

A Zip-code for Quarks, Leptons and Higgs Bosons



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In collaboration with D. K. Mayorga and H. P. Nilles
Based on arXiv:1209.6041; arXiv:1305.0566

Planck 2013

May, 22, 2013

The \mathbb{Z}_{6-II} Mini-Landscape

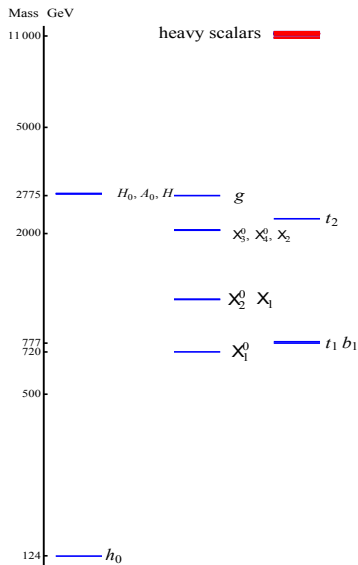
~ 200 realistic MSSM models based on the \mathbb{Z}_{6-II} orbifold geometry have been constructed in computer based searches

[Lebedev, Nilles, Raby, Ratz, et al '06]

Main features

- 1 The Higgs system
 - Higgs doublets are **untwisted** (Gauge-Higgs-Unification) [Fairlie, et al '79]
 - Shift symmetry in Kähler potential solves vacuum instability [Hebecker, et al '12]
 - μ -term forbidden by R-Symmetry [e.g. Raby, Ratz, Ross, Vaudrevange et al '11]
- 2 The Top-Quark
 - Top-Quark is **untwisted** with tree level Higgs Yukawa coupling
 \leftrightarrow Gauge-Top-Unification [Hosteins, Kappl, Ratz, Schmidt-Hoberger et al '07]
 - Third family is a patchwork, completed by fields from different locations.
- 3 The light families
 - **Twisted** fields localized at fixed points
 - Form a complete **16** of $SO(10)$

SUSY Breaking Pattern



Dilaton stabilization needs:

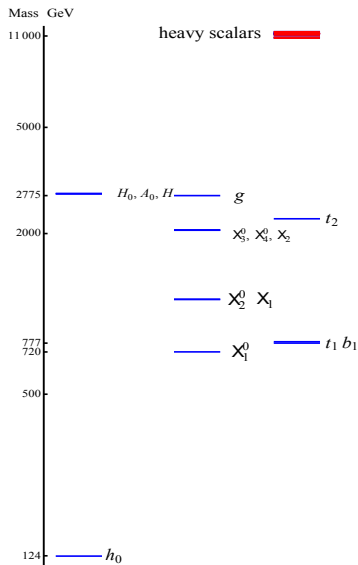
① Hidden sector gaugino condensation, favored

[Lebedev et al'07]

② Down-lifting of vacuum energy

$\Rightarrow m_{3/2}$ in multi-TeV range

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- 1 Hidden sector gaugino condensation, favored [Lebedev et al'07]
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- Fields at **Fixed Points** feel only $\mathcal{N} = 1$ SUSY
 \rightarrow Scalar masses $\sim m_{3/2}$
- Fields in **Bulk** and **Fixed Tori** feel remnants of $\mathcal{N} = 4, 2$ SUSY
 \rightarrow Superpartner masses suppressed by $\log(M_{\text{Pl}}/m_{3/2})$

\rightarrow **Natural SUSY**

[Krippendorff et al'12]

Motivation

- How **general** are these results?
- What can be expected in **different geometries**?

A Zip-code for Quarks, Leptons and Higgs Bosons

Outline:

- 1 $\mathbb{Z}_2 \times \mathbb{Z}_4$ Orbifold Geometry
- 2 $\mathbb{Z}_2 \times \mathbb{Z}_4$ Phenomenology and Examples
- 3 Conclusion

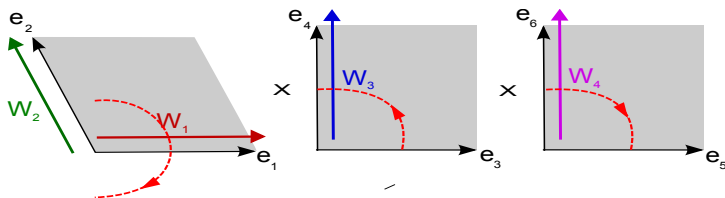
$\mathbb{Z}_2 \times \mathbb{Z}_4$ Geometry

