Fermion Mixing & Flavor Symmetries: Ideas & Models

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Outline

- Lepton mixing: parametrization and experimental results
- Lepton mixing from non-trivial breaking of G_f & CP

  (Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. ('12))
- Model with S_4 and CP for leptons ***(Feruglio et al. ('12,'13))***
- Conclusions

Parametrization of lepton mixing

Parametrization *(PDG)*

$$U_{PMNS} = \tilde{U}\ \mathrm{diag}(1, e^{i\alpha/2}, e^{i(\beta/2+\delta)})$$

with

$$\tilde{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

and $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Jarlskog invariant J_{CP}

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left[U_{PMNS,11}U^*_{PMNS,13}U^*_{PMNS,31}U_{PMNS,33}\right] \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta \end{aligned}$$

Experimental results on lepton mixing

Latest global fits ***(Gonzalez-Garcia et al. ('12))***

best fit and $1\,\sigma$ error	$3\,\sigma$ range
$\sin^2\theta_{13} = 0.0227^{+0.0023}_{-0.0024}$	$0.0156 \leq \sin^2\theta_{13} \leq 0.0299$
$\sin^2\theta_{12} = 0.302^{+0.013}_{-0.012}$	$0.267 \leq \sin^2\theta_{12} \leq 0.344$
$\sin^2\theta_{23} = \begin{cases} 0.413^{+0.037}_{-0.025} \\ 0.594^{+0.021}_{-0.022} \end{cases}$	$0.342 \leq \sin^2\theta_{23} \leq 0.667$
$\delta = 300^\circ{}^{+\ 66^\circ}_{-138^\circ}$	$0^\circ \leq \delta \leq 360^\circ$
$\alpha\ ,\ \ \beta$	unconstrained

Non-trivial breaking of G_f and CP

<u>Idea</u>:

Relate lepton mixing to how G_f and CP are broken

Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f and CP

(Feruglio et al. ('12,'13), Holthausen et al. ('12), Grimus/Rebelo ('95))

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos	charged leptons
G_ν	G_e

Non-trivial breaking of G_f and CP

Idea:

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Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f and CP

$$G_f$$

$$\swarrow \qquad \searrow$$

neutrinos assume 3 generations of Majorana neutrinos	charged leptons distinguish 3 generations

Non-trivial breaking of G_f and CP

Idea:

Relate lepton mixing to how G_f and CP are broken
Interpretation as mismatch of embedding of different sub-groups G_ν and G_e into G_f and CP

G_f & CP

$\swarrow$ $\searrow$

neutrinos	charged leptons
$G_\nu = Z_2 \times$ CP	$G_e = Z_N$ with $N \geq 3$

An example: $\mu\tau$ reflection symmetry ***(Harrison/Scott ('02,'04), Grimus/Lavoura ('03))***

Non-trivial breaking of G_f and CP

G_f & CP

$\swarrow$ $\searrow$

neutrinos

$G_\nu = Z_2 \times$ CP

charged leptons

$G_e = Z_N$ with $N \geq 3$

Further requirements

- two/three non-trivial mixing angles $\Rightarrow$ irred 3-dim rep of G_f
- "maximize" predictability of approach

Non-trivial breaking of G_f and CP

Consistency conditions have to be fulfilled:

- definition of generalized CP transformation (X being unitary and symmetric) ***(see e.g. Branco et al. ('11))***

$$\phi \xrightarrow{\text{CP}} X\phi^\star$$

- "closure" relation

$$(X^\star A X)^\star = A' \quad \text{with in general} \quad A \neq A' \quad \text{and} \quad A,\, A' \in G_f$$

- commutation of Z_2 and CP ($Z_2 \subset G_f$ generated by Z)

$$X Z^\star - Z X = 0$$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

neutrino mass term $\nu_a \, m_{\nu,ab} \, \nu_b$
is invariant under $\nu_\alpha \to Z_{\alpha\beta} \, \nu_\beta$
is invariant under generalized CP transformation $\nu_\alpha \to X_{\alpha\beta} \, \nu_\beta^\star$

- charged lepton sector: Z_N, $N \geq 3$, preserved

charged lepton mass term $e_a^c \, m_{e,ab} \, l_b$
is invariant under $l_\alpha \to Q_{e,\alpha\beta} \, l_\beta$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

