Hypercharge Flux and U(1) Symmetries in F-theory GUTs

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[EP ’12] [Mayrhofer, EP, Weigand ’12] [Mayrhofer, EP, Weigand ’13]
[Borchmann, Mayrhofer, EP, Weigand ’13]

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Supersymmetric Grand Unified Theories based on F-theory are well motivated proposals for the UV behaviour of the Standard Model of particle physics

• One of the (few) focus points in the string landscape following from few well motivated constraints:

1. The observed Top quark Yukawa ~O(1)
2. Supersymmetry + Gauge Coupling Unification → GUT

String theory: 5 10 10 come from adjoint of some simple group

Exceptional symmetries (E_6, E_7, E_8) → F-theory, M-theory, or Heterotic

• Additionally for F-theory:

1) Moduli Stabilisation relatively well (=best) understood
2) Mathematical machinery for constructing models
3) Use of hypercharge flux to break GUT

Why U(1) symmetries?

• Because they are a crucial part of BSM physics:

Global symmetries to control operators

Hidden forces and sectors

Non-decoupled U(1)s can influence MSSM parameters (Higgs tree-level mass)

…

• Because they are required for the simplest methods of generating chirality

• Because they are very constrained in F-theory due to the exceptional group structure implying an underlying E_8

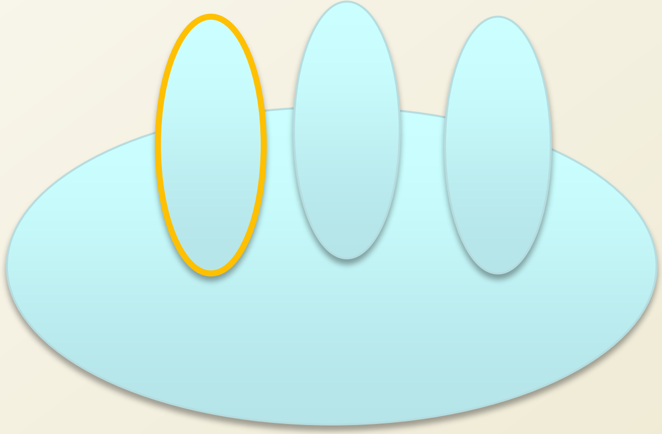
What is a U(1) symmetry (in F-theory)?

Field theory → By hand

String Theory → A single D-brane

In M/F-theory → Geometry

Type IIB String Theory



6D Calabi-Yau

F-theory

6D Base

Major progress over last year at understanding the geometry of the elliptic fibration for U(1)s in F-theory

Generic Weierstrass form in $P_{[2,3,1]}$ has 0 U(1)s

$$y^2 = x^3 + f x z^4 + g z^6$$

$$z = 0$$

The most general fibration with 1 U(1) is quartic in $P_{[1,1,2]}$ with one point blown up (s)

[Morrison,Park '12]

$$\mathrm{B}\,\mathrm{v}^2\,\mathrm{w} + s\,\mathrm{w}^2 = \mathrm{C}_3\,\mathrm{v}^3\,\mathrm{u} + \mathrm{C}_2\,s\,\mathrm{v}^2\,\mathrm{u}^2 + \mathrm{C}_1\,s^2\,\mathrm{v}\,\mathrm{u}^3 + \mathrm{C}_0\,s^3\,\mathrm{u}^4\,,$$

The most general fibration with 2 U(1)s is cubic in $P_{[1,1,1]}$ with two points blown up (s_0, s_1)

[Borchmann, Mayrhofer, EP, Weigand '13] [Cvetic, Klevers, Piragua'13]

$$\mathrm{v}\,\mathrm{w}(c_1\,\mathrm{w}\,s_1 + c_2\,\mathrm{v}\,s_0) + \mathrm{u}\,(b_0\,\mathrm{v}^2\,s_0^2 + b_1\,\mathrm{v}\,\mathrm{w}\,s_0\,s_1 + b_2\,\mathrm{w}^2\,s_1^2) + \mathrm{u}^2(d_0\,\mathrm{v}\,s_0^2\,s_1 + d_1\,\mathrm{w}\,s_0\,s_1^2 + d_2\,\mathrm{u}\,s_0^2\,s_1^2) = 0$$

Non-Abelian gauge symmetries are better understood as ADE singularities, so in model building often useful to start from the form of the U(1) then induce the required non-Abelian sector

For single U(1) there is toric classification of 5 possibilities

[Keitel, Braun, Grimm '13] [Borchmann, Mayrhofer, EP, Weigand '13]

