New Aspects of Heterotic/F-theory Duality

Lara B. Anderson

Harvard University

Work done in collaboration with:

W. Taylor (MIT)

PLANCK 2013 – Bonn

May 23rd, 2013

Motivation

- String theory is a powerful extension of quantum field theory, but extracting low-energy physics from string geometry is mathematically challenging...

  Higher dimensional geometry → String Comp. → $4d$ physics

- Need a good toolkit in any corner of string theory to extract the full low energy physics: (missing structure in the $N = 1$ lagrangian, coupings, moduli stabilization, etc.)

- String Pheno: What are the rules for "top down" model building? Patterns/Constraints/Predictions?

- Is it "Anything goes"? Or no viable models at all?

- Finiteness?

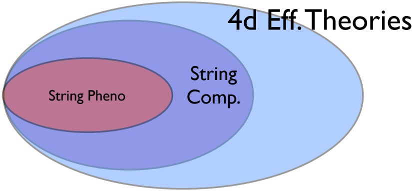
Much recent work: Classifying effective theories, scanning for models/patterns

(For this work: Taylor ($6d$ F-theory), LA, Gray and Lukas ($4d$ Heterotic))

- **Goal**: Combine two approaches.

  Consider $4d$ $N = 1$, Dual Heterotic-F-theory Vacua

  - Try to understand/classify how topology constrains effective theories
  - Complementary approach to large-scale scanning
  - Develop new tools for string pheno



A smooth $E_8 \times E_8$ heterotic model:

- The geometric ingredients include:
  - A Calabi-Yau 3-fold, X_3
  - Two holomorphic vector bundles, (V_1, V_2) on X (with structure group $G \subset E_8$)
- Compactifying on X leads to $\mathcal{N} = 1$ SUSY in $4D$, while V breaks $E_8 \to H \times G$. H_i are the structure groups of V_i and G_1 is the $4d$ GUT group (G_2 a hidden sector)
  - E.g. $H = SU(n)$, $n = 3, 4, 5$ leads to $G = E_6, SO(10), SU(5)$
- Matter and Moduli
  - H-charged matter, $H^1(X, V)$, $H^1(X, V^\vee)$, $H^1(X, \wedge^2 V)$, ...
  - $X \Rightarrow h^{1,1}(X)$ - Kähler moduli and $h^{2,1}(X)$ - Complex structure moduli
  - $V \Rightarrow h^1(X, End_0(V))$ Bundle moduli

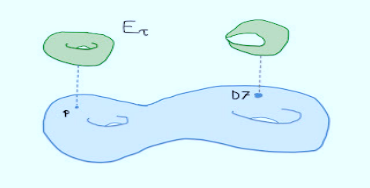
F-theory

- Geometric ingredients:
  - An elliptically fibered Calabi-Yau 4-fold, $\pi : Y_4 \xrightarrow{\mathbb{E}} B_3$
  - If the fibration has a section, Y_4 can be written in Weierstrass form

$$y^2 = x^3 + f(u_i)x + g(u_i)$$

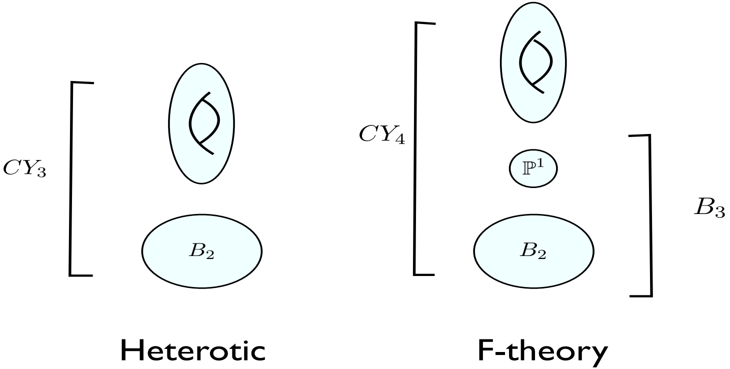
  u_i coords on B_3, $f \in H^0(B_3, K_{B_3}^{-4})$, $g \in H^0(B_3, K_{B_3}^{-6})$

