

Non-geometric String Compactifications

Ralph Blumenhagen

Max-Planck-Institut für Physik, München



(Bhg, Deser, Plauschinn, Rennecke, Schmid , arXiv:1304.2784)

(Bhg, Gao, Herschmann, Shukla, to appear)



Introduction

Introduction

String theory is described by **2D non-linear sigma model**

$$\mathcal{S} = \frac{1}{2\pi\alpha'} \int_{\Sigma} d^2z (G_{ab} + B_{ab}) \partial X^a \bar{\partial} X^b + \dots ,$$

where **conformal invariance** provides the string equations of motion, which are captured by the effective action

$$S = \frac{1}{2\kappa^2} \int d^n x \sqrt{-|G|} e^{-2\phi} \left(R - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) .$$

There exist **conformal field theories** which **cannot** be identified with such simple large radius geometries (asymmetric orbifolds).

Introduction

Introduction

Applying **T-duality** using the Buscher rules leads to the chain of fluxes (Shelton, Taylor, Wecht, hep-th/0508133)

$$H_{abc} \leftrightarrow F_{ab}{}^c \leftrightarrow Q_a{}^{bc} \overset{?}{\leftrightarrow} R^{abc} ,$$

with geometric flux

$$[e_a, e_b] = F_{ab}{}^c e_c$$

for n -bein $e_a = e_a^i \partial_i$. Q and R are **non-geometric** fluxes.

- What is the **formal** description of these **fluxes**?
- Mutual consistency conditions: **independence?**, **Bianchi** identities?
- Effects for **compactifications** with these fluxes turned on?

Superpotential and moduli potential

Superpotential and moduli potential

Via T-duality, it was argued that all these fluxes generate a 4D (type IIA) superpotential (Shelton, Taylor, Wecht, Aldazabal, Camara, Font, Ibanez, Villadoro, Zwirner, Dibitetto, Guarino, Roest, ...)

$$W = -i \int_X \bar{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \bar{G}.$$

Superpotential and moduli potential

Via T-duality, it was argued that all these fluxes generate a 4D (type IIA) superpotential (Shelton, Taylor, Wecht, Aldazabal, Camara, Font, Ibanez, Villadoro, Zwirner, Dibitetto, Guarino, Roest, ...)

$$W = -i \int_X \bar{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \bar{G}.$$

with

$$\begin{aligned} \bar{\mathfrak{H}}_{ijk}^C = & \bar{H}_{ijk} + 3 \bar{F}^m \underline{[ij} (-iJ_c)_{\underline{k]m}} + 3 \bar{Q}_{\underline{[i}{}^{mn} (-iJ_c)_{\underline{j]m}} (-iJ_c)_{\underline{k]n}} \\ & + \bar{R}^{mnp} (-iJ_c)_{im} (-iJ_c)_{jn} (-iJ_c)_{kp} \end{aligned}$$

- Ω^C depends on **complex structure** moduli
- $J_c = J + iB$ depends on **Kähler** moduli

Superpotential and moduli potential

Via T-duality, it was argued that all these fluxes generate a 4D (type IIA) superpotential (Shelton, Taylor, Wecht, Aldazabal, Camara, Font, Ibanez, Villadoro, Zwirner, Dibitetto, Guarino, Roest, ...)

$$W = -i \int_X \bar{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \bar{G}.$$

There are claims that orientifolds with non-geometric fluxes generically

- stabilize moduli and in particular can lead to de-Sitter vacua (Danielsson, Dibitetto, 1212.4984), (Damian, Loaiza-Brito, 1304.0792)
- Contribute “negatively” to R-R tadpoles.

Superpotential and moduli potential

Via T-duality, it was argued that all these fluxes generate a 4D (type IIA) superpotential (Shelton, Taylor, Wecht, Aldazabal, Camara, Font, Ibanez, Villadoro, Zwirner, Dibitetto, Guarino, Roest, ...)

$$W = -i \int_X \bar{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \bar{G}.$$

- What is the 10D origin of this 4D scalar potential?
- Perform the dimensional reduction 10d \rightarrow 4d a la Taylor/Vafa
- String phenomenology/cosmology with generic fluxes?