Curve on $\{w=0\}$	Matter representation
$\{b_1 = 0\}$	$\mathbf{10}_2 + \overline{\mathbf{10}}_{-2}$
$\{b_2 = 0\}$	$\mathbf{5}_6 + \overline{\mathbf{5}}_{-6}$
$\{b_1 c_{3,1} + b_2 c_{2,1} = 0\}$	$\mathbf{5}_{-4} + \overline{\mathbf{5}}_4$
$\{b_1^2 c_{0,4} - b_{0,2} b_1 c_{1,2} - c_{1,2}^2 = 0\}$	$\mathbf{5}_1 + \overline{\mathbf{5}}_{-1}$

For two U(1)s there is toric classification of 9 possibilities

[Borchmann, Mayrhofer, EP, Weigand '13]

Curve on $\{w=0\}$	Matter representation
$\{b_1=0\}$	$\mathbf{10}_{-1,-3}+\overline{\mathbf{10}}_{1,3}$
$\{b_2=0\}$	$\mathbf{5}_{-3,-4}+\overline{\mathbf{5}}_{3,4}$
$\{c_{1,1}=0\}$	$\mathbf{5}_{2,6}+\overline{\mathbf{5}}_{-2,-6}$
$\{c_{2,2}=0\}$	$\mathbf{5}_{2,-4}+\overline{\mathbf{5}}_{-2,4}$
$\{b_{0,2}b_1-c_{2,2}d_0=0\}$	$\mathbf{5}_{-3,1}+\overline{\mathbf{5}}_{3,-1}$
$\{b_2d_0^2+b_1(b_1d_2-d_0d_1)=0\}$	$\mathbf{5}_{2,1}+\overline{\mathbf{5}}_{-2,-1}$

There are also constructions that can not be seen through toric geometry which allow to recreate the general embedding of SU(5) inside E_8 which has up to 4 U(1)s : Factorised Tate Models

[Mayrhofer, EP, Weigand '12]

$$C_{\mathbf{10}^{(1)}}: \quad q_{\mathbf{10}^{(1)}}=-2, \qquad C_{\mathbf{10}^{(2)}}: \quad q_{\mathbf{10}^{(2)}}=3,$$

$$C_{\mathbf{5}^{(1)}}: q_{\mathbf{5}^{(1)}}=-6, \qquad C_{\mathbf{5}^{(2)}}: q_{\mathbf{5}^{(2)}}=4, \qquad C_{\mathbf{5}^{(3)}}: q_{\mathbf{5}^{(3)}}=-1,$$

$$C_{\mathbf{1}^{(1)}}: q_{\mathbf{1}^{(1)}}=-5.$$

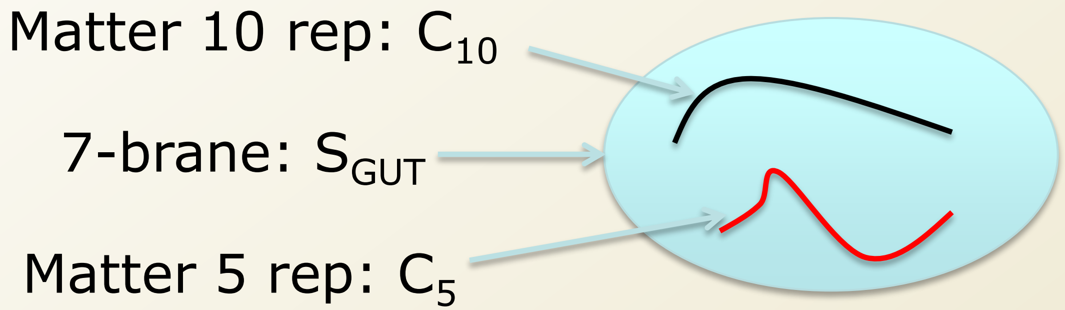
However the resolution of the singularities has only been performed for the case of 1 U(1) in these models

Why Hypercharge Flux?

• Because 4D GUTs are nice only until we have to break the GUT group: doublet triplet splitting?

[Buican et al '06, Beasley et al. '08, Donagi, Wijnholt '08]

Hypercharge flux can be used to break the GUT group in F-theory and induce doublet-triplet splitting



Hypercharge flux along the matter curve:

$$\int_{C_5} f_Y = \chi\left(\mathbf{1},\mathbf{2}\right)_{\frac{1}{2}} - \chi\left(\mathbf{3},\mathbf{1}\right)_{-\frac{1}{3}} = N$$

• Rare treasure in String Phenomenology: a well motivated approach to an essential aspect of GUTs that is intrinsically string theoretic in nature

• Flux quantisation explains proximity of GUT scale to Planck scale

Hypercharge flux and gauge coupling unification

Hypercharge flux splits the gauge couplings at the GUT scale in IIB string theory