Orbifold identification under twist $\theta(k_1, k_2)$

$$\theta(k_1, k_2) = \text{Diag}(e^{2\pi i (k_1 v_2^i + k_2 w^i)})$$

Geometric Definitions

- Choose factorizable lattice $SU(2)^2 \times SO(4) \times SO(4)$
- Choose the shifts $v_2 = (0, \frac{1}{2}, -\frac{1}{2}, 0)$ $v_4 = (0, 0, \frac{1}{4}, -\frac{1}{4})$
 - 8 sectors: $T_{(k_1, k_2)}$ $k_1 = 0, 1$ and $k_2 = 0, 3$
 - 4 Wilson lines of order two: W_1, W_2, W_3, W_4



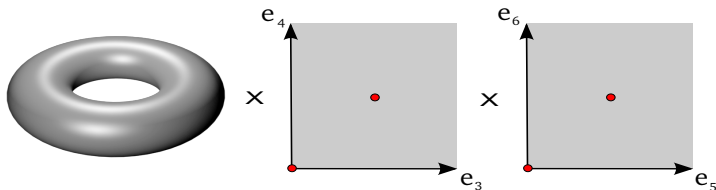
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Twisted Sectors

- $T_{(0,1)}/T_{(0,3)}$ Twisted sector
 - 4 Fixed Tori



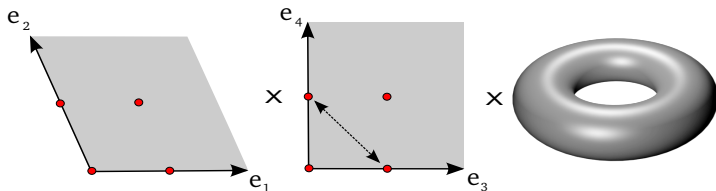
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Twisted Sectors

- $T_{(1,0)}$ Twisted Sector
 - $\boxed{12} = 8 + 4$ Fixed Tori
 - \mathbb{Z}_4 identification: Gauge enhancement at special fixed tori
 - Broken degeneracy without Wilson lines



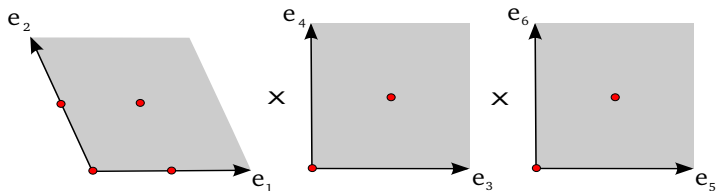
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Twisted Sectors

- $T_{(1,1)}/T_{(1,3)}$ Twisted Sector
 - 16 Fixed Points
 - Fixed points, high flexibility in breaking the degeneracy via Wilson lines



The Gauge Embedding

Modular invariance of the one-loop partition function

↔ Embedding of space group twist as translation in gauge lattice $\Lambda_{E_8 \times E_8}$

Construction of inequivalent Embeddings

Quotienting the automorphism group out of $\Lambda_{E_8 \times E_8}$, we classified all

144 (61 in \mathbb{Z}_{6-II})

inequivalent models and their brother model

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Gauge Spectrum

- 35 embeddings with $SO(10)$ gauge factors (13 in \mathbb{Z}_{6-II})
- 26 embeddings with E_6 gauge factors (16 in \mathbb{Z}_{6-II})
- 25 embeddings with $SU(5)$ gauge factors (4 in \mathbb{Z}_{6-II})
 ↪ Seems more fertile for Model building

Phenomenological Input and Constraints

Goal: Want to judge the fertility of this geometry and find preferred field localizations

Focus on $SO(10)$ breaking via Wilson lines to SM

- 1 Renormalizable Top-Yukawa from $\mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}$
- 2 Achieve doublet-triplet splitting via Wilson lines
- 3 Break the fixed point degeneracy via Wilson lines
↪ Get three families as complete as possible

Which models are compatible with those constraints?

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Constraints Higgs location to be Wilson line affected
- 3 Break the fixed point degeneracy via Wilson lines
 \hookrightarrow Get three families as complete as possible
Fixes Wilson configuration and family locations

Which models are compatible with those constraints?

