

PLANCK 2013

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Froggatt-Nielsen models with derived Z_4^R symmetry

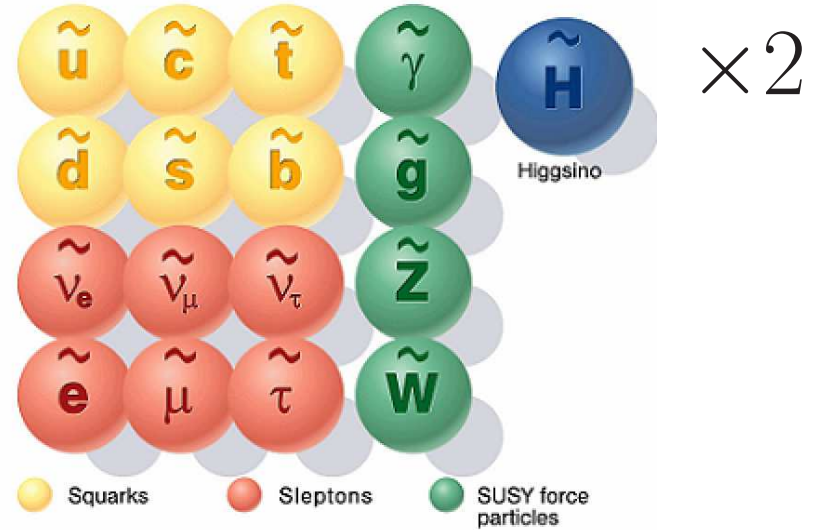
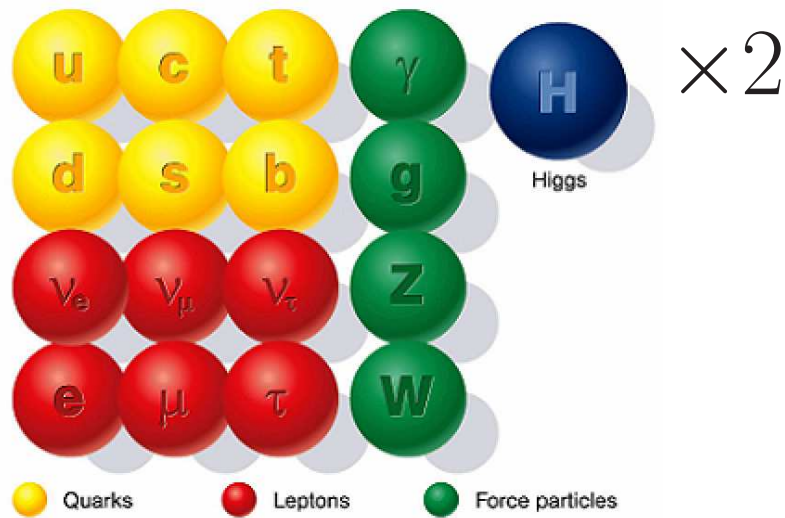
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Outline

- ▶ supersymmetric Standard Model
 - open questions
 - pattern of fermion masses and mixings
- ▶ Froggatt-Nielsen mechanism
 - $U(1)_{\text{FN}}$ family symmetry
 - residual discrete Z_N symmetries
 - extend to R -symmetries
- ▶ model realisation (finding viable charge assignments)
 - anomaly conditions
 - flavon VEV
 - μ -term
 - neutrino masses

Supersymmetric Standard Model



$$\begin{aligned}
 W_{\text{ren}} = & \mu H_u H_d + Y_{ij}^u H_u Q_i U_j^c + Y_{ij}^d H_d Q_i D_j^c + Y_{ij}^e H_d L_i E_j^c \\
 & + \kappa_i H_u L_i + \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c
 \end{aligned}$$

- (some) issues
- proton decay
 - family structure
 - μ -term
 - neutrino masses

Matter parity from $U(1)_{B-L}$

- matter parity
- forbids (renormalisable) B and L violation
 - introduced to stabilise proton
 - violated by quantum gravity effects [Krauss, Wilczek \(1989\)](#)
 - unless gauge origin, e.g. from breaking $U(1)_{B-L}$