$\rightarrow$ neutrino mass matrix m_ν fulfills

$$Z^T m_\nu Z = m_\nu \quad \text{and} \quad X m_\nu X = m_\nu^\star$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

  $\rightarrow$ neutrino mass matrix m_ν is diagonalized by

$$\Omega_\nu(X, Z)R(\theta)K_\nu$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

$$\Omega_e^\dagger(Q_e)m_e^\dagger m_e\Omega_e(Q_e) \quad \text{is diagonal}$$

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  $\to$ charged lepton mass matrix m_e fulfills

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- conclusion: PMNS mixing matrix reads

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu \quad \text{in} \quad \bar{l} \not{W}^- U_{PMNS} \, \nu$$

Non-trivial breaking of G_f and CP

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu$$

- 3 unphysical phases are removed by $\Omega_e \to \Omega_e K_e$
- U_{PMNS} contains one parameter θ
- permutations of rows and columns of U_{PMNS} possible

$$\Downarrow$$

Predictions:

Mixing angles and CP phases are predicted
in terms of one parameter θ only,
up to permutations of rows/columns

Study of S_4 and CP

Generators in rep. $\mathbf{3}'$: $\left(\omega = e^{2\pi i/3}\right)$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \; , \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \; , \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfill

$$S^2 = \mathbb{1} \; , \;\; T^3 = \mathbb{1} \; , \;\; U^2 = \mathbb{1} \; ,$$
$$(ST)^3 = \mathbb{1} \; , \;\; (SU)^2 = \mathbb{1} \; , \;\; (TU)^2 = \mathbb{1} \; , \;\; (STU)^4 = \mathbb{1}$$

Study of S_4 and CP

A transformation X in rep. $\mathbf{3}'$ for $Z = S$:

$$X_{\mathbf{3}'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfills

$$XX^\dagger = XX^\star = \mathbb{1}$$
$$(X^\star A X)^\star = A' \ , \quad XZ^\star - ZX = 0$$

Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and our X

(Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. ('12,'13))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

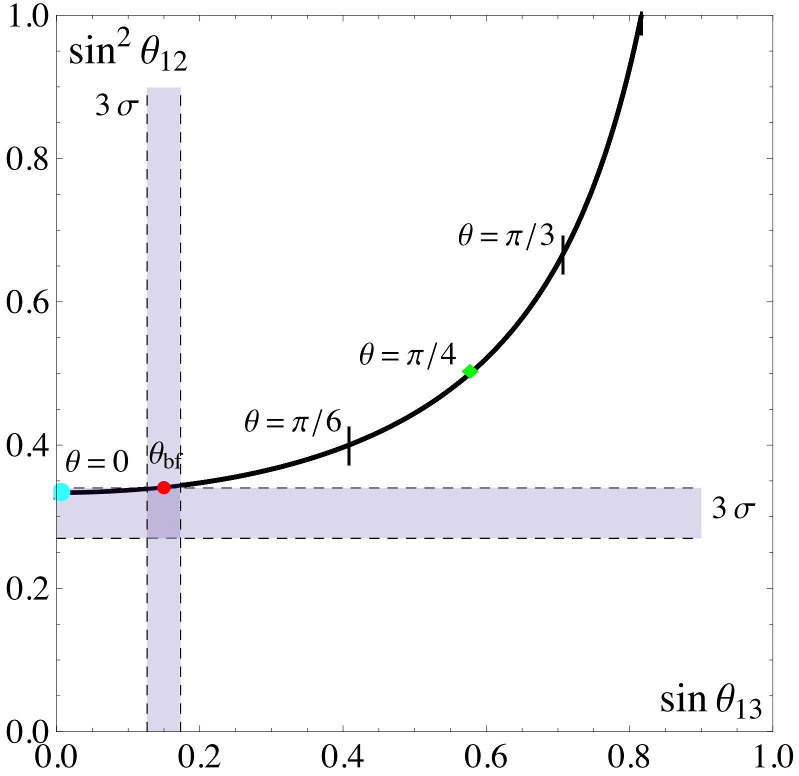
$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta\ , \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta}\ , \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1\ , \quad |J_{CP}| = \frac{|\sin 2\theta|}{6\sqrt{3}}\ , \quad \sin\alpha = 0\ , \quad \sin\beta = 0$$