[Blumenhagen ’08, Donagi, Wijnholt]

$$S_{CS} \supset \frac{1}{4}\int_{\mathbb{R}^{1,3}} C_0 \,\mathrm{tr}\left[F \wedge F \int_S f \wedge f\right]$$

$$\tfrac{1}{4}\left(\mathrm{tr}\left(F^2_{SU(3)}\right) N + \mathrm{tr}\left(F^2_{SU(2)}\right)[N + M] + \tfrac{5}{6}F^2_Y\,[N + \tfrac{3}{5}M] + \ldots\right)$$

$$N = \int_S \tilde{f}_S \wedge C_0 \tilde{f}_S \qquad M = \int_S f_Y \wedge \left(C_0 f_Y + 2C_0 \tilde{f}_S\right) \qquad \tilde{f}_S \equiv f_S - \tfrac{2}{5} f_Y$$

The F-theory analogue of this is not well understood, but would prefer not to rely on a cancellation to restore gauge coupling unification at strong coupling

Rough estimate of the size of the splitting: 5-10%

$$\delta \mathrm{Re} f_i = \frac{1}{2}\delta_i \int_S f_Y \wedge \left[C_0\left(2\tilde{f}_S + f_Y\right)\right] \qquad \int_S f_Y \wedge f_Y = -2\,. \qquad \tilde{f}_S = 0 \qquad g_s = 1$$

Result derived by a particular choice of twisting of U(1) flux by hypercharge flux which is required for integer quantisation

$$\begin{aligned}
\text{bulk}: \quad & (\mathbf{3},\mathbf{2})_{-5/6} \quad : -f_Y \, , \\
\mathbf{10}: \quad & (\mathbf{3},\mathbf{2})_{1/6} \quad : \frac{1}{5}f_Y + 2f_S \, , \quad (\bar{\mathbf{3}},1)_{-2/3} \, : -\frac{4}{5}f_Y + 2f_S \, , \quad (1,1)_1 \, : \frac{6}{5}f_Y + 2f_S \, , \\
\mathbf{5}: \quad & (\mathbf{3},1)_{-1/3} \quad : -\frac{2}{5}f_Y + f_S - f_i \, , \quad (1,\mathbf{2})_{1/2} \, : \frac{3}{5}f_Y + f_S - f_i \, .
\end{aligned}$$

$f_Y \, , \, \tilde{f}_S \equiv f_S - \frac{2}{5} f_Y \, , \, f_i$ integer

Alternative twisting, which is more appropriate in F-theory, implies a factor of 5 reduction in the splitting

$f_Y \;\; \tilde{f}_i = f_i + \frac{2}{5} f_Y \, , \, f_S$ integer

[Mayrhofer, EP, Weigand ’13]

$$\delta \mathrm{Re} f_i = \frac{1}{2}\delta_i \int_S f_Y \wedge \left[C_0 \left(2\tilde{f}_S + f_Y \right) \right) \longrightarrow \delta \mathrm{Re} f_i = \frac{1}{10}\delta_i C_0 \int_S f_Y \wedge f_Y$$

Splitting, 1-2%, is therefore well within MSSM 2-loop results

Hypercharge flux, Anomalies and Exotics

In order to keep the hypercharge generator massless hypercharge flux should have vanishing intersection with the pull-back of 2-forms in the base

Type IIB

$$\int_{D7} C_4 \wedge F \wedge F$$

$$S^2_{St} \simeq \int_{\mathbb{R}^{1,3}} \mathrm{tr}(T_Y^2) F_Y \wedge c_2^\alpha \int_S f_Y \wedge \iota^* \omega_\alpha$$

$$\int_S f_Y \wedge \iota^* \omega_\alpha \stackrel{!}{=} 0 \quad \forall \quad \omega_\alpha \in H^{1,1}_+(X_3)$$

F-theory

$$\int_{11D} C_3 \wedge G_4 \wedge G_4$$

$$\int_S f_Y \wedge \iota^* \mathrm{w}_\alpha \stackrel{!}{=} 0 \quad \forall \quad \mathrm{w}_\alpha \in H^{1,1}(B_3)$$

Implies hypercharge flux can not modify the Green-Schwarz anomaly cancellation mechanism

[Marsano '10]

Non-GUT anomalies and GUT ones must be proportional

$$\mathcal{A}_{SU(3)^2-U(1)} \propto \mathcal{A}_{SU(2)^2-U(1)} \propto \mathcal{A}_{U(1)_Y^2-U(1)} \propto \mathcal{A}_{\mathbf{SU(5)^2}-U(1)} \,,$$
$$\mathcal{A}_{SU(3)^2-SU(3)} \propto \ldots \propto \mathcal{A}_{U(1)_Y^2-U(1)_Y} \propto \mathcal{A}_{\mathbf{SU(5)^2-SU(5)}} = 0 \,,$$
$$\mathcal{A}_{U(1)_Y-U(1)^2} \propto \mathcal{A}_{SU(3)-U(1)^2} \propto \mathcal{A}_{SU(2)-U(1)^2} \propto \mathcal{A}_{\mathbf{SU(5)}-U(1)^2} = 0 \,.$$