	Q	U^c	D^c	L	E^c	H_u	H_d	ϕ
$U(1)_{B-L}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	1	0	0	$\pm\frac{2}{3}$
$3 \times U(1)_{B-L}$	1	-1	-1	-3	3	0	0	± 2
M_p	1	1	1	1	1	0	0	0

→ vev of **field** ϕ breaks $U(1)_{B-L}$ spontaneously down to M_p

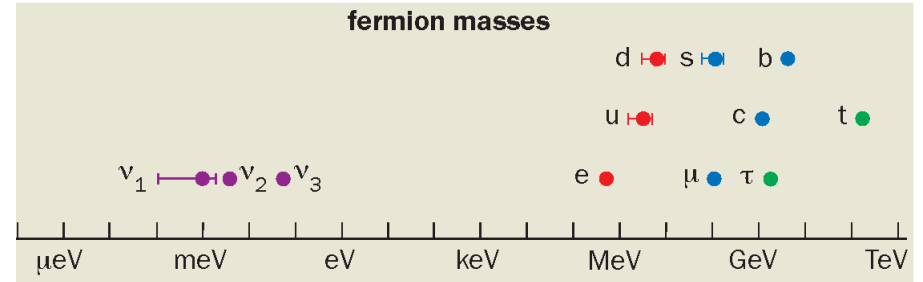
“Higgs”-type field

neutral under

SM gauge group

Fermion masses and mixings

- fermion mass terms: $M_{ij} \bar{\psi}_i \psi_j$
- masses = eigenvalues of M
- with $\lambda \approx 0.22$



► quarks

$$m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$$

$$m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1$$

$$U_{\text{CKM}} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

► leptons

$$m_e : m_\mu : m_\tau \sim \lambda^{4 \text{ or } 5} : \lambda^2 : 1$$

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \begin{cases} \lambda^x : \lambda : 1 \\ 1 : 1 : \lambda^x \\ 1 : 1 : 1 \end{cases}$$

$$U_{\text{PMNS}} \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

Gonzalez-Garcia et al. (2012)

Froggatt-Nielsen mechanism

- $U(1)_{\text{FN}}$ family symmetry
- spontaneously broken by flavon field ϕ

$$\boxed{Y_{ij} \bar{\psi}_i \psi_j H} \text{ forbidden} \longrightarrow \boxed{c_{ij} \bar{\psi}_i \psi_j H \left(\frac{\phi}{\Lambda}\right)^{x_{ij}}} \text{ allowed}$$

- flavon VEV generates Yukawas $Y_{ij} = c_{ij} \left(\frac{\langle\phi\rangle}{\Lambda}\right)^{x_{ij}}$
- $\mathcal{O}(1)$ coefficients c_{ij} not fixed