Generalized geometry

Generalized geometry

Formal developments have led to proposals for **effective field theories** describing this **non-geometric** regime of string theory

- **Generalized Geometry**: manifold M with generalized bundle $E = TM \oplus T^*M$ (Hitchin, Gualtieri, Grana, Minasian, Petrini, Waldram, Coimbra, Strickland-Constable, ...)
- **Double field theory**: doubled coordinates $X^M = (\tilde{x}_i, x^i)$ tree-level $O(D, D)$ covariant action (Siegel, Hull, Zwiebach, Hohm, Kwak, Aldazabal, Baron, Marques, Nunez, Berman, Blair, Malek, Perry, Musaev, Thompson, Jeon, Lee, Park, ...)
- Generalization to **M-theory**: U-duality covariant actions

Doubled differential geometry is non-standard.

Generalized geometry

Generalized geometry

- **Generalized** tangent bundle $E = TM \oplus T^*M$ over a D -dimensional manifold M , $(X + \xi) \in \Gamma(E)$
- The natural **bilinear form** on the bundle E is

$$\langle X + \xi, Y + \zeta \rangle = \xi(Y) + \zeta(X),$$

- The transformations $\mathcal{M}^t \eta \mathcal{M} = \eta$ form the group $O(D, D)$.

$$\mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

Briefly review formal **framework** for string actions in $O(D, D)$ redefined fields (Andriot, Hohm, Larfors, Lüst, arXiv:1202.3060+1204.1979), (Bhg, Deser, Plauschinn, Rennecke, arXiv:1210.1591+1211.0030+1304.2784)

Generalized metric

Generalized metric

Combine the metric G_{ab} and Kalb-Ramond field B_{ab} into the generalized metric

Generalized metric

Combine the metric G_{ab} and Kalb-Ramond field B_{ab} into the **generalized metric**

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}.$$

Mass formula of toroidal compactification

$$M^2 = P^M \mathcal{H}_{MN} P^N + \text{oscil.}$$

with $P^M = (w_i, p^i)$.

Generalized metric

Combine the metric G_{ab} and Kalb-Ramond field B_{ab} into the **generalized metric**

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}.$$

$O(D, D)$ act on \mathcal{H} as

$$\hat{\mathcal{H}} = \mathcal{M}^t \mathcal{H} \mathcal{M}.$$

Main examples

$$\begin{pmatrix} A & 0 \\ 0 & (A^t)^{-1} \end{pmatrix}, \quad \begin{pmatrix} \mathbb{1} & 0 \\ -d\xi & \mathbb{1} \end{pmatrix}, \quad \begin{pmatrix} \mathbb{1} & \beta \\ 0 & \mathbb{1} \end{pmatrix}$$

$G_{\text{geom}} = (\text{diffeomorphisms}, \quad \text{B-field gauge trafos}) \quad \beta\text{-transforms}$

Field redefinition

Field redefinition

Action invariant under G_{geom} :

$$S(G, B) = \frac{1}{2\kappa^2} \int d^n x \sqrt{|G|} e^{-2\phi} \left(R - \frac{1}{12} H^2 + 4\partial_a \phi \partial^a \phi \right)$$

String action for $O(D, D)$ transformed fields (G, B) ?

Field redefinition

Action invariant under G_{geom} :

$$S(G, B) = \frac{1}{2\kappa^2} \int d^n x \sqrt{|G|} e^{-2\phi} \left(R - \frac{1}{12} H^2 + 4\partial_a \phi \partial^a \phi \right)$$

String action for $O(D, D)$ transformed fields (G, B) ?

$$\mathcal{M}^t \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix} \mathcal{M} = \mathcal{H}'(G, B)$$

Field redefinition

Action invariant under G_{geom} :

$$S(G, B) = \frac{1}{2\kappa^2} \int d^n x \sqrt{|G|} e^{-2\phi} \left(R - \frac{1}{12} H^2 + 4\partial_a \phi \partial^a \phi \right)$$

String action for $O(D, D)$ transformed fields (G, B) ?

$$\mathcal{H}'(G, B) = \mathcal{M}^t \mathcal{H}(G, B) \mathcal{M} \stackrel{!}{=} \begin{pmatrix} g - bg^{-1}b & bg^{-1} \\ -g^{-1}b & g^{-1} \end{pmatrix}$$

Field redefinition

Action invariant under G_{geom} :

$$S(G, B) = \frac{1}{2\kappa^2} \int d^n x \sqrt{|G|} e^{-2\phi} \left(R - \frac{1}{12} H^2 + 4\partial_a \phi \partial^a \phi \right)$$