2 of these constraints were known to automatically arise from the geometry of local models

A $$\sum_i \int_S f_Y \wedge [C_{\mathbf{5}^{(i)}}] = \sum_j \int_S f_Y \wedge [C_{\mathbf{10}^{(j)}}] = 0 \,.$$ [Donagi, Wijnholt '08]

B $$\sum_i Q_A^i \int_S f_Y \wedge [C_{\mathbf{5}^{(i)}}] + \sum_j Q_A^j \int_S f_Y \wedge [C_{\mathbf{10}^{(j)}}] = 0 \,.$$ [Dudas, EP '10]

Conjecture constraint of geometry to satisfy 3rd equation

C $$\sum_i Q_i^A Q_i^B \int_S f_Y \wedge [C_i^{\mathbf{5}}] + 3 \sum_j Q_j^A Q_j^B \int_S f_Y \wedge [C_j^{\mathbf{10}}] = 0 \,,$$ [EP '12]

Strongly constrain possible U(1) symmetries (including those made massive by flux that remain as global symmetries)

Particularly sharp constraints on $U(1)_{PQ}$ symmetries which are defined by the Higgs pair having different charges

Important implications for the idea of NMSSM [XH_uH_d] coupling from U(1) global symmetry. (Also for dimension 5 proton decay.)

B → $U(1)_{PQ}$ and doublet-triplet splitting implies massless non-GUT exotics [Dudas, EP '10]

C → Only two types of models based on breaking E_8 can support any net hypercharge restriction [EP '12]

No viable doublet-triplet splitting with only 1 additional $U(1)_{PQ}$

Always find exotics in incomplete 10 representations which is bad for gauge coupling unification

Insight from IIB: hypercharge flux is not globally trivial in IIB with respect to the orientifold odd sector

$$\int_{D7} C_4 \wedge F \wedge F$$

$$S^2_{St} \simeq \int_{\mathbb{R}^{1,3}} \mathrm{tr}(T_Y^2) F_Y \wedge c_2^\alpha \int_S f_Y \wedge \iota^* \omega_\alpha$$

Orientifold even forms

$\int_S f_Y \wedge \iota^* \omega_a \neq 0$ for $\omega_a \in H^{1,1}_-(X_3)$ possible

[Mayrhofer, EP, Weigand '13]

Can contribute to anomaly cancellation through the terms

$$\int_{\mathbb{R}^{1,3}} c_a^0 F_Y \wedge F_S \int_S 2\mathrm{tr}(T_Y^2) f_Y \wedge \iota^* \omega_a +$$
$$\int_{\mathbb{R}^{1,3}} c_a^0 F_Y \wedge F_Y \int_S \left(\mathrm{tr}(T_Y^3) f_Y + \mathrm{tr}(T_Y^2) f_S\right) \wedge \iota^* \omega_a$$

$$\int_{D7} C_2 \wedge F \wedge F \wedge F$$

Implies that in IIB hypercharge flux can induce anomalies, and the constraints can be relaxed

Only if the U(1)s are geometrically massive from $\int_{D7} C_6 \wedge F$

$$(F_I \wedge c_2^\alpha) - (c_\alpha^0\, F_J \wedge F_K) \qquad \text{or} \qquad (F_I \wedge c_a^2) - (c_0^a\, F_J \wedge F_K).$$

Can construct models in IIB with $U(1)_{PQ}$, hypercharge flux doublet-triplet splitting, and no exotics

Uplift to F-theory not well understood; conjecture that they arise from reducing C_3 on non-closed 2-forms

$$C_3 = A_i \wedge \mathrm{w}_i, \qquad \mathrm{dw}_i \neq 0$$

[Grimm,Weigand] [Grimm,Kerstan,EP, Weigand] [Krause,Mayrhofer,Weigand]

Need to understand better as crucial for doublet-triplet splitting, hypercharge flux, U(1) symmetries, exotics…

Summary

Lots of progress in constructing F-theory GUT models that have U(1) symmetries

Hypercharge flux compatible with precision gauge coupling unification

Hints from IIB that it might be possible to induce doublet-triplet splitting by hypercharge flux in the MSSM+$U(1)_{PQ}$ without any exotics in F-theory

The geometric uplift to F-theory of the orientifold odd sector in IIB crucial for model building phenomenology

F-theory GUTs remain one of the best motivated UV completions of the SM, much learned and yet to learn, model building improving in phenomenology and in explicit constructions

Thank You