Example Model

Given by shifts

$$V_2 = (1, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0) (\frac{5}{4}, -\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, -\frac{1}{4}, \frac{1}{4})$$

$$V_4 = (\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0) (\frac{5}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, -\frac{1}{2})$$

Leading to $SO(10) \times SU(2) \times SU(2) \times SU(8) \times U(1)^2$ with spectrum

U	$1 (16, 2, 1, 1)_{0,-1}$
	$1 (16, 1, 2, 1)_{0,1}$
	$1 (10, 2, 2, 1)_{0,0}$
	$1 (1, 1, 1, 1)_{-12,0}$
	$1 (1, 1, 1, 1)_{12,0}$
	$1 (1, 1, 1, 28)_{6,0}$
	$1 (1, 1, 1, 28)_{6,0}$

$T(0, 1)$	$4 (1, 1, 1, 8)_{6,1}$
	$4 (1, 1, 1, 8)_{0,1}$
$T(0, 2)$	$10 (1, 2, 2, 1)_{6,0}$
	$10 (10, 1, 1, 1)_{-6,0}$
	$6 (1, 1, 1, 1)_{-6,-2}$
	$6 (1, 1, 1, 1)_{-6,2}$

$T(0, 3)$	$4 (1, 1, 1, 8)_{6,-1}$
	$4 (1, 1, 1, 8)_{0,-1}$
$T(1, 0)$	$4 (1, 1, 2, 8)_{-3,0}$
$T(1, 1)$	Empty
$T(1, 2)$	$4 (1, 2, 1, 8)_{-3,0}$
$T(1, 3)$	$16(16, 1, 1, 1)_{3,0}$
	$16 (1, 2, 1, 1)_{3,1}$
	$16 (1, 1, 2, 1)_{3,-1}$

Example Model

Given by shifts

$$V_2 = (1, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0)(\frac{5}{4}, -\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, -\frac{1}{4}, \frac{1}{4})$$

$$V_4 = (\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0)(\frac{5}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, -\frac{1}{2})$$

and Wilson lines

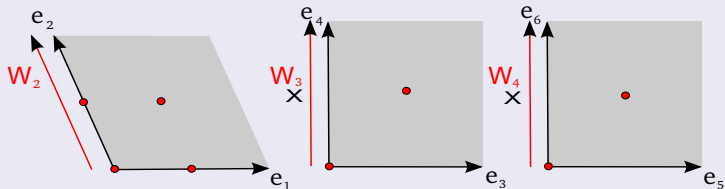
$$W_2 = (-\frac{1}{2}, \frac{1}{2}, -\frac{3}{2}, -\frac{1}{2}, 0, -1, -1, 2)(-\frac{3}{4}, -\frac{7}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{7}{4}, \frac{3}{4}, \frac{3}{4})$$

$$W_3 = (-1, \frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, 0, 2)(-\frac{1}{2}, 1, -2, 0, \frac{3}{2}, -1, -\frac{3}{2}, -\frac{3}{2})$$

$$W_4 = (-\frac{5}{4}, \frac{5}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{3}{4}, -\frac{1}{4}, \frac{5}{4}, \frac{9}{4})(0, 1, 1, 2, -1, -\frac{1}{2}, 2, \frac{3}{2})$$

Wilson line breaking

$$SO(10) \times SU(2) \times SU(2) \times SU(8) \times U(1)^2 \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3) \times SU(2) \times U(1)^9$$



Example Model

Given by shifts

$$V_2 = (1, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0) \left(\frac{5}{4}, -\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, -\frac{1}{4}, \frac{1}{4} \right)$$

$$V_4 = \left(\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0 \right) \left(\frac{5}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, -\frac{1}{2} \right)$$

and Wilson lines

$$W_2 = \left(-\frac{1}{2}, \frac{1}{2}, -\frac{3}{2}, -\frac{1}{2}, 0, -1, -1, 2 \right) \left(-\frac{3}{4}, -\frac{7}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{7}{4}, \frac{3}{4}, \frac{3}{4} \right)$$

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Break the degeneracy

Lower the degeneracy by factor of 8

Two local **16**-plets unaffected

→ Two light families

$T(0, 3)$	4 $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \bar{\mathbf{8}})_{6,-1}$ 4 $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{8})_{0,-1}$
$T(1, 0)$	4 $(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{8})_{-3,0}$
$T(1, 1)$	Empty
$T(1, 2)$	4 $(\mathbf{1}, \mathbf{2}, \mathbf{1}, \bar{\mathbf{8}})_{-3,0}$
$T(1, 3)$	16 $(\mathbf{16}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{3,0}$ 16 $(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{3,1}$ 16 $(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1})_{3,-1}$