String action for $O(D, D)$ transformed fields (G, B) ?

$$\mathcal{H}'(G, B) = \mathcal{M}^t \mathcal{H}(G, B) \mathcal{M} = \mathcal{H}(g, b)$$

One gets

$$g = \gamma^{-1} G (\gamma^{-1})^t \quad \text{with } \gamma = d + (G - B) b.$$

Field redefinition

Action invariant under G_{geom} :

$$S(G, B) = \frac{1}{2\kappa^2} \int d^n x \sqrt{|G|} e^{-2\phi} \left(R - \frac{1}{12} H^2 + 4\partial_a \phi \partial^a \phi \right)$$

String action for $O(D, D)$ transformed fields (G, B) ?

$$\mathcal{H}'(G, B) = \mathcal{M}^t \mathcal{H}(G, B) \mathcal{M} = \mathcal{H}(g, b)$$

One gets

$$g = \gamma^{-1} G (\gamma^{-1})^t \quad \text{with } \gamma = d + (G - B) b.$$

and similarly

$$b = \gamma^{-1} \left[\gamma (c + (G - B) a)^t - G \right] (\gamma^{-1})^t.$$

Quest for non-geometric actions

Quest for non-geometric actions

$$S(G, B) = \frac{1}{2\kappa^2} \int d^n x \sqrt{|G|} e^{-2\phi} \left(R - \frac{1}{12} H^2 + 4\partial_a \phi \partial^a \phi \right)$$

$$\begin{array}{c} \downarrow g(G, B) \\ \downarrow b(G, B) \\ S(g, b)? \end{array}$$

- Due to $\gamma = d + (G - B) b$, the computation is very cumbersome and the geometric symmetries mix in a complicated manner
- Need an **order** principle!
- Observation: The **differential geometry of Lie-algebroids** provides precisely that

Lie-algebroids

Lie-algebroids

Differential geometry is usually based on the tangent bundle. This can be generalized by introducing the notion of a **Lie-algebroid**:

Definition: Let M be a manifold, $E \rightarrow M$ a vector bundle together with a bracket $[\cdot, \cdot]_E : E \times E \rightarrow E$ satisfying the Jacobi identity, and a homomorphism $\rho : E \rightarrow TM$ called the **anchor-map**. Then $(E, [\cdot, \cdot]_E, \rho)$ is called Lie algebroid if the following **Leibniz rule** is satisfied

$$[s_1, f s_2]_E = f [s_1, s_2]_E + \rho(s_1)(f) s_2,$$

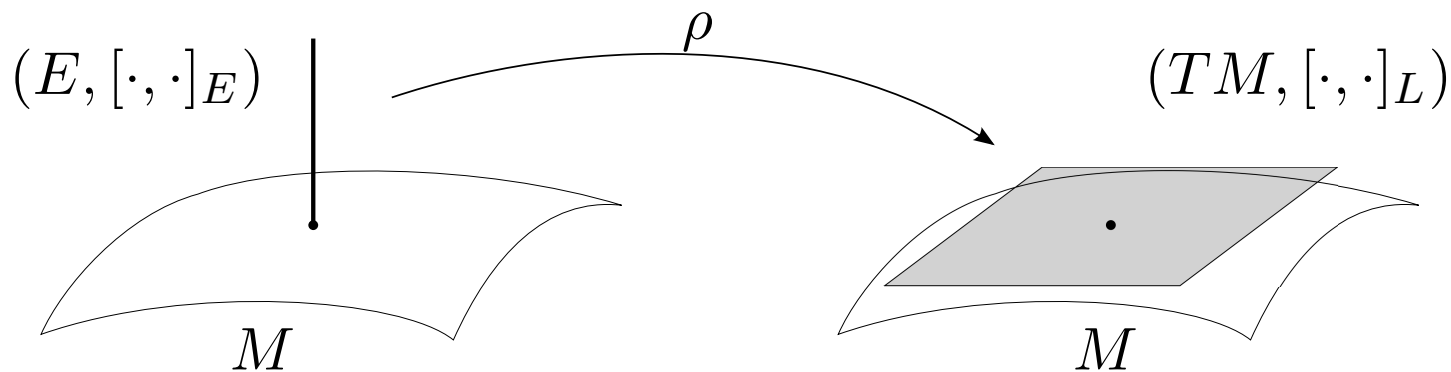
for $f \in C^\infty(M)$ and sections s_i of E .