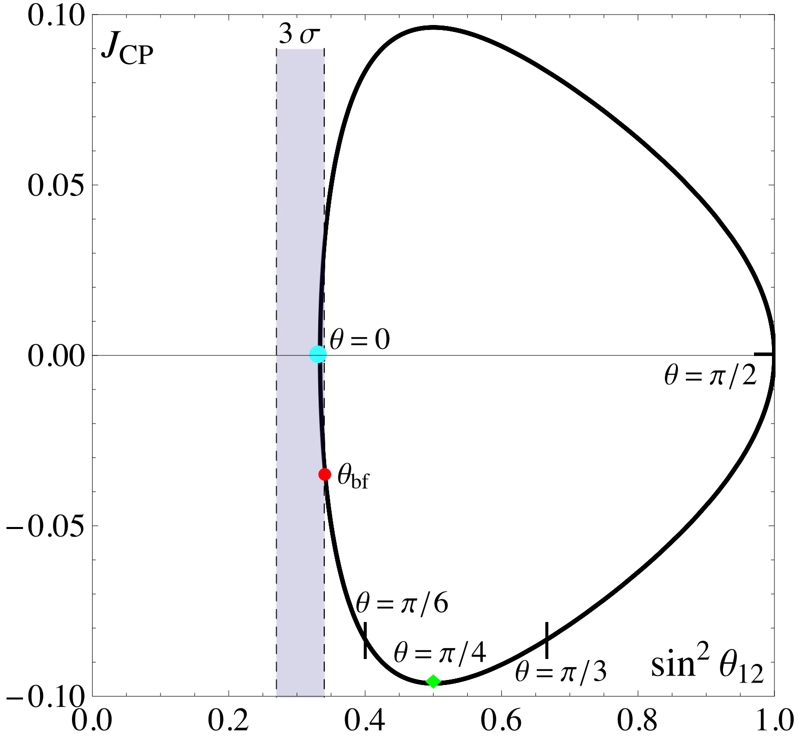
Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and our X



Study of $\boldsymbol{S_4}$ and $\boldsymbol{CP}$

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and our X



Study of S_4 and CP

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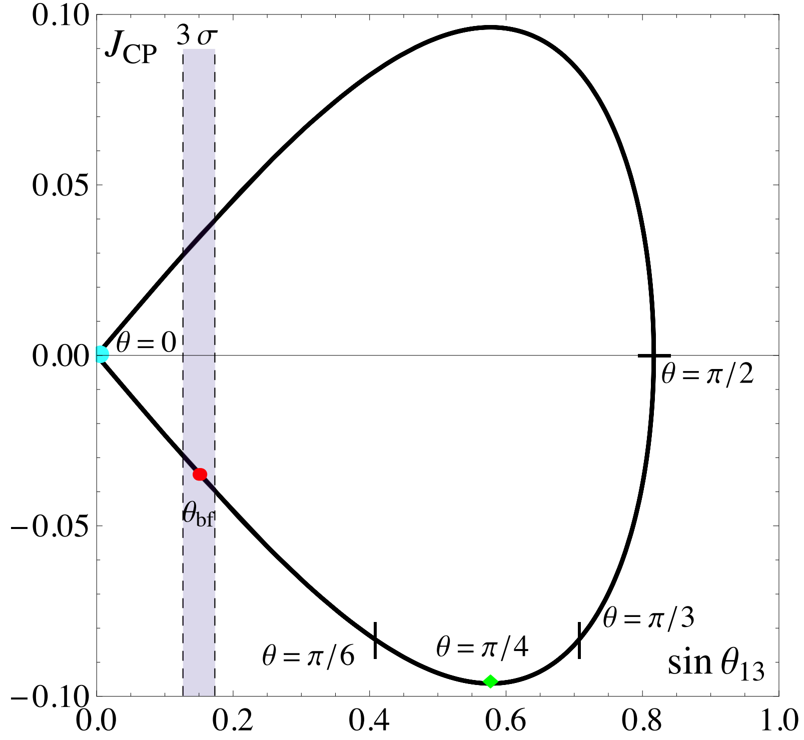

Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and our X

$$\theta_{\text{bf}} \approx 0.185 \;\;, \quad \chi^2_{\text{min}} \approx 18.4 \quad \text{for} \quad \theta_{23} < \pi/4$$

$$\sin^2\theta_{13}(\theta_{\text{bf}}) \approx 0.023 \;\;, \quad \sin^2\theta_{12}(\theta_{\text{bf}}) \approx 0.341 \;\;,$$
$$|J_{CP}(\theta_{\text{bf}})| \approx 0.0348$$