- Degenerations of $\mathbb{E}$-fiber encode positions of 7-branes. $\Delta = 4f^3 + 27g^2 = 0$



- Divisors $D \subset B_3 \Rightarrow$ GUT Symmetries. Curves, $C \subset B_3 \Rightarrow$ matter.
- Also G-flux $\in H^{2,2}(Y_4)$

Duality

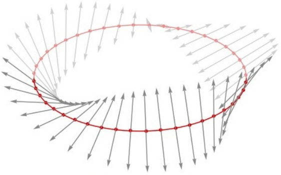


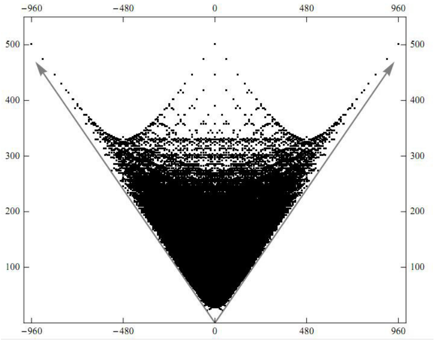
Heterotic on $\pi_h : X_n \xrightarrow{\mathbb{E}} B_{n-1} \quad \Leftrightarrow \quad$ F-theory on $\pi_f : Y_{n+1} \xrightarrow{K3} B_{n-1}$

Where these two theories are dual, there is a finite set of geometries to count

A finite class of geometries

- The number of elliptically fibered CY 3-folds, X_3, is finite (M. Gross)
- What about the number of vector bundles (V_1, V_2) over X_3?





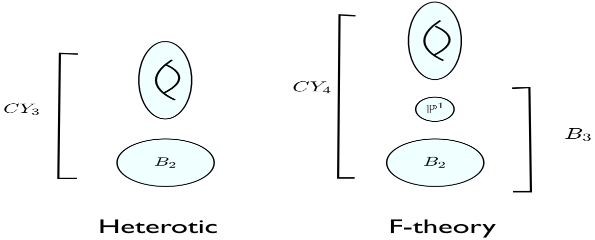
- $(h^{1,1}(X_3), h^{2,1}(X_3))$
- $\mathbb{E}$-fibered 3-folds "extremal" in known data set (Taylor, Candelas, Ooguri)

The topology of V: a total Chern class: $c(V) = (rank, c_1, c_2, c_3)(V)$

- Moduli Space: $\mathcal{M}_\omega(rk, c_1, c_2, c_3)$
- For *fixed topology* it is known that $\mathcal{M}$ is compact and has only finitely many components

  Bounds on Topology:
  - Sub-bundles of E_8: $rk < 8$ since $H \in E_8$
  - Spinors: $c_1 = 0 \pmod 2$
  - Anomaly cancellation $c_2(TX) = c_2(V_1) + c_2(V_2) + [W]_{eff} \Rightarrow$ $0 \leq c_2(V_i) \leq c_2(TX)$
  - For fixed c_2 can be shown that there are only finitely many values of c_3 compatible with $\mathcal{N} = 1$ supersymmetry (slope-stability of the bundle). (A. Langer)
- Hence, for bundles on elliptically fibered 3-folds X_3, we have, in principle, a finite set of compactification geometries to consider!

The Plan...



- Build dual (X_3, Y_4) pairs using dataset of $61,539$ toric surfaces, B_2 (Morrison + Taylor)
  - Caveats: All fibrations w/ section. B_3 constructed as a $\mathbb{P}^1$-bundle over B_2.
  - Only 16 of these B_2 lead to smooth X_3 $\Rightarrow$ Start with these.
- Use Y_4 to determine information about $\mathcal{M}(c(V))$ over X_3
- Use X_3 to further determine EFT assoc. to Y_4.