Lie-algebroids

Lie-algebroids

- In a Lie algebroid vector fields and their Lie bracket $[\cdot, \cdot]_L$ are replaced by **sections** in E and the corresponding bracket.
- The relation between the different brackets is established by the **anchor** preserving the algebraic structure

$$\rho([s_1, s_2]_E) = [\rho(s_1), \rho(s_2)]_L ,$$



Non-geometric string action

Non-geometric string action

Thus we found:

$$S(G, B) = \int d^n x \sqrt{|G|} e^{-2\phi} \left(R - \frac{1}{12} H^2 + 4\partial_a \phi \partial^a \phi \right)$$

$$\begin{array}{c} \downarrow g(G, B) \\ \downarrow b(G, B) \end{array}$$

$$S(g, b) = \int d^n x \sqrt{|g|} |\det \rho^*| e^{-2\phi} \left(\hat{R} - \frac{1}{12} \Theta_{\alpha\beta\gamma} \Theta^{\alpha\beta\gamma} + 4D_\alpha \phi D^\alpha \phi \right).$$

By construction, this action

- for β -transf. makes visible non-geometric fields
- is **invariant** under diffeos, 2-form gauge transf.
- but **lacks** the symmetries needed for a **global** description of non-geometric backgrounds

Double field theory

Double field theory

Doubled coordinates $X^M = (\tilde{x}_i, x^i) \rightarrow$ DFT action

$$S_{\text{DFT}} = -\frac{1}{2\kappa^2} \int d^D x d^D \tilde{x} e^{-2d} \left(\frac{1}{8} \mathcal{H}^{MN} (\partial_M \mathcal{H}^{KL}) (\partial_N \mathcal{H}_{KL}) \right. \\ \left. - \frac{1}{2} \mathcal{H}^{MN} (\partial_N \mathcal{H}^{KL}) (\partial_L \mathcal{H}_{MK}) - 2 (\partial_M d) (\partial_N \mathcal{H}^{MN}) \right. \\ \left. + 4 \mathcal{H}^{MN} (\partial_M d) (\partial_N d) \right).$$

Double field theory

Doubled coordinates $X^M = (\tilde{x}_i, x^i) \rightarrow$ DFT action

$$S_{\text{DFT}} = -\frac{1}{2\kappa^2} \int d^D x d^D \tilde{x} e^{-2d} \left(\frac{1}{8} \mathcal{H}^{MN} (\partial_M \mathcal{H}^{KL}) (\partial_N \mathcal{H}_{KL}) \right. \\ \left. - \frac{1}{2} \mathcal{H}^{MN} (\partial_N \mathcal{H}^{KL}) (\partial_L \mathcal{H}_{MK}) - 2 (\partial_M d) (\partial_N \mathcal{H}^{MN}) \right. \\ \left. + 4 \mathcal{H}^{MN} (\partial_M d) (\partial_N d) \right).$$

String world-sheet: **doubled** zero modes

$$X_L^a(z) = \frac{x^a + \tilde{x}^a}{2} + (p^a - \frac{1}{2} w^a) (\tau - \sigma) + \text{osc.}$$

$$X_R^a(z) = \frac{x^a - \tilde{x}^a}{2} + (p^a + \frac{1}{2} w^a) (\tau + \sigma) + \text{osc.}$$

Double field theory

Doubled coordinates $X^M = (\tilde{x}_i, x^i) \rightarrow$ **DFT** action

$$S_{\text{DFT}} = -\frac{1}{2\kappa^2} \int d^D x d^D \tilde{x} e^{-2d} \left(\frac{1}{8} \mathcal{H}^{MN} (\partial_M \mathcal{H}^{KL}) (\partial_N \mathcal{H}_{KL}) \right. \\ \left. - \frac{1}{2} \mathcal{H}^{MN} (\partial_N \mathcal{H}^{KL}) (\partial_L \mathcal{H}_{MK}) - 2 (\partial_M d) (\partial_N \mathcal{H}^{MN}) \right. \\ \left. + 4 \mathcal{H}^{MN} (\partial_M d) (\partial_N d) \right).$$

