R-Symmetries from Heterotic Orbifold Compactifications

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In collaboration with: N. Cabo Bizet, T. Kobayashi, S. Parameswaran, M. Schmitz and I. Zavala.

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Motivation

Orbifolds

- MSSM models can be obtained from the Heterotic orbifold models such as $\mathbb{Z}_{6\mathrm{II}}$, $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathbb{Z}_4$,...

  Blaszczyk, Buchmüller, Groot Nibbelink, Hamaguchi, Kim, Kobayashi, Kyae, Lebedev, Nilles, Oehlmann, Quevedo, Raby, Ramos-Sanchez, Ratz, Rühle, Trapletti, Vaudrevange, Wingerter, ...

- Couplings can be computed exactly since orbifold CFT is free. Vanishing of certain couplings can be related to a symmetry of the effective field theory (EFT)!

R-**Symmetries**

- An elegant way to forbid certain dangerous proton decay operators in SUSY models.
- Required in certain constructions where the μ-problem is solved.

  Casas, Muñoz'93; Lebedev *et. al.*'08, Kappl *et. al.*'09

- In models with extra dimensions, R-symmetries arise naturally as remnants of the Lorentz group in internal space.

Outline

- Heterotic Orbifolds and Lattice Automorphisms
- R-Symmeries from Correlation Functions
- Conclusions and Outlook

Heterotic Orbifolds and Lattice Automorphisms

- Assume the target space of the heterotic string to be of the form

$$\mathcal{M}_{10} = \mathcal{M}_{3,1} \times \frac{T^6}{P} = \mathcal{M}_{3,1} \times \frac{\mathbb{C}^3}{P \ltimes \Gamma_6} = \mathcal{M}_{3,1} \times \frac{\mathbb{C}^3}{S}$$

  P is an isometry of Γ_6 which we take as $\mathbb{Z}_N$, with $\theta = (\theta_1, \theta_2, \theta_3)$ its generating element.
  The orbifold is called **factorizable** if $\Gamma_6 = \Gamma_2 \times \Gamma_2' \times \Gamma_2''$.

- String boundary conditions:

$$Z(\sigma + \pi, \tau) = \theta^k Z(\sigma, \tau) + \lambda \qquad (\theta^k, \lambda) \in S \quad j = 1, 2, 3$$

- Action of P has some **fixed points** z_f, which fall into **conjugacy classes**

$$[z_\mathrm{f}] = \{z_\mathrm{f}' \,|\, z_\mathrm{f}' = h z_\mathrm{f} \text{ for some } h \in S\},$$

  supporting **twisted string states**.

Heterotic Orbifolds and Lattice Automorphisms

- We search for elements in $\mathrm{Aut}(\Gamma_6) \subset \mathrm{O}(6)$ which are consistent with orbifolding. We identify the following decomposition

$$\mathrm{Aut}(\Gamma_6) = G \ltimes F \ltimes E \ltimes D$$

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    $\rightarrow$ candidates for R-symmetries in the EFT.

Results:

- factorizable:

	Lattice	Twist	Orbifold Automorphisms
$\mathbb{Z}_3$	SU(3)×SU(3)×SU(3)	$\frac{1}{3}(1,1,-2)$	$\theta_1,\ \theta_2,\ \theta_3$
$\mathbb{Z}_4$	SO(4)×SO(4)×SO(4)	$\frac{1}{4}(1,1,-2)$	$\theta_1\theta_2,\ (\theta_1)^2,\ \theta_3$
$\mathbb{Z}_{6-\mathrm{I}}$	$\mathrm{G}_2 \times \mathrm{G}_2$×SU(3)	$\frac{1}{6}(1,1,-2)$	$\theta_1\theta_2,\ \theta_3$
$\mathbb{Z}_{6-\mathrm{II}}$	G_2×SU(3)×SO(4)	$\frac{1}{6}(1,2,-3)$	$\theta_1,\ \theta_2,\ \theta_3$