fields	Q_1	Q_2	Q_3	D_1^c	D_2^c	D_3^c	H_d	ϕ
$U(1)_{\text{FN}}$	6	4	0	5	3	3	-3	-2

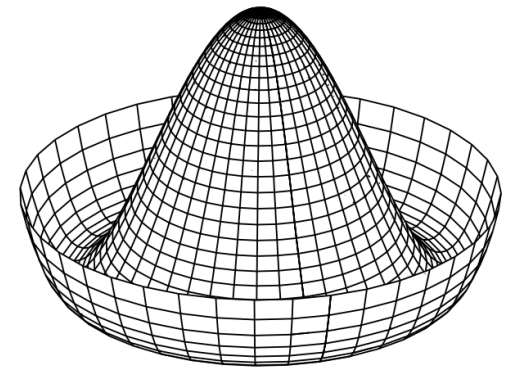
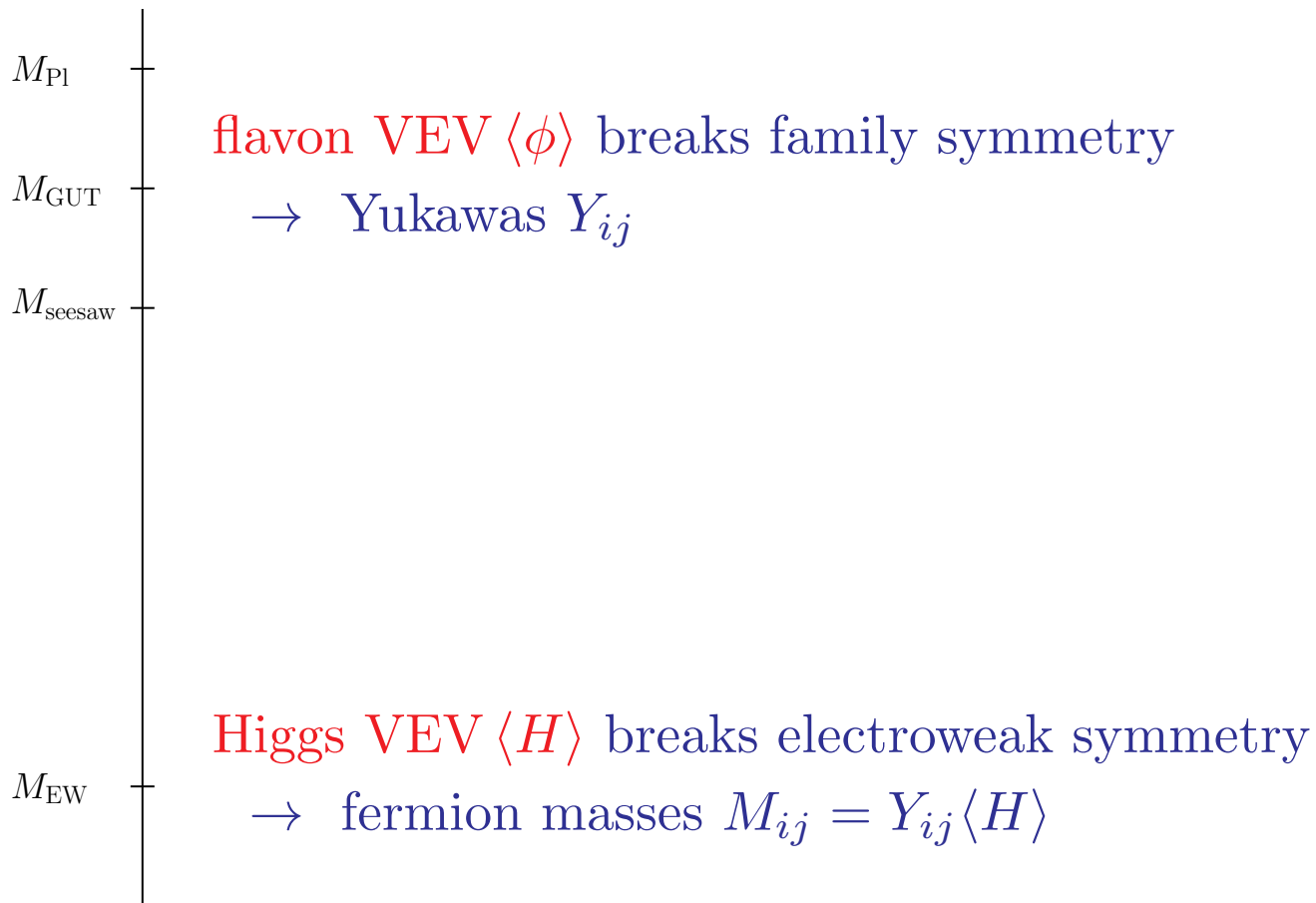
$$\frac{\langle\phi\rangle}{\Lambda} \sim \lambda$$

$$c_{ij} Q_i D_j^c H_d \left(\frac{\phi}{\Lambda}\right)^{x_{ij}} \longrightarrow c_{ij} Q_i D_j^c H_d \lambda^{x_{ij}} \longrightarrow Y_d \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}$$

- hierarchies arise from spontaneous breakdown of $U(1)_{\text{FN}}$
- often residual discrete Z_N symmetry, e.g. matter parity

Scale of family symmetry breaking

- models of flavour are typically formulated at high energies
- separate family symmetry from EW symmetry breaking