Effective action for correlators of **vertex operators**

$$V(z, \bar{z}) = (h + B)_{ab} \partial X^a \bar{\partial} X^b \exp \left(p \cdot X + \omega \cdot \tilde{X} \right)$$

Level matching: $L_0 - \bar{L}_0 = 0 \Rightarrow p \cdot w = 0.$

Double field theory

Doubled coordinates $X^M = (\tilde{x}_i, x^i) \rightarrow$ DFT action

$$S_{\text{DFT}} = -\frac{1}{2\kappa^2} \int d^D x d^D \tilde{x} e^{-2d} \left(\frac{1}{8} \mathcal{H}^{MN} (\partial_M \mathcal{H}^{KL}) (\partial_N \mathcal{H}_{KL}) \right. \\ \left. - \frac{1}{2} \mathcal{H}^{MN} (\partial_N \mathcal{H}^{KL}) (\partial_L \mathcal{H}_{MK}) - 2 (\partial_M d) (\partial_N \mathcal{H}^{MN}) \right. \\ \left. + 4 \mathcal{H}^{MN} (\partial_M d) (\partial_N d) \right).$$

Strong constraint

$$\partial_i A \tilde{\partial}^i B + \tilde{\partial}^i A \partial_i B = 0 .$$

Double field theory

Double field theory

Symmetries:

- Doubled diffeomorphisms, i.e.

$$(\tilde{x}_i, x^i) \rightarrow (\tilde{x}_i + \tilde{\xi}_i(X), x^i + \xi^i(X))$$

x^i dependence: standard diffeomorphisms and B -field gauge transformations

\tilde{x}_i dependence: β -field gauge transformations.

- Global $O(D, D)$ symmetry:

$$\mathcal{H}' = h^t \mathcal{H} h, \quad d' = d,$$

$$X' = hX, \quad \partial' = (h^t)^{-1} \partial,$$

T-duality in DFT

Nature of fluxes in DFT (Bhg, Gao, Herschmann, Shukla, to appear)

T-duality in DFT

Nature of fluxes in DFT (Bhg, Gao, Herschmann, Shukla, to appear)

Example: T^3 with $H = \text{const}$. Choose $B_{12} = hx_3$.

T-duality in DFT

Nature of fluxes in DFT (Bhg, Gao, Herschmann, Shukla, to appear)

Example: T^3 with $H = \text{const.}$ Choose $B_{12} = hx_3$.

Generalized metric

$$\mathcal{H} = \left(\begin{array}{ccc|ccc} 1 + (hx_3)^2 & 0 & 0 & 0 & hx_3 & 0 \\ 0 & 1 + (hx_3)^2 & 0 & -hx_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & -hx_3 & 0 & 1 & 0 & 0 \\ hx_3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

T-duality in DFT

Nature of fluxes in DFT (Bhg, Gao, Herschmann, Shukla, to appear)

Example: T^3 with $H = \text{const.}$ Choose $B_{12} = hx_3$.

T-duality along x_1 : $\mathcal{H}' = \mathcal{T}_1^t \mathcal{H} \mathcal{T}_1$ with

$$\mathcal{T}_1 = \left(\begin{array}{ccc|ccc} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

T-duality in DFT

Nature of fluxes in DFT (Bhg, Gao, Herschmann, Shukla, to appear)

Example: T^3 with $H = \text{const}$. Choose $B_{12} = hx_3$.

One gets

$$\mathcal{H}' = \left(\begin{array}{ccc|ccc} 1 & -hx_3 & 0 & 0 & 0 & 0 \\ -hx_3 & 1 + (hx_3)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 + (hx_3)^2 & hx_3 & 0 \\ 0 & 0 & 0 & hx_3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

corresponding to the metric

$$ds^2 = (dx_1 - hx_3 dx_2)^2 + dx_2^2 + dx_3^2$$

on the **twisted torus** with vanishing B -field (Buscher rules).

T-duality in DFT

Nature of fluxes in DFT (Bhg, Gao, Herschmann, Shukla, to appear)

Example: T^3 with $H = \text{const.}$ Choose $B_{12} = hx_3$.

