$U(1)_X$	SM spectrum
$10_{t_{1,2}}^{(1)}$	0	2	$2 \times (Q, u^c, e^c)$
$10_{t_3}^{(2)}$	1	1	$(1 \times Q, -, 2 \times e^c)$
$10_{t_5}^{(3)}$	-1	0	$(-, 1 \times u^c, 1 \times \bar{e}^c)$
$5_{-t_1-t_2}^{(0)}$	0	1	$1 \times (d, h_u)$
$5_{-t_{1,2}-t_3}^{(1)}$	0	-1	$1 \times (d^c, \ell)$
$5_{-t_{1,2}-t_5}^{(2)}$	0	-1	$1 \times (d^c, \ell)$
$5_{-t_{3,4}-t_5}^{(3)}$	-1	0	$1 \times (h_d, -)$
$5_{-t_3-t_4}^{(4)}$	1	-2	$(2 \times d^c, 1 \times \ell)$

For the sake of simplicity, we drop Yukawa coefficients and write the *charged Lepton mass matrix*

$$m_\ell = \begin{pmatrix} a^2 & b & a \\ a & c & 1 \\ a & -c & 1 \end{pmatrix},$$

with *diagonalising matrix* ($a = \sqrt{2} \tan \phi$),

$$V_\ell = \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) \\ -\frac{\sin(\varphi)}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\cos(\varphi)}{\sqrt{2}} \\ -\frac{\sin(\varphi)}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\cos(\varphi)}{\sqrt{2}} \end{pmatrix} \quad (3)$$

The Neutrino Sector

Left handed neutrinos are in the following fiveplets

$$\nu_1 \in \bar{5}_{t_3+t_4}, \nu_2 \in \bar{5}_{t_1+t_5}, \nu_3 \in \bar{5}_{t_1+t_3}$$

Their Right Handed partners can be sought among **KK-modes** of the singlet fields θ_{ij}^{KK} (*Antoniadis et al hep-th/0210263*)

In $F - SU(5)$ however,

$$\theta_{ij}^{KK} \rightarrow \nu^c; \theta_{ji}^{KK} \rightarrow \bar{\nu}^c, \Rightarrow \nu^c \neq \bar{\nu}^c$$

Remarkably, due to the monodromies (*Vafa et al 0904.1419*)

$$\theta_{12}^{KK} \equiv \theta_{21}^{KK} \rightarrow \nu_a^c = \bar{\nu}_a^c, \theta_{34}^{KK} \equiv \theta_{43}^{KK} \rightarrow \bar{\nu}_b^c = \nu_b^c$$

A convenient arrangement on matter curves:

$$\nu_1^c = \nu_a^c, \nu_2^c = \nu_b^c, \nu_3^c = \nu_b^c$$

★ The effective neutrino mass matrix

$$m_{\nu}^{eff} = m_{\nu D} M_R^{-1} m_{\nu D}^T$$

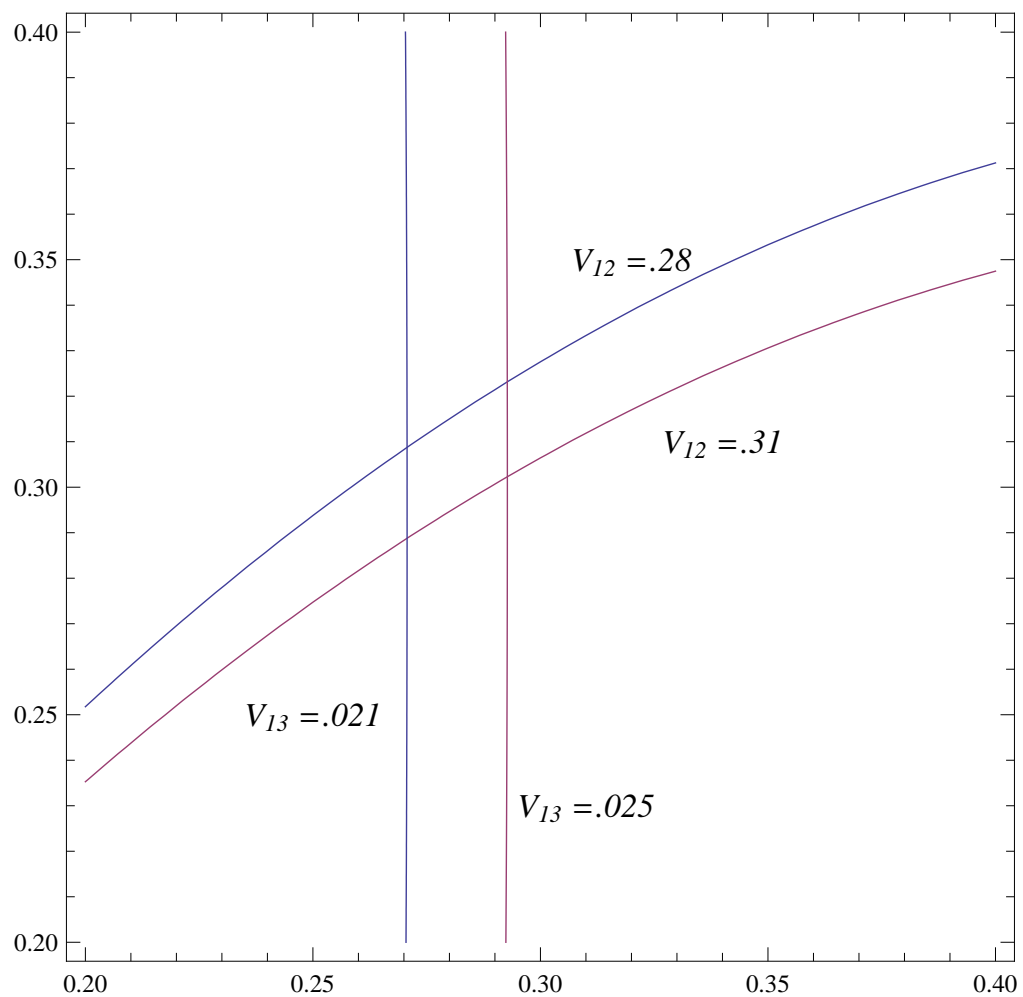
Assumptions:

- ▲ Kaluza-Klein scale \sim GUT scale, $M_{KK} \sim M_X$
- ▲ Singlet vevs $\frac{\langle \theta_{ij} \rangle}{M_X} \rightarrow a, b, c$, such that $r = \frac{b}{a} < a, c < 1$

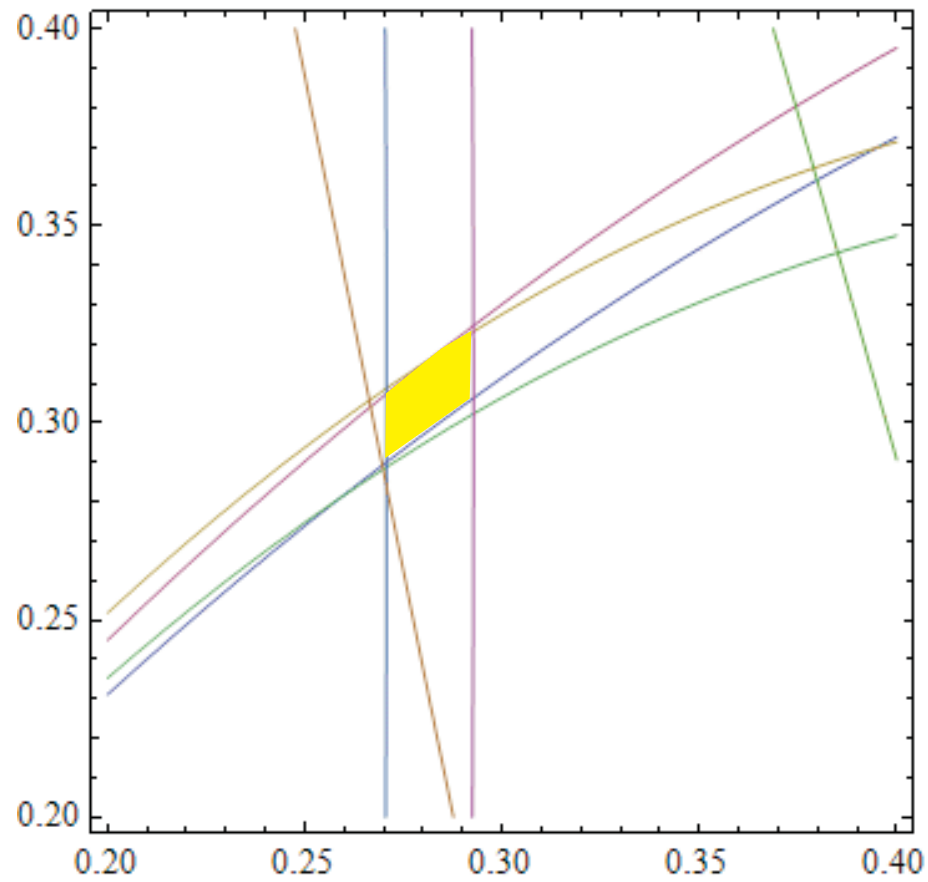
$$m_{\nu}^{eff} \approx \begin{pmatrix} 2a^2 & a(c+r) & a \\ a(c+r) & c^2 + r^2 & r \\ a & r & 1 \end{pmatrix} \frac{m_0^2}{M_X}$$

- To leading order, Mixing effects are linked to singlet vevs a, b, c
- ▲ Consistency check: mixing $(V_{\nu})_{ij}$ should be derived for $a, b, c < 1$

Most **stringent** restrictions arise from V_{12}, V_{13} elements (*case $b = 0$*)