η: Building bundles and B_3

- Idea: Choose topology of bundles $(V_1, V_2) \Leftrightarrow$ Build $\pi_1 : B_3 \to B_2$

Heterotic:

- Can expand:

  $c_2(V_i) = \eta_i \wedge \omega_0 + \zeta_i$,

  w/ η_i (resp. ζ_i) $\{1,1\}$ (resp. $\{2,2\}$) forms on B_2 and ω_0 dual to the zero section.

- Anomaly Cancellation $\Rightarrow$

$$\eta_{1,2} = 6c_1(B_2) \pm t$$

- Can build B_3 over B_2 by "twisting" the $\mathbb{P}^1$ fibration (analog of $\mathbb{F}_n$ surfaces in $6d$)

- $c_1(B_3) = c_1(B_2) + 2\Sigma + t$ where Σ is dual to the zero-section of the $\mathbb{P}^1$-fiber

Can be shown that in Het/F-dual pairs, two t's are the same (FMW, Grimm + Taylor)

$N = 1$ Supersymmetry

Heterotic:

- X_3 a smooth CY 3-fold
- Bundles, V_i satisfy the Hermitian-Yang-Mills Eq.s:

  $F_{ab} = F_{\bar{a}\bar{b}} = 0 \qquad g^{a\bar{b}}F_{a\bar{b}} = 0$
- By Donaldson-Uhlenbeck-Yau Thm, HYM Sol'n $\Leftrightarrow$ Slope-stable Vector bundles
- Bogomolov Bound: If V is stable, $\int_X c_2(V) \wedge \omega \geq 0 \Rightarrow \eta$ is an effective curve class in B_2.

F-theory:

- $\mathcal{N} = 1 \Leftrightarrow Y_4$ can be resolved into a smooth Calabi-Yau 4-fold
- Need vanishing degrees of $(f, g, \Delta) \leq (4, 6, 12)$ on every divisor in B_3 or too singular to admit CY resolution.
- Likewise, f, g cannot vanish to orders $4, 6$ on any curve.
- These conditions on $t \Rightarrow \eta$ an effective curve class in B_2.

Example:

- Consider $B_2 = \mathbb{F}_1$ the Hirzebruch surface ($\mathbb{P}^1$ fibered over $\mathbb{P}^1$) with $h^{1,1}(B_2) = 2$ spanned by S, F with $S^2 = -1$, $S \cdot F = 1$ and $F^2 = 0$
- With B_3 constructed via the "twist" $t = 3S + 9F$
- Here Y_4 is generically singular with E_6 symmetry over $\Sigma = 0$.
- This symmetry cannot be deformed away in the C.S. moduli space of Y_4 (i.e. no matter available to "Higgs" it)
- But this carries non-trivial information about $V_{1,2}$...
  - V_2 with $\eta_2 = 6c_1(B) - t = 9S + 9F$
  - $G = E_6$ symmetry means V_2 is an $H = SU(3)$ bundle
  - Unbreakable $E_6 \Rightarrow \mathcal{M}(r, 0, (9S + 9F) \wedge \omega_0 + \zeta, c_3) = \emptyset \; \forall r > 3$
- Only E_6 GUTs possible for this topology!

Upper bounds on the structure group, H

- Constructed 4983 bases $B_3 \Leftrightarrow$ Triples (X_3, V_1, V_2).
- Constraints arising from "generic" symmetries on Y_4 provide rank(V)-dependent criteria for $\mathcal{M}(c(V)) = \emptyset$
- First examples by Rajesh and Berglund & Myer ('90s).
- Non-trivial information about higher-rank Donaldson-Thomas Invariants on CY 3-folds (notoriously hard to compute)

H	$\eta \geq Nc_1(B_2)$ $N =$
$SU(n)$	$n \quad (n \geq 2)$
$SO(7)$	4
$SO(m)$	$\frac{m}{2} \quad (m \geq 8)$
$Sp(k)$	$2k \quad (k \geq 2)$
F_4	$\frac{13}{3}$
G_2	$\frac{7}{2}$
E_6	$\frac{9}{2}$
E_7	$\frac{14}{3}$
E_8	5

We can go further...