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

- left-handed leptons l are unified in $\mathbf{3}'$ of S_4
- right-handed charged leptons are singlets under S_4; add Z_3 in order to distinguish them: 1, ω, ω^2
- deviation in breaking pattern: $G_e = Z_3^{(D)}$: diagonal subgroup of $Z_3 \subset S_4$ (generated by T) and add. Z_3
- θ small due to small breaking $Z_2 \times Z_2 \times$ CP $\quad \rightarrow \quad Z_2 \times$ CP
- (not shown an auxiliary symmetry Z_{16} and $U(1)_R$ for vacuum alignment)

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	l	e^c	μ^c	τ^c	h_u	h_d
S_4	$\mathbf{3}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_3	1	1	ω	ω^2	1	1

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	χ_E	φ_E	ξ_N	χ_N	φ_N	ξ'_N
S_4	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}'$
Z_3	ω	ω	1	1	1	1

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

Charged lepton sector

- breaking to $G_e = Z_3^{(D)}$ via

$$\langle \chi_E \rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad , \quad \langle \varphi_E \rangle \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

- lowest order couplings

$$y_\tau (l \varphi_E) \tau^c h_d / \Lambda + y_{\mu,1} (l \varphi_E^2) \mu^c h_d / \Lambda^2 + y_{\mu,2} (l \chi_E \varphi_E) \mu^c h_d / \Lambda^2$$

SUSY model with S_4 and CP for leptons

Charged lepton sector

- lead to non-zero muon and tau lepton mass

$$m_\mu = \left| (2\, y_{\mu,1} v_{\varphi_E} + y_{\mu,2} v_{\chi_E}) \, \frac{v_{\varphi_E}}{\Lambda^2} \right| \, \langle h_d \rangle \quad m_\tau = \left| y_\tau \frac{v_{\varphi_E}}{\Lambda} \right| \, \langle h_d \rangle$$

for $\langle \chi_E \rangle \, , \, \langle \varphi_E \rangle \sim \lambda^2 \, \Lambda \quad \Rightarrow \quad m_\mu / m_\tau \approx \lambda^2 \, , \quad m_\tau \approx \lambda^2 \, \langle h_d \rangle$

- electron mass vanishes; generated by higher order terms
- charged lepton mass matrix is diagonal $\Rightarrow \quad \Omega_e = \mathbb{1}$

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

Neutrino sector

- breaking to $Z_2 \times Z_2 (\times \text{CP})$ via

$$\langle \xi_N \rangle = v_{\xi_N} \ , \quad \langle \chi_N \rangle = v_{\chi_N} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \ , \quad \langle \varphi_N \rangle = v_{\varphi_N} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

and v_{ξ_N} , v_{χ_N} , v_{φ_N} have same phase ($\pm\pi$)

- with $\langle \xi'_N \rangle \in i\mathbb{R}$ breaking to $Z_2 \times$ CP
- lowest order couplings

$$y_{\nu,1} (ll) \xi_N h_u^2/\Lambda^2 + y_{\nu,2} (ll \varphi_N) h_u^2/\Lambda^2 + y_{\nu,3} (ll \chi_N \xi'_N) h_u^2/\Lambda^3$$

SUSY model with $\boldsymbol{S_4}$ and $\boldsymbol{CP}$ for leptons

Neutrino sector

- neutrino mass matrix contains three real parameters $t_\nu \propto v_{\xi_N}/\Lambda$, $u_\nu \propto v_{\varphi_N}/\Lambda$ and $x_\nu \propto \langle \xi'_N \rangle v_{\chi_N}/\Lambda^2$
- they have order: $t_\nu\,,\, u_\nu \sim \lambda$ and $x_\nu \sim \lambda^2$ for $\langle \Phi_N \rangle \sim \lambda\,\Lambda$
- form of neutrino mass matrix m_ν

$$m_\nu = \begin{pmatrix} t_\nu + 2\,u_\nu & -u_\nu - i x_\nu & -u_\nu + i x_\nu \\ -u_\nu - i x_\nu & 2\,u_\nu + i x_\nu & t_\nu - u_\nu \\ -u_\nu + i x_\nu & t_\nu - u_\nu & 2\,u_\nu - i x_\nu \end{pmatrix} \frac{\langle h_u \rangle^2}{\Lambda}$$