T-duality in the **non-isometric** x_3 direction.

$$\mathcal{H}'' = \left(\begin{array}{ccc|ccc} 1 + (h\tilde{x}_3)^2 & 0 & 0 & 0 & h\tilde{x}_3 & 0 \\ 0 & 1 + (h\tilde{x}_3)^2 & 0 & -h\tilde{x}_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & -h\tilde{x}_3 & 0 & 1 & 0 & 0 \\ h\tilde{x}_3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

i.e. a **flat three-torus** with B -field $B_{12} = h\tilde{x}_3$. The **geometric flux** should contain a piece $F^a{}_{bc} = \tilde{\partial}^a B_{bc} + \dots$

DFT Fluxes

DFT Fluxes

Introduce **non-holonomic** basis

$$\partial_a = e_a^i \partial_i \quad \tilde{\partial}^a = e_i^a \tilde{\partial}^i$$

The e_a^i depend on (\tilde{x}_i, x^i) .

DFT Fluxes

Introduce **non-holonomic** basis

$$\partial_a = e_a^i \partial_i \quad \tilde{\partial}^a = e_i^a \tilde{\partial}^i$$

The e_a^i depend on (\tilde{x}_i, x^i) .

For the **commutator** of two partial derivatives one gets

$$[\partial_a, \partial_b] = f^c_{ab} \partial_c$$

with

$$f^c_{ab} = e_i^c \left(\partial_a e_b^i - \partial_b e_a^i \right) .$$

DFT Fluxes

Introduce **non-holonomic** basis

$$\partial_a = e_a^i \partial_i \quad \tilde{\partial}^a = e_i^a \tilde{\partial}^i$$

The e_a^i depend on (\tilde{x}_i, x^i) .

Analogously, for the partial **winding** derivatives one finds

$$[\tilde{\partial}^a, \tilde{\partial}^b] = \tilde{f}_c^{ab} \tilde{\partial}^c$$

with

$$\tilde{f}_a^{bc} = e_a^i \left(\tilde{\partial}^b e_i^c - \tilde{\partial}^c e_i^b \right).$$

DFT Fluxes

Introduce **non-holonomic** basis

$$\partial_a = e_a^i \partial_i \quad \tilde{\partial}^a = e_i^a \tilde{\partial}^i$$

The e_a^i depend on (\tilde{x}_i, x^i) .

Consider the two **DFT vector fields**

$$\mathcal{D}_a = \partial_a + B_{am} \tilde{\partial}^m, \quad \tilde{\mathcal{D}}^a = \tilde{\partial}^a + \beta^{am} \mathcal{D}_m$$

and compute

$$[\mathcal{D}_a, \mathcal{D}_b] = F^c{}_{ab} \mathcal{D}_c + H_{abc} \tilde{\mathcal{D}}^c$$

$$[\mathcal{D}_a, \tilde{\mathcal{D}}^b] = Q_a{}^{bc} \mathcal{D}_c - F^b{}_{ac} \tilde{\mathcal{D}}^c$$

$$[\tilde{\mathcal{D}}^a, \tilde{\mathcal{D}}^b] = R^{abc} \mathcal{D}_c + Q_c{}^{ab} \tilde{\mathcal{D}}^c$$

see also ([Geissbuhler, Marquez, Nunez, Penaz, arXiv:1304.1472](#))

DFT Fluxes

DFT Fluxes

with the DFT flux

DFT Fluxes

with the DFT flux

$$H_{abc} = 3 \left(\partial_{[\underline{a}} B_{\underline{bc}]} + f^m_{[\underline{ab}} B_{\underline{c}]m} + B_{[\underline{am}} \tilde{\partial}^m B_{\underline{bc}]} + B_{[\underline{am}} B_{\underline{bn}} \tilde{f}_{\underline{c}]}^{mn} \right),$$

DFT Fluxes

with the DFT flux

$$F^c_{ab} = f^c_{ab} + \tilde{\partial}^c B_{ab} + \tilde{f}_a^{cm} B_{mb} + \tilde{f}_b^{cm} B_{am} + \beta^{cm} H_{mab},$$

DFT Fluxes

with the DFT flux

$$\begin{aligned} Q_c^{ab} = & \tilde{f}_c^{ab} + \partial_c \beta^{ab} + f^a_{cm} \beta^{mb} + f^b_{cm} \beta^{am} \\ & + B_{cm} \tilde{\partial}^m \beta^{ab} + 2\beta^{m[a} \tilde{\partial}^{b]} B_{mc} \\ & + 2B_{cm} \tilde{f}_n^{m[a} \beta^{b]n} + 2\beta^{m[a} \tilde{f}_c^{b]n} B_{mn} + \beta^{am} \beta^{bn} H_{mnc} \end{aligned}$$

