Moduli Stabilization and Cosmology with Poly-Instanton Corrections

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Ralph Blumenhagen, Xin Gao, Thorsten Rahn and PS:
JHEP 1206 (2012) 162 ; JHEP 1211 (2012) 101.

Xin Gao and PS: JHEP 1303 (2013) 061,

Some General Motivations

Requirements for (semi)realistic inflationary model building in string theoretic framework

- Moduli stabilization (and AdS to dS uplifting)
- Looking for available flat-directions
- UV sensitivity: η problem (protecting flatness against higher order operators)
- Consistent realization of cosmological observables from the point of view of present/future experimental constraints;
  - No. of e-foldings, $N_e \sim \mathcal{O}(60)$
  - Almost scale invariant power spectrum, $n_S \sim 1$
  - Signatures of non-Gaussianities, $f_{NL} \sim \mathcal{O}(1)$
  - tensor-to-scalar ratio, $r > 0.01$, Very Hard !

More-involed issues

- Post-inflationary aspects, e.g. reheating, moduli thermalizations
- Finite temperature corrections
- Dark Energy and Dark Matter

One has to be more careful ! (local/global effects interplay)

- Challenges in moduli stabilization
  - Chirality conflict, Tadpoles, open string moduli, ...
- Challenges in (anti-$D3$ brane) uplifting mechanism
- Challenges in post inflationary regime

On top of all: EFT is a valid description ? ; Control over (un)known α' and g_s corrections ? .

Setup & Plan

We proceed with

- Type IIB compactified on an orientifold of a swiss-cheese Calabi Yau threefold (with $O3/O7$ planes)
- Inclusion of perturbative α' corrections to the Kähler potential and the standard non-perturbative superpotential $\longrightarrow$ LARGE volume scenarios
- Inclusion of poly-instanton corrections on top of the standard non-perturbative superpotential contribution.

The Plan of the talk is threefold:

- Moduli Stabilization
- A new class of (single-field) Kähler moduli inflation
- Roulette poly-instanton inflation and the possibilities of realizing large f_{NL} values in beyond slow-roll regime.

Poly-instanton corrections

- Correction of an Euclidian D-brane instanton action S_a by the presence of other D-brane instantons b, gives a superpotential [Blumenhagen, Schmidt-Sommerfeld' 08]

$$W_{\rm np} = A_a \exp\left(-S_a + A_b\, e^{-S_b}\right).$$

- To have such a correction in type IIB CY orientifolds, the zero-mode analysis shows that the instanton b has to wrap a so-called 'Wilson' divisor (W) inside the CY which is a $\mathbb{P}^1$ fibration over $\mathbb{T}^2$ [Blumenhagen, Gao, Rahn, PS' 12].
- The equivariant co-homology of this divisor is such that $h^{1,0}(W) = h^{1,0}_+(W/\sigma) = 1$ and volume of this W divisor appear in a peculiar way in the expression of CY volume.
- Zero modes of a and b are different; good for moduli stabilization.

Minimal poly-instanton framework

Let us start with the simplest poly-instanton motivated ansatz for the Kähler and superpotential

$$K = -2\,\ln\left(\mathcal{V}(T_\alpha) + C_{\alpha'}\right)\ ,$$

$$W = W_0 + A_s e^{-a_s T_s} + A_s A_w\, e^{-a_s T_s - a_w T_w} + ...$$

$$\mathcal{V}(T_\alpha) = \xi_b (T_b + \bar{T}_b)^{\frac{3}{2}} - \xi_s (T_s + \bar{T}_s)^{\frac{3}{2}} - \xi_{sw}\big((T_s + \bar{T}_s) + (T_w + \bar{T}_w)\big)^{\frac{3}{2}}$$

where $C_{\alpha'}$ is a shift in overall CY volume due to the perturbative ${\alpha'}^3$-corrections [Becker^2, Haack, Louis ' 02].

$$C_{\alpha'} = -\frac{\chi(\mathcal{M})\,(\tau - \bar{\tau})^{\frac{3}{2}}\zeta(3)}{4(2\pi)^3\,(2i)^{\frac{3}{2}}}$$

The leading contributions to F-term scalar potential are

$$V(\mathcal{V}, \tau_s, \tau_w;\ \rho_s, \rho_w) = V_{\alpha'} + V_{\rm np1} + V_{\rm np2}$$

Moduli Stabilization

We follow a three step moduli stabilization.

- In first step, the complex structure moduli along with axion-dilaton are stabilized by the GVW superpotential at leading order.
- In second step, some of the Kähler moduli are stabilized in the absence of poly-instanton corrections [Balasubramanian, Berglund, Conlon, Quevedo ' 05],

$$V^{\rm LVS} = \frac{3\,\mathcal{C}_{\alpha'}\,|W_0|^2}{2\,\mathcal{V}^3} + \frac{4\,a_s\,A_s\,e^{-a_s\tau_s}\,\tau_s\cos(a_s\rho_s)W_0}{\mathcal{V}^2} + \frac{2\sqrt{2}\,a_s^2\,A_s^2\,e^{-2a_s\tau_s}\sqrt{\tau_s}}{3\,\xi_s\,\mathcal{V}}$$

$$\text{Standard LVS}: \; a_s\overline{\rho}_s = N\pi, \qquad \overline{\tau}_s \sim \mathcal{C}_{\alpha'}^{2/3} \sim \mathcal{O}\left(\frac{1}{g_s}\right), \qquad \overline{\mathcal{V}} \sim |W_0|\,e^{a_s\overline{\tau}_s}$$

- The τ_w direction corresponding to the Wilson divisor volume modulus still remains flat and is stabilized by sub-leading poly-instanton effects in third step [Blumenhagen, Gao, Rahn, PS'12], [Lüst, Zhang'13].