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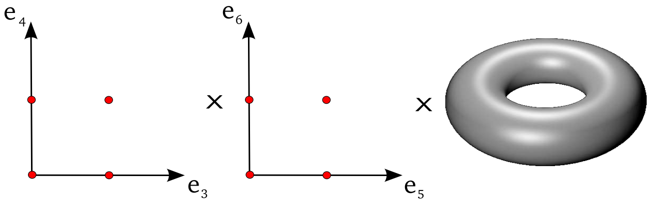
Example: $\mathbb{Z}_{\mathbf{4}}$ θ^2 sector



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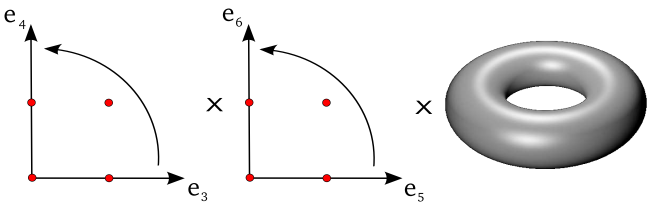

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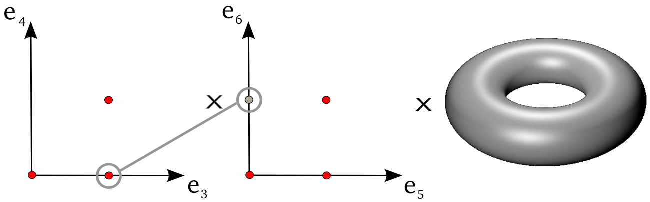

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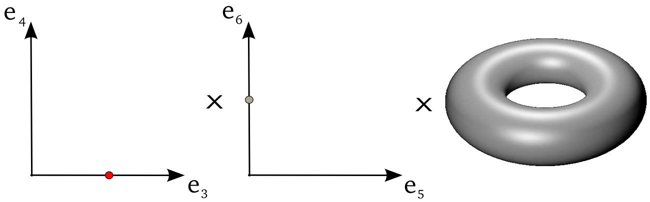

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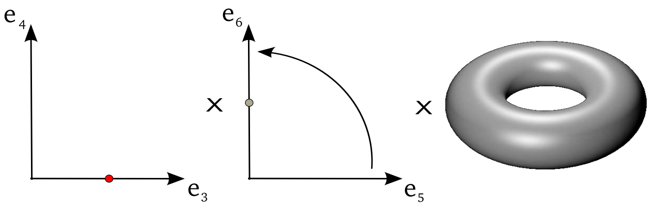

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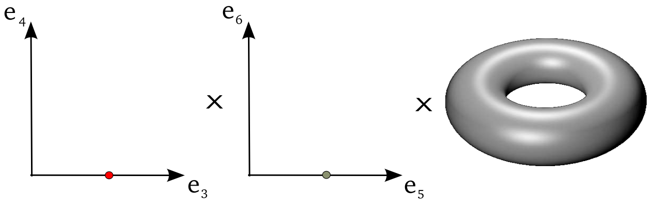

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- non-factorizable:

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$\mathbb{Z}_4$	SU(4)×SU(4)	$\frac{1}{4}(1,1,-2)$	$\theta,\ (\theta_1)^2$
$\mathbb{Z}_{6-\mathrm{II}}$	SU(6)×SU(2)	$\frac{1}{6}(1,2,-3)$	θ
$\mathbb{Z}_7$	SU(7)	$\frac{1}{7}(1,2,-3)$	θ
$\mathbb{Z}_{8-\mathrm{I}}$	SO(5)×SO(9)	$\frac{1}{8}(2,1,-3)$	$\theta,\ (\theta_1)^2$
$\mathbb{Z}_{8-\mathrm{II}}$	SO(8)×SO(4)	$\frac{1}{8}(1,3,-4)$	$\theta,\ \theta_3$
$\mathbb{Z}_{12-\mathrm{I}}$	SU(3)×F_4	$\frac{1}{12}(4,1,-5)$	$\theta,\ \theta_1$
$\mathbb{Z}_{12-\mathrm{II}}$	F_4×SO(4)	$\frac{1}{12}(1,5,-6)$	$\theta,\ \theta_3$