Discrete symmetries from $U(1)_{\text{FN}}$

- simple conditions to obtain a particular Z_N symmetry from $U(1)_{\text{FN}}$
- in $U(1)_{\text{FN}}$ charge normalisation with $X_\phi = -1$

	$X_{H_d} - X_{L_1}$	$3X_{Q_1} + X_{L_1}$
M_p	integer $- \frac{1}{2}$	integer
P_6	integer $- \frac{1}{2}$	integer $\pm \frac{1}{3}$
B_3	integer	integer $\pm \frac{1}{3}$

Dreiner, Luhn, Murayama,
Thormeier (2006)

M_p lightest SUSY particle (LSP) is stable \rightarrow dark matter candidate
allows for non-renormalisable operator $QQQL \rightarrow$ proton decay

P_6 LSP is stable \rightarrow dark matter candidate
forbids $QQQL \rightarrow$ proton stabilised

B_3 LSP not stable \rightarrow less missing E_T
forbids $QQQL \rightarrow$ proton stabilised
renormalisable L violation \rightarrow neutrino masses without seesaw

A unique Z_4^R symmetry for the MSSM

- no μ -term in superpotential
- (discrete) anomaly cancellation adopting Green-Schwarz mechanism
- compatible with $SU(5)$ [$SO(10)$]

→ requires a discrete R -symmetry

- minimal [unique] solution

	Q	U^c	D^c	L	E^c	H_u	H_d	θ
Z_4^R	1	1	1	1	1	0	0	1

Lee, Raby, Ratz, Ross,
Schieren, Schmidt-Hoberg
Vaudevange (2010)

- superspace variable θ charged under R -symmetry
- forbids $H_u H_d$ as well as $QQQL$
- both generated when Z_4^R breaks down to M_p

Z_4^R from $U(1)_{\text{FN}}^R$

- define $U(1)_{\text{FN}}^R$ charges on superfields, e.g. $Q = \tilde{q} + \theta q + \theta^2 F_q$
- fix normalisation such that

$$\boxed{X_\phi = -1 \quad \rightarrow \quad X_\theta = R}$$

- terms allowed by $U(1)_{\text{FN}}^R$ have

$$X_{\text{term}} = \begin{cases} 2R + \text{integer} & \text{(superpotential)} \\ 0 + \text{integer} & \text{(Kähler potential)} \end{cases}$$

- require: superpotential \rightarrow Yukawas $Q_i U_j^c H_u, Q_i D_j^c H_d, L_i E_j^c H_d$
Kähler potential \rightarrow μ -term $H_u H_d$

- each term $\left\{ \begin{array}{l} \text{allowed} \\ \text{forbidden} \end{array} \right\}$ by Z_4^R also $\left\{ \begin{array}{l} \text{allowed} \\ \text{forbidden} \end{array} \right\}$ by $U(1)_{\text{FN}}^R$ if

$X_{H_d} - X_{L_1}$	$3X_{Q_1} + X_{L_1}$	$X_\theta = R$	X_ϕ
$\text{integer} \pm \frac{1}{4}$	integer	$\text{integer} \pm \frac{1}{4}$	-1

Dreiner, Luhn,
Opferkuch
(in preparation)

Anomalies and $U(1)_{\text{FN}}^R$ breaking

- anomaly cancellation via Green-Schwarz
- gauge coupling unification $g_C = g_W = \sqrt{\frac{5}{3}}g_Y$

$$\mathcal{A}_{CCR} = \mathcal{A}_{WWR} = \frac{3}{5} \mathcal{A}_{YYR} \neq 0 \quad \mathcal{A}_{YRR} = 0$$

- $U(1)_{\text{FN}}^R$ charges enter with, e.g., $X_{Q_1} - R$

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- “anomalous” nature of $U(1)_{\text{FN}}^R$
 - Dine-Seiberg-Wen-Witten mechanism
(Fayet-Iliopoulos term and unbroken supersymmetry)
 - induces VEV of flavon field

$$\frac{\langle \phi \rangle}{M_{\text{grav}}} = \frac{g_C}{4\pi} \sqrt{\mathcal{A}_{CCR}} \stackrel{!}{\approx} \lambda \approx 0.22$$