Moduli stabilization continued...

- After stabilizing the heavier volume moduli (and axions) at their respective minimum, we have

$$V(\tau_w) = V^{\rm LVS}(\overline{\mathcal{V}}, \overline{\tau}_s, \overline{\rho}_s) + e^{-a_w \tau_w}\left(\gamma_1 + \gamma_2\, \tau_w\right)$$

$$\text{where } \gamma_1 = \frac{\gamma_2\, \overline{\tau}_s\left(a_s(3+4a_w\overline{\tau}_s) - 4a_w\right)}{a_w(-1+4a_s\overline{\tau}_s)} \sim \mathcal{O}\left(\frac{1}{\overline{\mathcal{V}}^3}\right)$$

$$\gamma_2 = \frac{24\sqrt{2}\, A_w\, a_w\, \xi_s\, |W_0|^2 \sqrt{\overline{\tau}_s}(-1+a_s\overline{\tau}_s)}{a_s\, \overline{\mathcal{V}}^3\, (1-4a_s\overline{\tau}_s)} \sim \mathcal{O}\left(\frac{1}{\overline{\mathcal{V}}^3}\right).$$

- The minimization conditions for τ_w boil down to

$$\partial_{\tau_w} V(\tau_w)|_{\overline{\tau}_w} = 0 \Rightarrow a_w\overline{\tau}_w = 1 + \frac{\overline{\tau}_s\left(a_s(3+4a_w\overline{\tau}_s) - 4a_w\right)}{1-4a_s\overline{\tau}_s} \Rightarrow \overline{\tau}_w < 1$$

$$\partial^2_{\tau_w} V(\tau_w)|_{\overline{\tau}_w} = \frac{24\sqrt{2}a_w^2\, A_w\, \xi_s\, |W_0|^2}{a_s\, \overline{\mathcal{V}}^3\, (-1+4a_s\overline{\tau}_s)} \sqrt{\overline{\tau}_s}(-1+a_s\overline{\tau}_s)\, e^{-a_w\overline{\tau}_w} > 0$$

Racetrack poly-instanton framework

- The poly-instanton corrected racetrack ansatz for superpotential is,

$$W = W_0 + A_s\, e^{-a_s T_s} + A_s\, A_w\, e^{-a_s T_s - a_w T_w}$$
$$- B_s\, e^{-b_s T_s} - B_s\, B_w\, e^{-b_s T_s - b_w T_w}$$

- Following the same strategy of minimal framework, the scalar potential can be written again in terms of three types of contributions, and two step moduli stabilization for Kähler moduli are followed.
- The scalar potential in the absence of poly-instanton effects has a flat direction τ_w and is given as,

$$\mathbf{V}(\mathcal{V}, \tau_s, \rho_s) \simeq \mathbf{V}_{\alpha'}(\mathcal{V}) + \mathbf{V}_{\rm np1}(\mathcal{V}, \tau_s, \rho_s) + \mathbf{V}_{\rm np2}(\mathcal{V}, \tau_s, \rho_s)\,,$$

Revisiting Moduli Stabilization

- After stabilizing the heavier volume moduli and axions at their respective minimum, the potential for modulus τ_w becomes,

$$\mathbf{V}(\tau_w) = V_0 + e^{-a_w \tau_w} \left(\mu_1 + \mu_2\, \tau_w\right)$$

where V_0, μ_1, μ_2 are constants crucially depending on model dependent parameters.

- The minimization condition $\partial_{\tau_w} \mathbf{V}(\tau_w)|_{\overline{\tau}_w} = 0$ demands

$$a_w \overline{\tau}_w = 1 - a_w \frac{\mu_1}{\mu_2}\,, \text{i.e. } \tau_w > 1 \text{ would need } \frac{\mu_1}{\mu_2} < 0$$

$$\partial^2_{\tau_w} \mathbf{V}(\tau_w)|_{\overline{\tau}_w} = -a_w\, \mu_2\, \exp\left(-1 + \frac{a_w\, \mu_1}{\mu_2}\right) > 0, \;\; \text{i.e. } \mu_2 < 0.$$