SUSY model with S_4 and CP for leptons

Neutrino sector

- PMNS mixing matrix from neutrino sector only and is of form

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

- the parameter θ is

$$\tan 2\theta = \frac{x_\nu}{\sqrt{3}\, u_\nu} \sim \lambda$$

- and the lepton mixing angles read

$$\sin^2\theta_{13} \approx \frac{2}{3}\lambda^2 \; , \;\; \sin^2\theta_{12} \approx \frac{1}{3} + \frac{2}{9}\,\lambda^2 \; , \;\; \sin^2\theta_{23} = \frac{1}{2}$$

SUSY model with S_4 and CP for leptons

Neutrino sector

- neutrino mass spectrum is normally ordered;
  masses depend on two parameters t_ν and $\tilde{u}_\nu = f(u_\nu, x_\nu)$
  (in units $\langle h_u \rangle^2/\Lambda$)

$$m_1 \propto |t_\nu + \tilde{u}_\nu| \ , \ \ m_2 \propto |t_\nu| \ , \ \ m_3 \propto |t_\nu - \tilde{u}_\nu|$$

- for best fit values of $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$

$$m_1 \approx 0.016\,{\rm eV} \ , \ \ m_2 \approx 0.018\,{\rm eV} \ , \ \ m_3 \approx 0.052\,{\rm eV}$$

SUSY model with S_4 and CP for leptons

NLO contributions for charged leptons

- electron mass is generated
  - ... through operators with five fields Φ_N,
    e.g. $l e^c \xi_N^3 \chi_N \varphi_N h_d / \Lambda^5$, if we consider shifts in $\langle \Phi_N \rangle$
  - ... through operators with six fields Φ_N,
    e.g. $l e^c \xi_N^4 \xi'_N \varphi_N h_d / \Lambda^6$ with LO VEVs
  - $\langle \Phi_N \rangle \sim \lambda \, \Lambda$ and $\delta \langle \chi_N \rangle \sim \lambda \, \langle \chi_N \rangle \quad \Rightarrow \quad m_e \approx \lambda^6 \, \langle h_d \rangle$
- charged lepton mass matrix is non-diagonal at NLO;
  however, induced mixing angles are very small: $\theta_{ij}^l \sim \lambda^4$

SUSY model with S_4 and CP for leptons

NLO contributions for neutrinos

- largest correction from VEV shift of field χ_N

$$\langle \chi_N \rangle = v_{\chi_N} \left(\begin{array}{c} 1 + i\,\alpha\,\lambda \\ 1 - i\,\alpha\,\lambda \end{array} \right)$$

  which still preserves $Z_2 \times$ CP $\Rightarrow$ parameter $p_\nu \sim \lambda^3$:
  no effect on mixing; small shift in neutrino masses

- other corrections to m_ν are max. order λ^6 in units $\langle h_u \rangle^2 / \Lambda$
  ... negligible

Conclusions

- Scenarios with G_f and CP predict mixing angles and CP phases in terms of one parameter θ
- Explicit model with S_4 and CP ...
  - predicts θ_{23} and δ maximal as well as α and β trivial due to symmetry breaking pattern

Conclusions

- Scenarios with G_f and CP predict mixing angles and CP phases in terms of one parameter θ
- Explicit model with S_4 and CP ...
    - explains θ_{13} small via naturally small θ,
    - leads to normally ordered light neutrino masses,
    - predicts absolute neutrino mass scale,
    - generates charged lepton masses of correct order

Conclusions

- Scenarios with G_f and CP predict mixing angles and CP phases in terms of one parameter θ
- Explicit model with S_4 and CP constructed
- Consider models with other G_f? Extension to quarks?

Thank you for your attention.

Back up

Study of S_4 and CP

For the other representations with generators S, T and U being:

$$\mathbf{1}: \quad S = 1\,, \qquad T = 1\,, \qquad U = 1\,,$$

$$\mathbf{1}': \quad S = 1\,, \qquad T = 1\,, \qquad U = -1\,,$$

$$\mathbf{2}: \quad S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \qquad U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\mathbf{3}: \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Study of S_4 and CP

the corresponding X reads:

$$X_{\mathbf{1}} = 1\ ,\ X_{\mathbf{1}'} = -1\ ,\ X_{\mathbf{2}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\ ,\ X_{\mathbf{3}} = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$