R-Symmetries from Correlation Functions

- The strength of the L point coupling $\psi\psi\phi^{L-2}$, is given by $\langle V_{\mathrm{F}} V_{\mathrm{F}} V_{\mathrm{B}} \ldots V_{\mathrm{B}} \rangle$. $\Rightarrow$ Correlators can be used to construct $\mathcal{W} \supset \Phi^L$.
- The emission vertices for strings twisted by θ^k are given by

$$V_{\mathrm{B}} = e^{-\phi} \prod_{i=1}^{3} (\partial X^i)^{\mathcal{N}_{\mathrm{L}}^i} \, (\partial \bar{X}^i)^{\bar{\mathcal{N}}_{\mathrm{L}}^i} \; e^{\mathrm{i} q_{sh}^m H^m} \, e^{\mathrm{i} p_{sh}^I X^I} \, \sigma^i_{(k,\psi)}$$

$$V_{\mathrm{F}} = \mathrm{e}^{\phi/2} V_{\mathrm{B}}(q_{sh} = q_{sh}^{(f)})$$

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$$V_{\rm F} = {\rm e}^{\phi/2} V_{\rm B}(q_{sh} = q_{sh}^{(f)})$$

- **Twist fields** $\sigma_{(k,\psi)}$, create twisted vacua out of the untwisted one.

$$\boxed{\sigma_{(k,\psi)} = \sum_{r=0}^{l-1} e^{-2\pi {\rm i} r \gamma} \sigma_{(k,\theta^r f)}}$$

Lauer, Mas, Nilles'91; Erler, Jungnickel, Lauer, Mas'92

cf. $\theta\sigma_{(k,\psi)} = e^{-2\pi{\rm i}\gamma}\sigma_{(k,\psi)}$, with l: smallest integer s.t. $\theta^l f = f + \lambda$.

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Correlation function factorizes as:

$$\mathcal{F} = \left\langle \mathrm{e}^{\mathrm{i}\sum_{\alpha=1}^{L} p^{l}_{\mathrm{sh},\alpha}\cdot X^{l}(z_\alpha)} \right\rangle \times \left\langle \mathrm{e}^{\mathrm{i}\sum_{\alpha=1}^{L} q^{m}_{\mathrm{sh},\alpha}\cdot H^{m}(z_\alpha)} \right\rangle$$

$$\times \prod_{i=1}^{3} \left\langle (\partial X^i)^{\sum_\alpha \mathcal{N}^i_{\mathrm{L},\alpha}} (\partial \bar{X}^i)^{\sum_\alpha \bar{\mathcal{N}}^i_{\mathrm{L},\alpha}} (\bar{\partial} \bar{X}^i)^{\sum_\alpha \bar{\mathcal{N}}^i_{\mathrm{R},\alpha}} \sigma^i_{(k_1,\psi_1)} \sigma^i_{(k_2,\psi_2)} \ldots \sigma^i_{(k_L,\psi_L)} \right\rangle$$

Dixon, Friedan, Martinec, Shenker'87; Hamidi, Vafa'87; Font, Ibañez, Nilles, Quevedo'88

R-Symmetries from Correlation Functions

Correlation function factorizes as:

$$\mathcal{F} = \left\langle \mathrm{e}^{\mathrm{i} \sum_{\alpha=1}^{L} p_{\mathrm{sh},\alpha}^{I} \cdot X^{I}(z_\alpha)} \right\rangle \times \left\langle \mathrm{e}^{\mathrm{i} \sum_{\alpha=1}^{L} q_{\mathrm{sh},\alpha}^{m} \cdot H^{m}(z_\alpha)} \right\rangle$$
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Selection rules for non vanishing couplings

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- Space group selection rule.
- Rule 5: Depending on $\{k_\alpha\}$ and classical solutions $\Rightarrow$ restrictions on $\mathcal{N}_{\mathrm{L}}$, $\bar{\mathcal{N}}_{\mathrm{L}}$ and $\bar{\mathcal{N}}_{\mathrm{R}}$. Specific to each particular coupling!