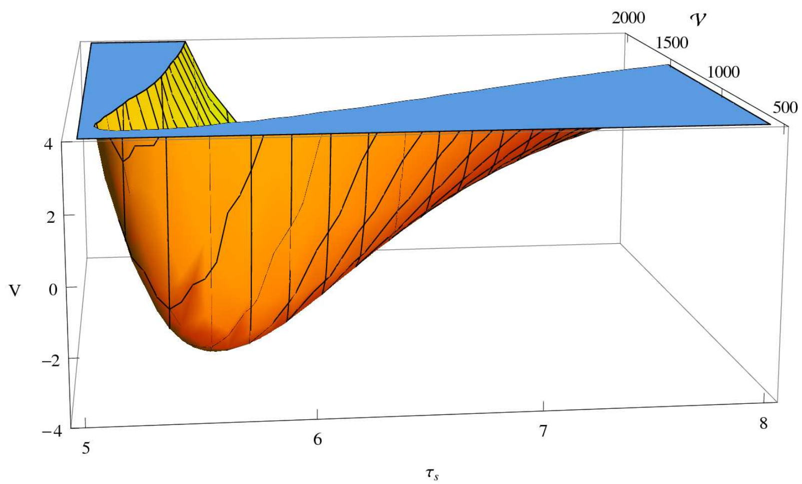
Fitting of parameters

	A_s	B_s	A_w	B_w	a_s	b_s	g_s	W_0	$\frac{\mu_1}{10^9}$	$\frac{\mu_2}{\mu_1}$	$\frac{V_0}{\mu_1}$	p
$\mathcal{B}_1$	3	2	0.5	1.749	$\frac{2\pi}{7}$	$\frac{2\pi}{6}$	0.12	-20	30	-0.7	-11.4	1.84
$\mathcal{B}_2$	6	0.5	0.1	4.771	$\frac{2\pi}{6}$	$\frac{2\pi}{5}$	0.10	-10	$.58$	-1.0	-117	1.72
$\mathcal{B}_3$	8	0.8	0.1	6.5075	$\frac{2\pi}{5}$	$\frac{2\pi}{4}$	0.11	-10	$.51$	-1.0	-85.0	1.61
$\mathcal{B}_4$	8	0.8	0.1	17.143	$\frac{2\pi}{4}$	$\frac{2\pi}{3}$	0.12	-5	$.11$	-1.0	-62.1	1.52

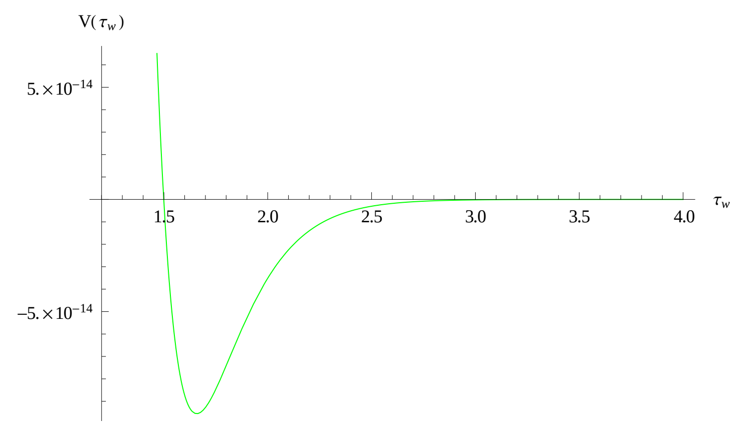
	$\overline{\tau}_s$	$\overline{\tau}_w$	$\overline{\mathcal{V}}$
$\mathcal{B}_1$	5.684	1.658	905.1
$\mathcal{B}_2$	6.607	1.129	941.0
$\mathcal{B}_3$	5.977	1.161	1015.3
$\mathcal{B}_4$	5.440	1.124	1093.2

All moduli stabilized within geometric regime along with sufficiently large CY volume !

The Scalar Potential



The scalar potential $V(\mathcal{V}, \tau_s)$ $(\times 10^7)$ vs $\mathcal{V}$ and τ_s for $\mathcal{B}_1$ without poly-instanton effects.



The effective scalar potential $V(\tau_w)$ at stabilized value of the heavier moduli for $\mathcal{B}_1$.

Moduli Mass Hierarchy and Inflation

- In LARGE volume limit, the scaling in moduli masses are

$$M_{\chi_1} \sim \mathcal{O}(1)\,\frac{M_p}{\mathcal{V}^{\frac{3}{2}}}\,,\; M_{\chi_2} \sim \mathcal{O}(1)\,\frac{M_p}{\mathcal{V}}\,,\; M_{\chi_3} \sim \mathcal{O}(1)\,\frac{M_p}{\mathcal{V}^{\frac{2+p}{2}}}$$

$$\mathcal{V} \sim \exp\Big(\sqrt{\tfrac{3}{2}}\,\chi_1\Big)\,,\;\; \tau_s \sim \exp\Big(\sqrt{\tfrac{2}{3}}\,\chi_1\Big)\,\chi_2^{\frac{4}{3}}\,,\;\; \tau_w + \tau_s \sim \exp\Big(\sqrt{\tfrac{2}{3}}\,\chi_1\Big)\,\chi_3^{\frac{4}{3}}\,.$$

For all benchmark models $\mathcal{B}_i$, we have $1.5 < p < 1.8$.

- Now shifting τ_w (i.e. χ_3) away from its minimum (and after considering a suitable uplifting), one gets,

$$V_{\rm inf}(\chi) \simeq -\frac{g_s\, e^{K_{\rm CS}}\, \mu_2\, e^{-a_w \overline{\tau}_w}}{8\pi a_w}\left[1 - \left(1 + \frac{\overline{\mathcal{V}}^{\frac{2}{3}}\, a_w}{\lambda_{sw}}\left(\chi^{\frac{4}{3}} - \overline{\chi}^{\frac{4}{3}}\right)\right)\right.$$
$$\left.\times \exp\left(-\frac{\overline{\mathcal{V}}^{\frac{2}{3}}\, a_w}{\lambda_{sw}}\left(\chi^{\frac{4}{3}} - \overline{\chi}^{\frac{4}{3}}\right)\right)\right]$$

where $\hat{\tau}_w = \tau_w - \overline{\tau}_w$.