Yukawas and μ -term

- encode fermion mass hierarchies in $U(1)_{\text{FN}}^R$ charges, e.g.

$$Y_{11}^u \approx \lambda^8 \quad \longrightarrow \quad X_{Q_1} + X_{U_1^c} + X_{H_u} = 8 + 2R$$

$$Y_{22}^u \approx \lambda^4 \quad \longrightarrow \quad X_{Q_2} + X_{U_2^c} + X_{H_u} = 4 + 2R$$

- μ -term from Kähler potential via Giudice-Masiero mechanism
- introduce $U(1)_{\text{FN}}^R$ neutral hidden sector field Z

$$\mathcal{L} \sim \int d^2\theta \int d^2\bar{\theta} \left(\frac{Z^*}{M_{\text{grav}}} \right) \cdot \left(\frac{\phi}{M_{\text{grav}}} \right)^{X_{H_u} + X_{H_d}} H_u H_d$$

- SUSY broken by $\langle F_Z \rangle = m_{\text{soft}} M_{\text{grav}} \quad [Z_4^R \rightarrow M_p]$

$$\int d^2\theta \int d^2\bar{\theta} \underbrace{\left(\frac{\langle F_Z \rangle^* \bar{\theta}^2}{M_{\text{grav}}} \right)}_{m_{\text{soft}}} \cdot \lambda^{|X_{H_u} + X_{H_d}|} H_u H_d$$

Neutrino sector

► without right-handed neutrinos

- $H_u L_i, L_i Q_j D_k^c, L_i L_j E_k^c$ - forbidden by Z_4^R ☹️
- $\frac{1}{M_{\text{grav}}} L_i H_u L_j H_u$ - too small ☹️

► with right-handed neutrinos

- seesaw mechanism

$$m_\nu = m_D \cdot M_M^{-1} \cdot m_D^T$$

- Dirac term $L_i N_j^c H_u$ from superpotential

$$(X_{L_i} + X_{N_j^c} + X_{H_u} - 2R) \in \mathbb{N} \quad \rightarrow \quad m_D^{ij} \sim \langle H_u \rangle \cdot \lambda^{X_{L_i} + X_{N_j^c} + X_{H_u} - 2R}$$

- Majorana term $N_i^c N_j^c$ from super- or Kähler potential

$$(X_{N_i^c} + X_{N_j^c} - 2R) \in \mathbb{N} \quad \rightarrow \quad M_M^{ij} \sim M_{\text{grav}} \cdot \lambda^{X_{N_i^c} + X_{N_j^c} - 2R}$$

$$(X_{N_i^c} + X_{N_j^c}) \in \mathbb{Z} \quad \rightarrow \quad M_M^{ij} \sim m_{\text{soft}} \cdot \lambda^{|X_{N_i^c} + X_{N_j^c}|}$$



Example & summary

i	X_{Q_i}	$X_{U_i^c}$	$X_{D_i^c}$	X_{L_i}	$X_{N_i^c}$	$X_{E_i^c}$
1	$\frac{337}{76}$	$\frac{419}{76}$	$\frac{47}{76}$	$\frac{53}{76}$	$\frac{9}{4}$	$\frac{407}{76}$
2	$\frac{261}{76}$	$\frac{191}{76}$	$-\frac{29}{76}$	$\frac{53}{76}$	$\frac{9}{4}$	$\frac{179}{76}$
3	$\frac{109}{76}$	$\frac{39}{76}$	$-\frac{29}{76}$	$-\frac{23}{76}$	$\frac{9}{4}$	$\frac{103}{76}$

X_ϕ	=	-1
X_θ	=	$-\frac{1}{4}$
X_{H_u}	=	$-\frac{93}{38}$
X_{H_d}	=	$-\frac{21}{38}$

$\text{MSSM} + U(1)_{\text{FN}}^R \left\{ \begin{array}{l} \blacktriangleright \text{discrete symmetry } Z_4^R \\ \blacktriangleright \text{family structure} \end{array} \right.$
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- proton stabilised
- μ -term from Giudice-Masiero
- neutrino masses from seesaw

Thank you