DFT Fluxes

with the DFT flux

$$\begin{aligned}
 R^{abc} = & 3 \left(\tilde{\partial}^{[a} \beta^{bc]} + \tilde{f}_m^{[ab} \beta^{c]m} \right) + 3 \left(\beta^{[am} \partial_m \beta^{bc]} + \beta^{[am} \beta^{bn} f^{c]}_{mn} \right) \\
 & + 3 \left(B_{mn} \beta^{[am} \tilde{\partial}^n \beta^{bc]} + \beta^{[am} \beta^{bn} \tilde{\partial}^c] B_{mn} + \right. \\
 & \left. 2 \beta^{[am} \beta^{bn} \tilde{f}_{[m}^{c]k} B_{kn]} \right) + \beta^{am} \beta^{bn} \beta^{cp} H_{mnp} .
 \end{aligned}$$

DFT Fluxes

with the DFT flux

$$\begin{aligned}
 R^{abc} = & 3 \left(\tilde{\partial}^{[a} \beta^{bc]} + \tilde{f}_m^{[ab} \beta^{c]m} \right) + 3 \left(\beta^{[am} \partial_m \beta^{bc]} + \beta^{[am} \beta^{bn} f^{c]}_{mn} \right) \\
 & + 3 \left(B_{mn} \beta^{[am} \tilde{\partial}^n \beta^{bc]} + \beta^{[am} \beta^{bn} \tilde{\partial}^c] B_{mn} + \right. \\
 & \left. 2 \beta^{[am} \beta^{bn} \tilde{f}_{[m}^{c]k} B_{kn]} \right) + \beta^{am} \beta^{bn} \beta^{cp} H_{mnp} .
 \end{aligned}$$

In a given patch with geometric frame, one can turn on **all** components of H, F, Q .

Bianchi identities

Bianchi identities

The Jacobi identities for the brackets generate the 5 **Bianchi identities** for the fluxes.

$$\begin{aligned}
 & \mathcal{D}_{[a} H_{bcd]} - \frac{3}{2} H_{m[ab} F^m{}_{cd]} = 0 \\
 & -\frac{1}{3} \tilde{D}^d H_{abc} + \mathcal{D}_{[a} F^d{}_{bc]} + F^m{}_{[bc} F^d{}_{a]m} + H_{m[ab} Q_{c]}{}^{md} = 0 \\
 & 2\tilde{D}^{[c} F^{d]}{}_{[ab]} + 2\mathcal{D}_{[a} Q_{b]}{}^{[cd]} \\
 & \quad - F^m{}_{[ab} Q_m{}^{[cd]} + 4 F^{[c}{}_{m[a} Q_{b]}{}^{d]m} - H_{abm} R^{mcd} = 0 \\
 & -\frac{1}{3} D_d R_{abc} + \tilde{D}^{[a} Q_d{}^{bc]} + Q_m{}^{[bc} Q_d{}^{a]m} + R^m{}^{[ab} F^c]{}_{md} = 0 \\
 & \quad \tilde{D}^{[a} R^{bcd]} - \frac{3}{2} R^m{}^{[ab} Q_m{}^{cd]} = 0.
 \end{aligned}$$

Orientifold projection

Orientifold projection

Under **world-sheet parity** $\Omega : (\sigma, \tau) \rightarrow (-\sigma, \tau)$

$$\Omega : \begin{cases} \partial_a \rightarrow \partial_a, & \tilde{\partial}^a \rightarrow -\tilde{\partial}^a \\ B_{ab} \rightarrow -B_{ab}, & \beta^{ab} \rightarrow -\beta^{ab} \\ f^a{}_{bc} \rightarrow f^a{}_{bc}, & \tilde{f}_a{}^{bc} \rightarrow -\tilde{f}_a{}^{bc} \end{cases}$$

so that the fluxes transform as

$$\Omega : \begin{cases} H_{abc} \rightarrow -H_{abc} \\ F^a{}_{bc} \rightarrow F^a{}_{bc} \\ Q_a{}^{bc} \rightarrow -Q_a{}^{bc} \\ R^{abc} \rightarrow R^{abc} \end{cases}$$

All components of all **Ω -invariant** fluxes can be turned on.