Cosmological observables-I

- In the large volume limit, the slow-roll inflationary conditions ($\epsilon \ll 1, \eta \ll 1$) are easily satisfied, and

$$\epsilon \ll |\eta| \sim \frac{1}{N_e},$$

	$\hat{\chi}^{\rm end}$ ($\times 10^{-2}\ M_p$)	$\hat{\chi}^*$	ϵ ($\times 10^{-10}$)	η ($\times 10^{-2}$)	N_e	$M_{\rm inf}$ ($\times 10^{14}$ GeV)
$\mathcal{B}_1$	1.29	3.08	3.2	−1.644	60.84	3.5
$\mathcal{B}_2$	1.26	3.00	2.9	−1.607	62.21	3.2
$\mathcal{B}_3$	1.24	2.95	2.6	−1.544	64.79	3.0
$\mathcal{B}_4$	1.22	2.90	2.4	−1.532	65.27	2.2

The table shows that our model is a small field and high scale inflationary model.

Cosmological observables-II

	n_S	$\mathcal{A}_{\rm COBE}$	r	M_{τ_s}	$M_{\mathcal{V}}$ ($\times 10^{14}$ GeV)	M_{τ_w}
$\mathcal{B}_1$	0.9671	7.1×10^{-7}	4.0×10^{-9}	297	3.49	0.28
$\mathcal{B}_2$	0.9679	5.4×10^{-7}	3.6×10^{-9}	201	1.48	0.19
$\mathcal{B}_3$	0.9691	4.7×10^{-7}	3.2×10^{-9}	214	1.23	0.12
$\mathcal{B}_4$	0.9694	1.5×10^{-7}	3.0×10^{-9}	120	0.52	0.03

- The number of e-foldings N_e, the scalar perturbation amplitudes $A_{\rm COBE}$ and the spectral index n_S are in perfect agreement with observational constraints.
- All tensor mode signatures (r, A_T, n_T) are negligibly small in this model.

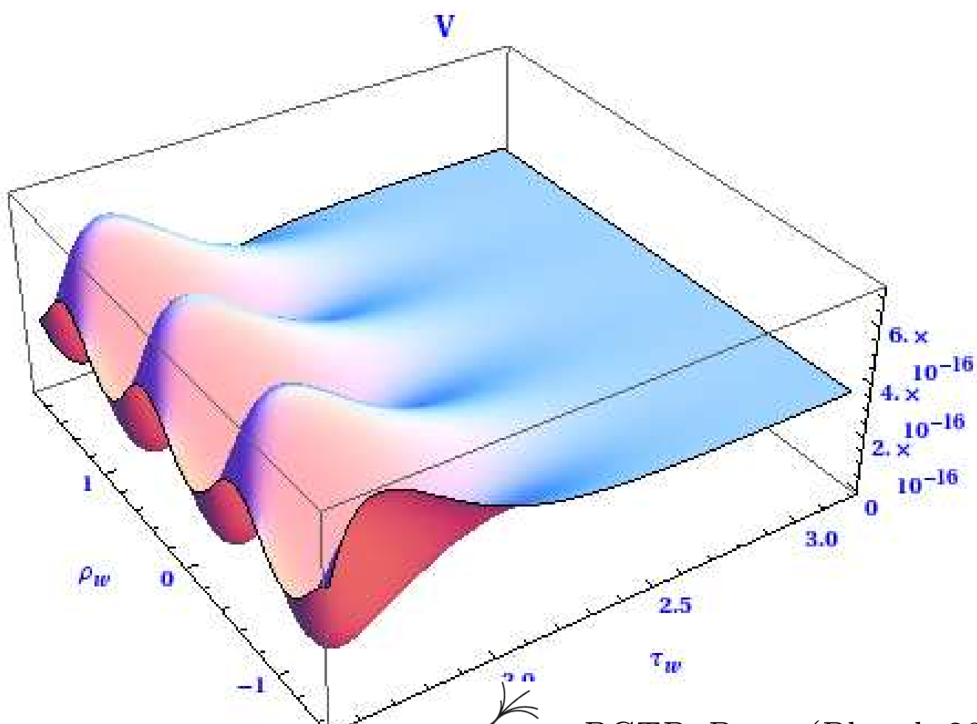
Two-field Model

The uplifted two-field potential is [Gao, PS' 13]

$$V_{\rm inf}(\phi_1,\phi_2) \simeq -\frac{\mu_2\, e^{-1+\frac{a_w\,\mu_1}{\mu_2}}}{a_w} + e^{-a_w\phi_1}\,(\mu_1+\mu_2\,\phi_1)\,\cos(a_w\phi_2)$$

For one set of sampling parameters:

$\mu_1 = 2.9\times10^{-8}$, $\mu_2 = -1.9\times10^{-8}$, $a_w = 2\pi, g_s = 0.12$, $\overline{\mathcal{V}} = 905, \overline{\tau}_s = 5.7, \xi_{sw} = 1/(6\sqrt{2})$