(a, c) restricted region, from all V_{ij} elements (*only case $b = 0$*)



A few remarks

★ Current **F-Theory** Models provide a dictionary between:

Manifold Singularity \Leftrightarrow **Gauge Symmetry** G_{GUT}

$$\mathcal{E}_8 \rightarrow \mathbf{G}_{GUT} \times \mathcal{C}_{\text{spectral cover}}$$

Spectral Cover provides **Additional Structure beyond GUTs**



★ $\mathcal{C}_{\text{spectral cover}} \rightarrow$ *Finite Groups such as* $A_4, D_4, V_4 = Z_2 \times Z_2$

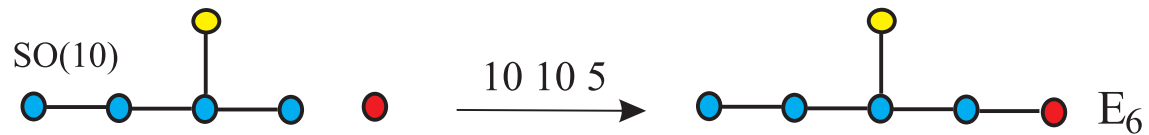
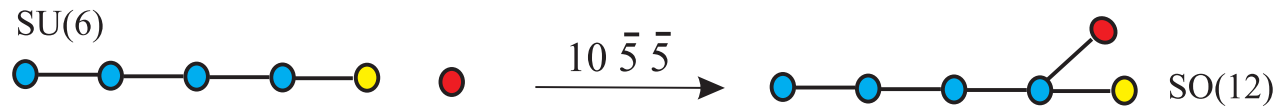
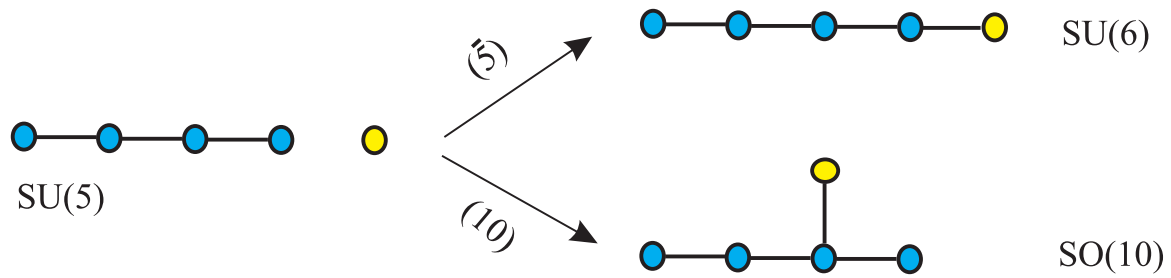


★ A natural way to interpret the peculiar **Neutrino properties**

*This way, the **neutrino physics** is linked directly to the topological properties of the internal manifold*

Supporting Material...

Symmetry enhancements for $SU(5)$.



Matter Parity from Geometry?

topological properties are encoded in b_k coefficients

Consider the phase transformation (*Hayashi et al, 0910:2762*)

$$s \rightarrow s e^{i\phi}, \quad b_k \rightarrow b_k e^{i(\xi - (6-k)\phi)}$$

Let's apply to $SU(5)$ case:

...spectral cover equation picks up an overall phase

$$\mathcal{C}_5 : \sum_k b_k s^{5-k} \rightarrow e^{i(\xi - \phi)} \sum_k b_k s^{5-k}$$

★ Z_2 -parity: $\phi = \pi$:

$$s \rightarrow -s, \quad b_k \rightarrow (-1)^k e^{i\xi} b_k$$

★ Communicating **Matter Parity** to **Matter Curves** ★

Example: Consider relations in \mathcal{Z}_2 monodromy:

$$b_k = \sum a_l a_m a_n, \quad l + m + n = N - k, \quad N = 17 \quad (4)$$

Choose a_n to transform as

$$a_n \rightarrow a_n e^{i(\zeta - n\phi)}$$

$$\rightarrow b_k \propto a_l a_m a_n \rightarrow a_l a_m a_n e^{3\zeta - (N-k)\phi} \quad (5)$$

(4) & (5) consistent for $\xi = \phi = \pi, \zeta = 0,$

$$a_n \rightarrow (-1)^n a_n$$

★ Implications on matter curves:

$$5^{(0)} \sim a_6 a_7 \rightarrow (-1)^{(6+7)} = (-)$$

... associate this to matter parity!

$$\begin{pmatrix} 2a^4 & a^2(b+ac) & a^3 \\ a^2(b+ac) & \frac{-2bcM_X a^3 + c^2 M a^2 + b^2 M}{M - a^2 M_X} & \frac{abM - a^4 c M_X}{M - a^2 M_X} \\ a^3 & \frac{abM - a^4 c M_X}{M - a^2 M_X} & \frac{a^2 M}{M - a^2 M_X} \end{pmatrix} m_{\nu_0}$$