Superpotential

Superpotential

For toroidal background, oxidize the scalar potential induced by the superpotential to 10D

$$W = -i \int_X \bar{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \bar{G}.$$

First results: The NS-NS part can be compactly written as

$$S_{\text{DFT}} = \int d^n x d^n \tilde{x} e^{-2d} \left[\mathcal{F}_{MNP} \mathcal{F}_{M'N'P'} \left(\frac{1}{4} \mathcal{H}^{MM'} \eta^{NN'} \eta^{PP'} - \frac{1}{12} \mathcal{H}^{MM'} \mathcal{H}^{NN'} \mathcal{H}^{PP'} + \dots \right) \right]$$

with $\mathcal{F}_{abc} = H_{abc}$, $\mathcal{F}^a{}_{bc} = F^a{}_{bc}$, $\mathcal{F}^{ab}{}_c = Q_c{}^{ab}$, $\mathcal{F}^{abc} = R^{abc}$.

Superpotential

As for the **GVW** superpotential $W = \int \Omega \wedge G_3$ (Taylor/Vafa), reducing the action

$$S_{\text{DFT}} = \int d^n x d^n \tilde{x} e^{-2d} \left[\mathcal{F}_{MNP} \mathcal{F}_{M'N'P'} \left(\frac{1}{4} \mathcal{H}^{MM'} \eta^{NN'} \eta^{PP'} - \frac{1}{12} \mathcal{H}^{MM'} \mathcal{H}^{NN'} \mathcal{H}^{PP'} + \dots \right) \right],$$

leads to the required F-term and a **D-term**, which is proportional to the **R-R tadpole** conditions.

Summary and outlook

Summary and outlook

- Briefly reviewed some recent **formal** developments in the description of non-geometric fluxes.
- Further future studies of de Sitter vacua and **non-geometric model building** should be performed (no runaway directions and no flux/anti-brane, anti-flux/brane config.)
- Further future developments: **Non-associative geometry** for R -flux?

Outlook: non-associativity

Outlook: non-associativity

In the non-geometric frames I+II, there exist a **quasi-Poisson** structure β^{ab} so that

$$\{f, g\} = \beta^{ab} (\partial_a f) (\partial_b g) ,$$

where $f, g \in \mathcal{C}^\infty(M)$. In general, this bracket does not satisfy the **Jacobi identity** but one finds

$$\{\{f, g\}, h\} + \text{cycl.} = R^{abc} (\partial_a f) (\partial_b g) (\partial_c h) .$$

Here, the non-geometric **R-flux** is given by

$$R^{abc} = 3 \beta^{[am} \partial_m \beta^{bc]}$$

The non-geometric flux **Q-flux** is defined as $Q_c^{ab} = \partial_c \beta^{ab}$.

Quantum non-associativity

Quantum non-associativity

Given a VEV to the fluxes, the quantum version of the classical Poisson relations for the coordinates becomes

$$[x^a, x^b] = \oint_{C_x} Q_c^{ab} dy^c, \quad [x^a, x^b, x^c] = R^{abc}.$$

Non-commutativity: Wilson line of Q -flux

Non-associativity: local R -flux

(Bouwknegt, Hannabuss, Mathai, arXiv:0412.092), (Bhg, Plauschinn, arXiv:1010.1263),
(Lüst, arXiv:1010.1361), (Bhg, Deser, Lüst, Plauschinn, Rennecke, arXiv:1106.0316),
(Mylonas, Schupp, Szabo, arXiv:1207.0926)

Quantum non-associativity

Given a VEV to the fluxes, the quantum version of the classical Poisson relations for the coordinates becomes

$$[x^a, x^b] = \oint_{C_x} Q_c^{ab} dy^c, \quad [x^a, x^b, x^c] = R^{abc}.$$

Non-commutativity: Wilson line of Q -flux

Non-associativity: local R -flux

(Bouwknegt, Hannabuss, Mathai, arXiv:0412.092), (Bhg, Plauschinn, arXiv:1010.1263),
(Lüst, arXiv:1010.1361), (Bhg, Deser, Lüst, Plauschinn, Rennecke, arXiv:1106.0316),
(Mylonas, Schupp, Szabo, arXiv:1207.0926)

In analogy to Seiberg-Witten theory, can one formulate a quantum non-commutative/non-associative gravity theory?