Roulette Poly-Instanton Inflation

The Evolution of inflationary trajectories on a non-flat background are governed by

$$\frac{d^2}{dN^2}\phi^a + \Gamma^a{}_{bc}\frac{d\phi^b}{dN}\frac{d\phi^c}{dN} + \left(3 + \frac{1}{H}\frac{dH}{dN}\right)\frac{d\phi^a}{dN} + \frac{\mathcal{G}^{ab}\partial_b V}{H^2} = 0,$$
$$H^2 = \frac{1}{3}\left(V(\phi^a) + \frac{1}{2}H^2\,\mathcal{G}_{ab}\frac{d\phi^a}{dN}\frac{d\phi^b}{dN}\right).$$

The field redefinitions [Yokoyama, Suyama, Tanaka'07]

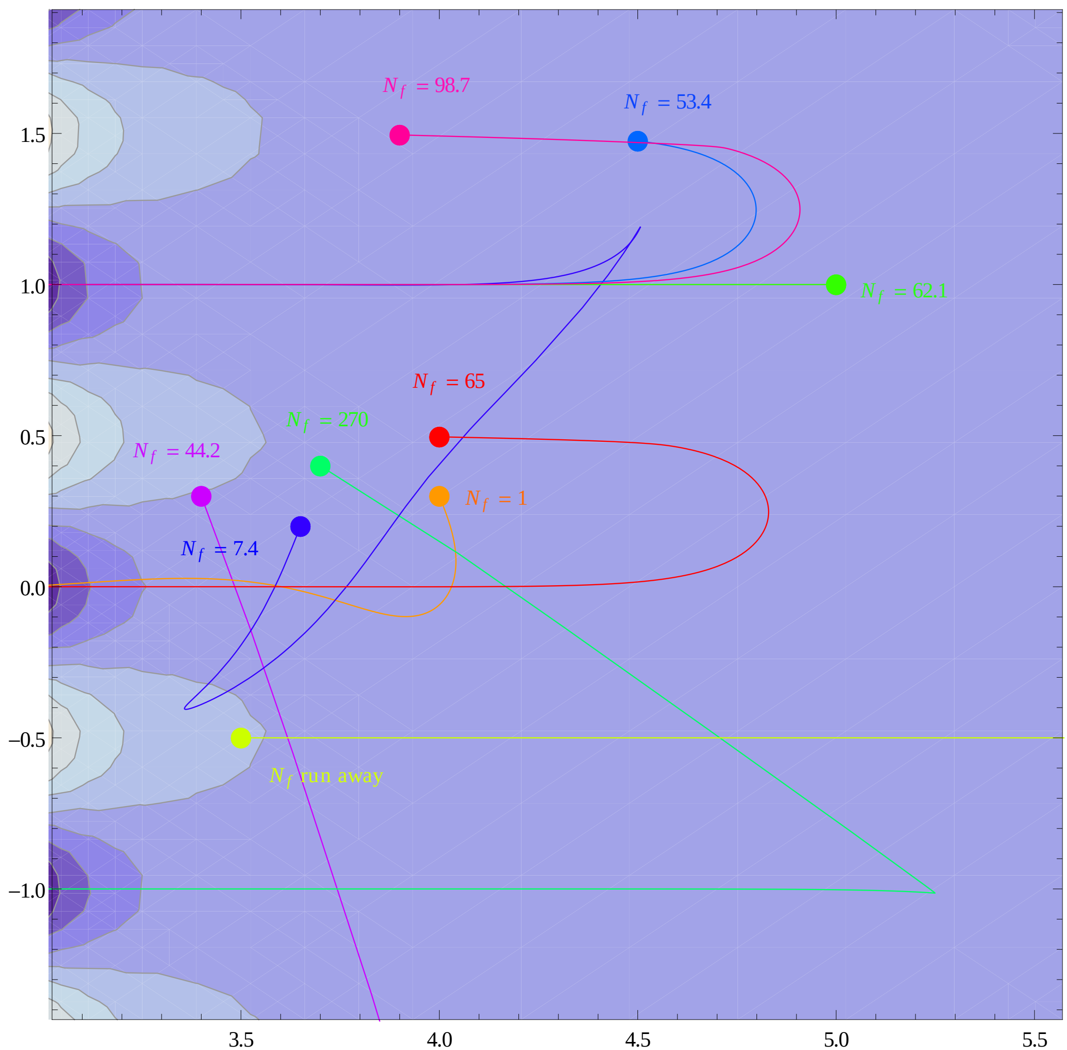
$$\varphi_1^a \equiv \phi^a, \qquad \varphi_2^a \equiv \dot{\phi}^a = \left(\frac{d\phi^a}{dt}\right), \quad \text{where } a = 1,\, 2.$$

translates the second-order background equations of motions into two first-order ODEs as follows

$$F_1^a \equiv \frac{d\varphi_1^a}{dN} = \left(\frac{d\phi^a}{dN}\right) = \frac{\varphi_2^a}{H},\; F_2^a \equiv \frac{D\varphi_2^a}{dN} = -3\varphi_2^a - \mathcal{G}^{ab}\frac{V_b}{H}$$
$$\text{subject to}: \; H^2 = \frac{1}{3}\left(V + \frac{1}{2}\,\mathcal{G}_{ab}\varphi_2^a\varphi_2^b\right);\; D\varphi_2^a = d\varphi_2^a + \Gamma^a{}_{bc}\varphi_2^b d\varphi_1^c$$

Evolution of inflationary trajectories

Similar to [Bond, Kofman, Prokushkin, Vaudrevange'06], we have four classes of inflationary trajectories:



Estimates for Non-Gaussinaities

Following the strategy of [Yokoyama, Suyama, Tanaka'07] based on δN formlism, we compute the non-linear parameters $f_{NL}, \tau_{NL}, g_{NL}$ in slow-roll regime and beyond.

$$f_{NL} = \frac{5}{6}\,\frac{1}{(N^*_{\mathcal{D}}\,\Theta^{\mathcal{D}}_*)^2}\left[N^F_{\mathcal{A}\mathcal{B}}\,\Theta^{\mathcal{A}}_F\,\Theta^{\mathcal{B}}_F + \int_{N_*}^{N_F} N_{\mathcal{A}}\,Q^{\mathcal{A}}_{\mathcal{B}\mathcal{C}}\,\Theta^{\mathcal{B}}\,\Theta^{\mathcal{C}}\,dN\right]$$

$$\tau_{NL} = \frac{1}{(N^*_{\mathcal{D}}\,\Theta^{\mathcal{D}}_*)^3}\left[A^{\mathcal{A}\mathcal{B}}\,\Omega_{\mathcal{A}}(N_*)\,\Omega_{\mathcal{B}}(N_*)\right]$$

$$g_{NL} = \frac{25}{54}\,\frac{1}{(N^*_{\mathcal{D}}\,\Theta^{\mathcal{D}}_*)^3}\left[N^F_{\mathcal{A}\mathcal{B}\mathcal{C}}\,\Theta^{\mathcal{A}}_F\,\Theta^{\mathcal{B}}_F\,\Theta^{\mathcal{C}}_F + 3\int_{N_*}^{N_F} \Omega_{\mathcal{A}}\,Q^{\mathcal{A}}_{\mathcal{B}\mathcal{C}}\,\Theta^{\mathcal{B}}\,\Theta^{\mathcal{C}}\,dN \right.$$
$$\left. + \int_{N_*}^{N_F} N_{\mathcal{A}}\,\mathcal{Q}^{\mathcal{A}}_{\mathcal{B}\mathcal{C}\mathcal{D}}\,\Theta^{\mathcal{B}}\,\Theta^{\mathcal{C}}\,\Theta^{\mathcal{D}}\,dN\right]$$

- All the quantities except $N_{\mathcal{A}}(N), \Theta^{\mathcal{A}}(N)$ and $\Omega_{\mathcal{A}}(N)$ are known in terms of various derivatives of potential and subsequent solutions of trajectory evolution equations.

Sketch of $f_{NL}, \tau_{NL}, g_{NL}$ computations

The intermediate quantities $\Theta^{\mathcal{A}}(N)$, $N_{\mathcal{A}}(N)$, $\Omega_{\mathcal{A}}(N)$ can be obtained by solving the following set of first order ODEs,

$$\frac{D}{dN} N_{\mathcal{A}}(N) = -P^{\mathcal{A}}_{\mathcal{B}}(N)\, N_{\mathcal{B}}(N),$$

$$\frac{D}{dN} \Theta^{\mathcal{A}}(N) = P^{\mathcal{A}}_{\mathcal{B}}(N)\, \Theta^{\mathcal{B}}(N),$$

$$\frac{D}{dN} \Omega_{\mathcal{A}}(N) = -\Omega_{\mathcal{B}}(N)\, P^{\mathcal{B}}{}_{\mathcal{A}}(N) - N_{\mathcal{B}}(N)\, Q^{\mathcal{B}}{}_{\mathcal{AC}}(N)\, \Theta^{\mathcal{C}}(N).$$

$$BCs : N_{\mathcal{A}}(N_F) = N^F_{\mathcal{A}},\ \Theta^{\mathcal{A}}(N_*) = A^{\mathcal{AB}}(N_*)\, N_{\mathcal{B}}(N_*),\ \Omega_{\mathcal{A}}(N_F) = N^F_{\mathcal{AB}}\, \Theta^{\mathcal{B}}(N_F)$$

- Here, quantities $N^F_{\mathcal{A}}, P^{\mathcal{A}}_{\mathcal{B}}(N), N^F_{\mathcal{A}}, N^F_{\mathcal{AB}}, A^{\mathcal{AB}}(N)$ are known from various derivatives of potential and subsequent solutions of trajectory evolutions.
- The problem simplifies into solving first order ODEs.
- However, it is not sufficient enough to proceed analytically for a given generic multi-field potential.

Numerical Estimates

Slow-roll regime:

τ_w	ρ_w	N_e	$\lvert f_{NL}\rvert$	g_{NL}	τ_{NL}
3.9	1.495	96.51	0.020	-0.010	6.26×10^{-4}
4	0.496	62.60	0.030	-0.009	1.35×10^{-3}
4.5	1.475	51.17	0.035	-0.010	1.89×10^{-3}
5	1	60	0.017	-0.0097	4.54×10^{-4}

Beyond slow-roll regime:

τ_w	ρ_w	N_e	$\lvert f_{NL}\rvert$	g_{NL}	τ_{NL}
3.9	1.495	98.74	9.8	592.2	138.3
4	0.496	65	377	4.39×10^{8}	2.05×10^{5}
4.5	1.475	53.42	48.6	4.27×10^{5}	3404.5
5	1	62.1	0.14	-1.255	0.0284

We observe significant enhancements in non-linearity parameters towards the end of inflation in beyond slow-roll regime.