Kobayashi, Parameswaran, Ramos-Sánchez, Zavala '11

R-Symmetries from Correlation Functions

- Upon splitting $\partial X = \partial X_{\mathrm{cl}} + \partial X_{\mathrm{qu}}$ between instantons ($\bar{\partial}\partial X_{\mathrm{cl}} = 0$) and quantum parts, the correlator simplifies to:

$$\mathcal{F} = \sum_{r_1=0}^{l_1} \cdots \sum_{r_L=0}^{l_L} e^{-2\pi \mathrm{i} \sum_{\alpha=1}^{L} r_\alpha \gamma_\alpha} \prod_{i=1}^{3} \mathcal{F}^i_{\mathrm{aux}}$$

with

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- Very Important Remarks:
  - In general, phases γ_α can not be split into 2D contributions.
  - $\partial X_{\rm cl}$ **enjoy symmetries** from $D \subset \mathrm{Aut}(\Gamma_6)$ (Only proven for factorizable orbifolds).

R-Symmetries from Correlation Functions

- Upon splitting $\partial X = \partial X_{\rm cl} + \partial X_{\rm qu}$ between instantons ($\bar\partial\partial X_{\rm cl} = 0$) and quantum parts, the correlator simplifies to:

$$\mathcal{F} = \sum_{r_1=0}^{l_1} \cdots \sum_{r_L=0}^{l_L} e^{-2\pi \mathrm{i} \sum_{\alpha=1}^{L} r_\alpha \gamma_\alpha} \prod_{i=1}^{3} \mathcal{F}^i_{\rm aux}$$

with

$$\mathcal{F}^i_{\rm aux} = \sum_{X^i_{\rm cl}} \mathrm{e}^{-S^i_{\rm cl}} (\partial X^i_{\rm cl})^{\mathcal{N}^i_{\rm L} - \bar{\mathcal{N}}^i_L - \bar{\mathcal{N}}^i_R} f(\underbrace{|\partial X^i_{\rm cl}|^2, \partial X^i_{\rm cl} \partial \bar{X}^i_{\rm cl}}_{D \text{ Invariant Blocks}}, \text{Quantum Pieces})$$

- Very Important Remarks:
  - In general, phases γ_α can not be split into 2D contributions.
  - $\partial X_{\rm cl}$ **enjoy symmetries** from $D \subset \mathrm{Aut}(\Gamma_6)$ (Only proven for factorizable orbifolds).
  - Quantum pieces **are independent** of the position of the fixed points

R-Symmetries from Correlation Functions

Using the elements of D we obtain:

- prime planes:

$$\mathcal{F}^j \sim (1)^{(\mathcal{N}^j_{\mathrm{L}} - \bar{\mathcal{N}}^j_{\mathrm{L}} - \bar{\mathcal{N}}^j_{\mathrm{R}})} + (\theta_j)^{(\mathcal{N}^j_{\mathrm{L}} - \bar{\mathcal{N}}^j_{\mathrm{L}} - \bar{\mathcal{N}}^j_{\mathrm{R}})} + \cdots + (\theta_i^{(N^j - 1)})^{(\mathcal{N}^j_{\mathrm{L}} - \bar{\mathcal{N}}^j_{L} - \bar{\mathcal{N}}^j_{R})}$$

$$\Rightarrow \boxed{\sum_\alpha \left(q^j_{\mathrm{sh}} - \mathcal{N}^j_{\mathrm{L}} + \bar{\mathcal{N}}^j_{\mathrm{L}} \right)_\alpha = 1 \mod \mathrm{N}^j}$$

Kobayashi, Raby, Zhang'04

R-Symmetries from Correlation Functions

Using the elements of D we obtain:

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Kobayashi, Raby, Zhang'04

- non-prime planes:

$$\mathcal{F} \sim \prod_{i\neq j} \sum_{|X^i_{cl}|} \sum_{n=0}^{N-1} e^{-S^i_{cl}} (|\partial X^i_{\rm cl}|\theta_i^n)^{(\mathcal{N}^i_{\rm L}-\bar{\mathcal{N}}^i_{\rm L}-\bar{\mathcal{N}}^i_{\rm R})}\, e^{-2\pi \mathrm{i}\, n \sum_{\alpha=1}^{L} \gamma_\alpha}$$

$$\Rightarrow \boxed{\sum_\alpha \left(\sum_{i\neq j} v^i \left(q^i_{\rm sh} - \mathcal{N}^i_{\rm L} + \bar{\mathcal{N}}^i_{\rm L}\right)_\alpha + \gamma_\alpha\right) = \left(\sum_{i\neq j} v^i\right) \mod 1}$$

R-symmetries are still obtained, but the R-charges need to be redefined to include the contribution of the γ phases.

Conclusions and Outlook

- From the **symmetries among instanton solutions** we could read of the R-symmetries expected for factorizable orbifolds.

- We conjectured which R-symmetries are to be expected in the non-factorizable case, but the explicit CFT still needs to be worked out!

- Traditional R-symmetries apply only for **prime planes** in factorizable orbifolds.

- In general it gets a contribution from the γ-**phases**.
  $\Rightarrow$ Redefinition of R-charges of the fields!
  $\Rightarrow$ Generically more couplings allowed!

- In special cases there are further "coupling dependent" conditions $\rightarrow$ 'Rule 6'.

Conclusions and Outlook

- From the **symmetries among instanton solutions** we could read of the R-symmetries expected for factorizable orbifolds.
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- In general it gets a contribution from the γ-**phases**.
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- In special cases there are further "coupling dependent" conditions $\rightarrow$ 'Rule 6'.

thank you

Rule 6

Consider a toy **example**: $\mathrm{T}^2/\mathbb{Z}_6$ on G_2 lattice:

- θ-**action**: $\theta e_1 = -e_1 - e_2$, $\theta e_2 = 3e_1 + 2e_2$
- θ^2 sector **fixed points**: $z_\mathrm{f} = 0\,,\quad e_2/2\,,\quad 2e_2/3$
- $\theta^2\theta^2\theta^2$ coupling has two contributions:

$$\mathcal{F} = e^{-2\pi\mathrm{i}\gamma_3}\sum_{X_\mathrm{cl}} e^{-S_\mathrm{cl}}(\partial X_\mathrm{cl})^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}}\left\langle\sigma_{(\theta^2,0)}\sigma_{(\theta^2,e_1/3)}\sigma_{(\theta^2,\theta e_1/3)}\right\rangle$$
$$+e^{-2\pi\mathrm{i}\gamma_2}\sum_{X_{cl}} e^{-S_{cl}}(\partial X_{cl})^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}}\left\langle\sigma_{(\theta^2,0)}\sigma_{(\theta^2,\theta e_1/3)}\sigma_{(\theta^2,e_1/3)}\right\rangle\,,$$

- **overall factor**

$$\mathcal{F} \sim e^{-2\pi\mathrm{i}\gamma_3}\left((1)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}} + (\theta^2)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}} + (\theta^4)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}}\right)$$
$$+e^{-2\pi\mathrm{i}\gamma_2}\left((\theta)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}} + (\theta^3)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}} + (\theta^5)^{\mathcal{N}_\mathrm{L}-\bar{\mathcal{N}}_\mathrm{L}}\right)$$

- Selection rule:

$$\boxed{\sum_{\alpha=1}^{3}\mathcal{N}_{\mathrm{L}\,\alpha} - \bar{\mathcal{N}}_{\mathrm{L}\,\alpha} = 0 \mod 3}$$