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Project out unwanted triplets

U	$1 (16, 2, 1, 1)_{0,-1}$
	$1 (16, 1, 2, 1)_{0,1}$
	$1 (10, 2, 2, 1)_{0,0}$
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	$1 (1, 1, 1, \mathbf{28})_{6,0}$
	$1 (1, 1, 1, \mathbf{\bar{28}})_{6,0}$

$$(16, 2, 1, 1)_{0,-1} \rightarrow (1, 1, 1, 1)_{-1, \dots} + \underbrace{(\bar{3}, 1, 1, 1)_{1/3, \dots}}_{\bar{u}}$$

$$(16, 1, 2, 1)_{0,1} \rightarrow \underbrace{(3, 2, 1, 1)_{-1/6, \dots}}_Q$$

$$(10, 2, 2, 1)_{0,0} \rightarrow \underbrace{(1, 2, 1, 1)_{-1/2, \dots}}_{H_u} + \underbrace{(1, 2, 1, 1)_{1/2, \dots}}_{H_d}$$

Example Model

Yukawa Couplings

U	1 (16, 2, 1, 1) _{0,-1}
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Trilinear Top-Yukawa:

$$\leftrightarrow (10, 2, 2, 1) \cdot (16, 1, 2, 1) \cdot (16, 2, 1, 1) \xrightarrow{W_2, W_3, W_4} H_u Q \bar{U}$$

Vector-like Higgs pair protected by R-Symmetry:

$$\leftrightarrow (10, 2, 2, 1) \cdot (10, 2, 2, 1) \xrightarrow{W_2, W_3, W_4} H_u H_d$$

Example Model

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Specific Model Details

- Net MSSM spectrum
- Non-Anomalous Hypercharge
- One unique pair of Higgs doublets
- All MSSM Exotics are vector-like
- Find realistic VEV configuration hard

The General Story

The Higgs System

- 1 Untwisted Higgs seems favored due to high flexibility to couple trilinearly
- 2 Same solution for μ -problem as in the \mathbb{Z}_{6-II} orbifold due to \mathbb{Z}_2 twist

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The Three Families

- ① Models with three all local **16**-plets, **NOT** incompatible with heavy top
- ② Models with 2 local **16**-plets favored
- ③ A Heavy Top Quarks suggests its location in bulk as well.

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Properties of Untwisted Sector crucial ingredient

Untwisted sectors with these features are **frequent: $\sim 75\%$ of all embeddings**

Conclusion

We investigated the MSSM matter properties found in the \mathbb{Z}_{6-II} and extended them to $\mathbb{Z}_2 \times \mathbb{Z}_4$

What we did

- Considered the factorisable $\mathbb{Z}_2 \times \mathbb{Z}_4$ geometry
- Constructed all gauge embeddings
- Strategy to construct good models

Our findings

The $\mathbb{Z}_2 \times \mathbb{Z}_4$ confirms the picture of the \mathbb{Z}_{6-II} Mini-Landscape

- Higgs and Top Quark are favorably untwisted fields
- \mathbb{Z}_2 twist induces a vector-like Higgs pair, μ -term absent by its R-Symmetry
- Light families originating from localized **16**-plets

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Thank you!

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Example Model: Full Spectrum

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times \underbrace{SU(3) \times SU(2) \times U(1)}_{\text{Hidden}}^9$$

#	Rep.	label	#	Rep.	label
3	$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1})_{\frac{2}{3}}$	\bar{u}	69	$(\mathbf{1}, \mathbf{1}, \mathbf{1})_0$	n
3	$(\mathbf{1}, \mathbf{1}, \mathbf{1})_{-1}$	\bar{e}	32	$(\mathbf{1}, \mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	r
3	$(\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{-\frac{1}{6}}$	q	4	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	b
4	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	l	30	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	\bar{r}
1	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	\bar{l}	4	$(\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_0$	s
9	$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{-\frac{1}{3}}$	\bar{d}	10	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_0$	\check{v}
6	$(\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{\frac{1}{3}}$	d	8	$(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1})_0$	\bar{s}
6	$(\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{-\frac{1}{6}}$	f	2	$(\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{2}}$	χ
8	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0$	ν	5	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	\bar{b}
1	$(\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{-\frac{1}{6}}$	m	2	$(\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{-\frac{1}{2}}$	$\check{\chi}$
8	$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{\frac{1}{6}}$	\bar{f}			