Desired Improvements

We have discussed several nice features of (roulette-) poly instanton inflation model. However, it is just a toy model and the following corners need scrutiny/improvements,

On technical grounds

- Uplifting Mechanism ? [SPhT Saclay Group, ...]
- Effects of string loop-corrections [Berg, Haack, Kors]
  Only a conjectural (not explicit) form is known for CY orientifold compactifications [Cicoli, Conlon, Quevedo].
- Effects of recently proposed α'-corrections via F-theory [Grimm, Savelli, Weissenbacher] ?

On phenomenological grounds

- What happens to inflaton (energy) after inflation ?, Reheating issues [Cicoli, Copeland, Mazumdar, ...]

Conclusions

In a LARGE volume type IIB orientifold setup equipped with poly-instanton corrections, we

- discussed a two-step moduli stabilization mechanism for Kähler moduli and respective axions.
- presented a *new class* of Kähler moduli inflation in which inflaton is a volume mode corresponding to a so-called 'Wilson' divisor (and not a blowup or fibre mode).
- presented four benchmark models which consistently reproduce the values of known cosmological observables.
- extended the single-field model with the inclusion of C_4 axion modulus and realize a 'roulette'-type inflation.
- realized large detectable values of non-linear parameters f_{NL}, τ_{NL} and g_{NL} in beyond slow-roll regime of 'roulette'-type inflation.

Extra Slides

Zero-mode analysis

- The instanton zero modes are related to the equivariant cohomologies of divisors which E3-instanton is wrapping.
- The zero mode structure required for the poly-instanton contribution to the superpotential is

class	Instanton a	Instanton b		
$H^{0,0}_{+}(E)$	1	1		
$H^{1,0}_{+}(E)$	0	$1 \mid 0$		✓
$H^{2,0}_{+}(E)$	0	$0 \mid 1$	e.g. $K3$	✗
$H^{n,0}_{-}(E)$	0	0		

- A divisor D: $H^{2,0}_{+}(D, \mathcal{O}) = 1$ would imply that a section of the normal bundle of D changes sign under involution and hence $E3$ instanton wrapping D would not be $O(1)$!

Zero-mode analysis

- Simple suitable complex surfaces with $h^{10} \neq 0$ can be, for examples, a $\mathbb{T}^4$ or a $\mathbb{P}^1$ fibrations over $\mathbb{T}^2$ surface.
- $\mathbb{T}^4$ has too many zero-modes which need to be soaked up via some non-standard mechanism.
- One example with $h^{00} = 1 = h^{10}$ and $h^{20} = 0$ is,

$W \equiv$

	x_1	x_2	x_3	x_4	x_5
0	1	1	0	0	0
3	0	-1	1	1	1

;

		1		
	1		1	
0		2		0
	1		1	
		1		

The fibre: $F = \mathbb{P}^1 \sqcup \mathbb{P}^1 \sqcup \mathbb{P}^1$; The base: $\Sigma = \{x_2 = 0\}$, which is a torus.

- This complex two-D surface after embedding in a CY would be what we call a Wilson-divisor.

Embedding W in concrete CY orientifolds

Model A

- The Calabi-Yau threefold $\mathcal{M}$ is given by a hypersurface in a toric variety with defining data

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
2	-1	0	1	1	0	0	0	1
4	-2	0	2	2	1	0	1	0
2	-3	0	2	1	1	1	0	0
2	1	1	0	0	0	0	0	0

- Stanley-Reisner ideal is

$$\mathrm{SR} = \{x_1\, x_2, x_4\, x_7, x_5\, x_7, x_1\, x_4\, x_8, x_2\, x_5\, x_6, x_3\, x_4\, x_8, x_3\, x_5\, x_6, x_3\, x_6\, x_8\}$$

- Hodge numbers: $(h^{21}, h^{11}) = (72, 4)$.
- D_1 is a $\mathbb{P}^2$ surface, D_7 is a dP_7 surface and the Wilson divisor W is given by D_8.

Model A

- The intersection form in the basis of smooth divisors as $\{D_1, D_6, D_7, D_8\}$

$$I_3 = 9D_1^3 - 3D_1^2 D_6 - 4D_6 D_8^2 + D_1 D_6^2 - 3D_6^3 + 2D_7 D_8^2 + 2D_6 D_7 D_8 + 2D_6^2 D_7 - 2D_7^2 D_8 - 2D_6 D_7^2 + 2D_7^3$$

- Using the Kähler form: $J = t_1 D_1 + t_6 D_6 + t_7 D_7 + t_8 D_8$,

$$\mathcal{V} \equiv \frac{1}{3!}\int_{\mathcal{M}} J \wedge J \wedge J = \frac{1}{6}\Big(9t_1^3 - 9t_1^2 t_6 + 3t_1 t_6^2 - 3t_6^3 + 6t_6^2 t_7 - 6t_6 t_7^2 + 2t_7^3 + 12t_6 t_7 t_8 - 6t_7^2 t_8 - 12t_6 t_8^2 + 6t_7 t_8^2\Big).$$

- The Kähler cone generators are,

$$K_1 = 2D_6 + 2D_7 + D_8,\ K_2 = 2D_6 + 3D_7 + D_8,$$
$$K_3 = D_1 + 4D_6 + 4D_7 + 2D_8,\ K_4 = D_1 + 5D_6 + 5D_7 + 2D_8$$