Lower Bounds on the structure group, H

- For a bundle with $\eta = 9S + 9F$ on $\pi : X_3 \to \mathbb{F}_1$, can't build more than $H = SU(3)$. Can we build less?
- If the complex structure of Y_4 is specialized to try to produce say, E_7 symmetry (sending $H = SU(3) \to SU(2)$) then the manifold becomes too singular for the CY condition.
- Hence, no $SU(2)$ bundles exist w/ $\eta = 9S + 9F$ either.
- The symptom of this in B_3 are "exotic" matter curves, $C = \Sigma \cap S$ with $E_6 \to E_8$ enhancement.
- If we try to tune $H = SU(3) \to SU(2)$,

  $V_3 \to \mathcal{O}_{X_3}^{\oplus 3} + \mathcal{I}_{\eta}$, Small Instantons ($M5$-branes wrapping η)
- Harder-Narasimhan Filtrations of stable bundles $\Leftrightarrow$ Exotic F-theory matter curves.

Constraints

- Thus, this $B_3 \Leftrightarrow (X_3, V_1, V_2)$ is only compatible with E_6 symmetry.
- This is an example of topology which is only compatible with a single choice of gauge symmetry. Can be studied systematically (200 of 4000 examples)
- Also similar story with only certain matter spectra compatible with η.
- These observations help in understanding which geometries are compatible with Standard Model symmetries and particle spectra.

base B_2		$h_{1,1}$	# B_3's	NB (1)	NB (2)	F_4	$SO(8)$	$SU(3)$	$SU(2)$
(1, 1, 1)	($\mathbb{P}^2$)	1	19	0	0	0	0	0	0
(0, 0, 0, 0)	($\mathbb{F}_0$)	2	169	0	0	0	0	0	0
(1, 0, -1, 0)	($\mathbb{F}_1$)	2	163	0	0	0	0	0	0
(2, 0, -2, 0)	($\mathbb{F}_2$)	2	31	18	0	2	1	0	1
(0, 0, -1, -1, -1)	(dP_2)	3	595	0	0	0	0	0	0
(1, -1, -1, -2, 0)		3	196	111	0	9	7	0	7
(-1, -1, -1, -1, -1, -1)	(dP_3)	4	474	0	0	0	0	0	0
(0, -1, -1, -2, -1, -1)		4	378	204	0	22	16	2	6
(0, 0, -2, -1, -2, -1)		4	400	273	42	44	32	19	10
(1, 0, -2, -2, -1, -2)		4	72	40	25	7	6	3	4
(-1, -1, -2, -1, -2, -1, -1)		5	1266	851	140	156	123	70	46
(0, -1, -1, -2, -2, -1, -2)		5	446	253	150	51	43	23	30
(-1, -1, -2, -1, -2, -2, -1, -2)		6	379	175	185	58	53	31	26
(-1, -2, -1, -2, -1, -2, -1, -2)		6	289	171	69	56	38	23	2
(0, -2, -1, -2, -2, -2, -1, -2)		6	89	23	59	15	7	9	7
(-1, -2, -2, -1, -2, -2, -1, -2, -2)		7	36	8	26	0	5	4	5
total			4983	2127	696	420	331	184	144

Conclusions

- Dual $N = 1$ Heterotic/F-theory geometries are a fruitful arena for classifying/enumerating (a finite set) of phenomenologically relevant string vacua
- Y_4 provides non-trivial vanishing conditions for $\mathcal{M}(c(V))$ on X_3
- Upper and lower bounds on H for a given η
- Novel $4d$ features to be explored
  - Multiple components to the moduli space $\mathcal{M}(c(V)) \Rightarrow$ topologically equivalent non-diffeomorphic Y_4
  - Obstructed small instanton transitions (bundles which cannot be dissolved into 5-branes) $\Leftrightarrow$ G-flux and non-commutive $D3$-branes.
- Patterns/Predictions for how to select phenomenologically relevant string vacua

The End