Model A

- The four-cycle volumes

$$\tau_1 = \frac{1}{2}(-3t_1 + t_6)^2,$$
$$\tau_6 = -\frac{3}{2}t_1^2 + t_1 t_6 - \frac{3}{2}t_6^2 + 2t_6 t_7 - t_7^2 + 2t_7 t_8 - 2t_8^2,$$
$$\tau_7 = (t_6 - t_7 + t_8)^2,$$
$$\tau_8 = (2t_6 - t_7)(t_7 - 2t_8)$$

- Using Kähler cone constraints result in a strong swiss-cheese like volume form

$$\mathcal{V} = \frac{1}{9}\left(\frac{1}{\sqrt{2}}(\tau_1 + 3\tau_6 + 6\tau_7 + 3\tau_8)^{3/2} - \sqrt{2}\tau_1^{3/2} - 3\tau_7^{3/2} - 3(\tau_7 + \tau_8)^{3/2}\right)$$

- The geometry $D_7 + D_8$ is a singular surface $x_7\, x_8$ which is the intersection of a dP_7 with the W divisors. It is a genus one curve in the Calabi-Yau threefold.
- The LARGE volume limit: $\tau_6 \to \infty$ while keeping the other four-cycles volumes small.

Model A: Orientifold involution

- We have to ensure $h^{10}(W) = 1$ to be in the orientifold even sector.
- Two inequivalent involutions $\sigma : \{x_7 \leftrightarrow -x_7, x_4 \leftrightarrow -x_4\}$, which have $h^{11}_{-}(CY_3) = 0$.
- The fixed point set for first involution

$$\mathrm{Fixed}_{x_7 \leftrightarrow -x_7} = \{D_5\,,\ D_7\,,\ D_1 D_4 D_6\,,\ D_2 D_4 D_6\}$$

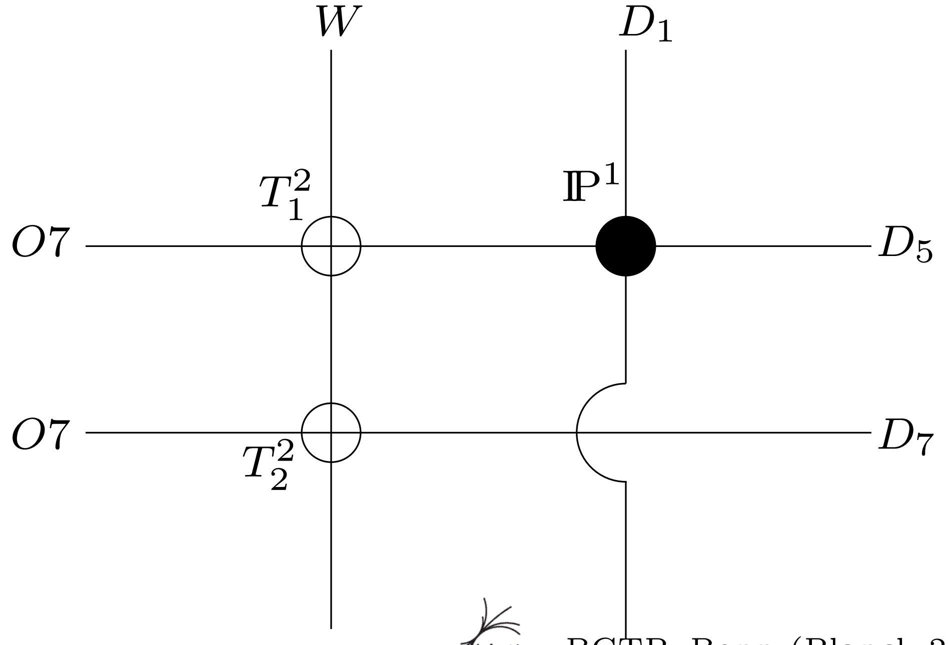
- Two $O7$-components and five $O3$-planes
- The $D7$-brane tadpole cancellation condition is met by placing eight $D7$-branes right on top of the $O7$-plane.
- The $D3$-brane tadpole is canceled via

$$N_{D3} + \frac{N_{\text{flux}}}{2} + N_{\text{gauge}} = \frac{N_{O3}}{4} + \frac{\chi(D_{O7})}{4} = \frac{5 + (10 + 25)}{4} = 10$$

Model A with $\sigma : x_7 \leftrightarrow -x_7$

divisor	$(h^{00}, h^{10}, h^{20}, h^{11})$	intersection curve
$D_7 = dP_7$	$(1_+, 0, 0, 8_+)$	$W : C_{g=1}$
D_5	$(1_+, 0, 1_+, 21_+)$	$W : C_{g=1}$
$D_8 = W$	$(1_+, 1_+, 0, 2_+)$	$D_5 : C_{g=1}, \quad D_7 : C_{g=1}$
$D_1 = \mathbb{P}^2$	$(1_+, 0, 0, 1_+)$	$D_5 : C_{g=0}$

Divisors and their equivariant cohomology. The first two lines are $O7$-plane components and the second two divisors can support $E3$ instantons. The D_7 divisor also supports gaugino condensation.



Model A with $\sigma : x_7 \leftrightarrow -x_7$

- Finally, one gets the following form of superpotential

$$W = A_1 \exp(-2\pi T_1) + A_1 A_8 \exp(-2\pi T_1 - 2\pi T_8) + A_7 \exp(-a_7 T_7) + A_7 A_8 \exp(-a_7 T_7 - 2\pi T_8) + \dots .$$