One-loop effective action in Quantum Gravity

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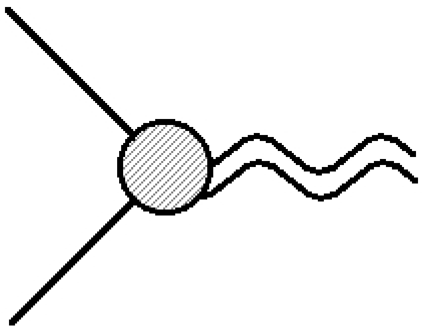
Outline

- Motivation
- Effective field theory of Gravitation
- Non-local heat kernel methods
- Gravitation and scalar field model computation
- Generalized picture
- Conclusions

Motivation

- Low energy QG is valid as QFT with a cutoff
- Despite nonrenormalizability predictions are possible and calculable in EFT
- New tools: Effective Average Action (EAA) and exact RG flow equations
- Simplest model: Gravitation + matter (scalar field minimally coupled)



Motivation

- Corrections to Newton, Schwarzschild, Kerr classical gravitational solutions
- Dynamical situations: gravitational scattering amplitudes
- Three-point vertex functions and gravitational form-factors
- Nonanalytic terms are to be obtained by integrating the RG flow of EAA from UV to IR
- Comparison with standard perturbative computations (Feynman diagrams)

GR as EFT

- Quantum field theory of small low-energetic fluctuations of metric degrees of freedom around flat Minkowski spacetime (Donoghue et al '94-'00) $g_{\mu\nu}=\eta_{\mu\nu}+\kappa h_{\mu\nu}$
- Standard quantization using Faddeev-Popov trick
- Expansion of lagrangian in number of derivatives and in powers of graviton field $L_{grav}=2\kappa^{-2}R(h)+o(R^2)$
- Feynman diagrams for gravitons interacting with matter
- General covariance as a gauge symmetry on the linearized level
- Perturbative calculus with dimensionful coupling $\kappa^2=32\pi G_N$

GR as EFT

- No dependence on UV completion of Quantum Gravitation (for leading quantum corrections)

- Gravitons around flat spacetime are massless ⇐ gauge symmetry

- Virtual massless excitations lead to nonanalytic behavior of low energy quantities

- For a scalar matter coupling to energy-momentum tensor

$$<p'|T_{\mu\nu}|p> = F_1(q^2)[p_\mu p'_\nu + p'_\mu p_\nu + 1/2\eta_{\mu\nu}q^2] + F_2(q^2)[q_\mu q_\nu - \eta_{\mu\nu}q^2]$$

- Gravitational formfactors:

$$F_1(q^2) = 1 + \frac{\kappa^2}{32\pi^2}q^2\left[-\frac{3}{4}\log(-q^2) + \frac{1}{16}\frac{\pi^2 m}{\sqrt{-q^2}}\right] \quad F_2(q^2) = \frac{\kappa^2 m^2}{32\pi^2}\left[-\frac{4}{3}\log(-q^2) + \frac{7}{8}\frac{\pi^2 m}{\sqrt{-q^2}}\right]$$

Heat kernel

- Manifestly covariant and covariantly computed effective action
- Straightforward way to implement nonlocal terms in the action
- EFT without divergences (no need for regularization of divergent integral, neither in UV, nor in IR) (Barvinsky, Ostrovsky, Vilkovisky)
- No use of non gauge covariant Green functions
- Orthogonal expansion in generalized curvatures of HK
- Easy generalization to theories on *d*-dimensional spacetimes
- Powerful tool for exact RG flows computations

Heat kernel

- One-loop effective action for QG (Codello, Mazzitelli, Satz)
- Truncation ansatz for scale-dependent effective action

$$\Gamma_k = \int d^4x \sqrt{g} \left(\frac{2}{\kappa^2} R + \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} m^2 \phi^2 \right)$$

- Exact RG flow equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k$$

- Inverse propagator $\Delta = \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \Psi^2}$
- Cutoff shape function (Litim) $R_k(z) = (k^2 - z)\,\theta(k^2 - z)$

Heat kernel

- Functional traces of the form $\mathrm{Tr}[W(\Delta)]$ $\quad \Delta = -\Box - \hat{P} + \frac{1}{6} R \hat{1}$
- Heat kernel of the operator Δ
$$\mathrm{Tr}[W(\Delta)] = \int_0^\infty ds\, \mathrm{Tr}[K(s)]\, \tilde{W}(s)$$
- Anti-Laplace transform of *W* $\quad \tilde{W}(s)$
- Explicitly $\mathrm{Tr}[K(s)] = \frac{1}{(4\pi s)^2} \int d^4x \sqrt{g}\, \mathrm{tr}\Big[\hat{1} + s\hat{P} + s^2 \hat{1}\Big(R_{\mu\nu}\phi_1(-s\Box)R^{\mu\nu} + R\phi_2(-s\Box)R\Big)$
$$+ s^2\Big(\hat{P}\phi_3(-s\Box)R + \hat{P}\phi_4(-s\Box)\hat{P} + \Omega_{\mu\nu}\phi_5(-s\Box)\Omega^{\mu\nu}\Big) + o(R^3)\Big]$$
- Generalized curvatures in HK: $\hat{P}$ $\quad \hat{\Omega}_{\mu\nu} \sim [\nabla_\mu, \nabla_\nu]$
- Spacetime curvatures: R, $R_{\mu\nu}$

Computation

- One-loop average effective action for gravitation+scalar field
- Function of the operator $W(\Delta)=\frac{\partial_t R_k(\Delta)}{\Delta+R_k(\Delta)}$
- Nonlocal structure functions
- Basic heat kernel non-local form factor $f(x)=\int_0^1 d\xi\, e^{-\xi(1-\xi)x}$
- HK methods to second order in generalized curvatures
- Flow equations for form-factors in EAA

Computation

- Integration of the flow from UV to IR ⇒ Quantum effective action (*k*=0) $F_{i,\Lambda}(x)-F_{i,k}(x)=\int_k^{\Lambda}\frac{dk'}{k'}\partial_t F_{i,k}(x)$
- Proper boundary conditions at UV remove the cutoff dependence Λ
- Possible finite renormalizations of couplings
- Usage of scalar and gravitational EOMs
- Third variational derivative = 3-point vertex of interaction $V_{\mu\nu}=\frac{\delta^3\Gamma}{\delta g^{\mu\nu}\delta\phi\delta\phi}$
- Flat space limit and no scalar background $g_{\mu\nu}\to\eta_{\mu\nu}$ $\phi\to 0$
- In momentum space, conservation law at vertex $\partial_{x,\alpha}\delta_{x,x_j}\to i\,p_{j,\alpha}$ $\sum_{j=1}^{3}p_j=0$

Γ_0

$$\Gamma_0 = \frac{1}{32\pi^2}\int d^4x\sqrt{g}\left[\frac{71}{30}R_{\mu\nu}XR^{\mu\nu} + \frac{71}{60}RXR + \right.$$

$$+\frac{5}{8}\kappa^4 m^4\phi^2 X\phi^2 - \kappa^2 m^4\phi X\phi - \frac{13}{6}\kappa^2 m^2 RX\phi^2 -$$

$$-\frac{1}{6}m^2 R + \frac{1}{2}m^4 + \frac{5}{8}\kappa^4(\nabla\phi)^2 X(\nabla\phi)^2 + \frac{1}{4}\kappa^4 m^2\phi^2 X(\nabla\phi)^2 -$$

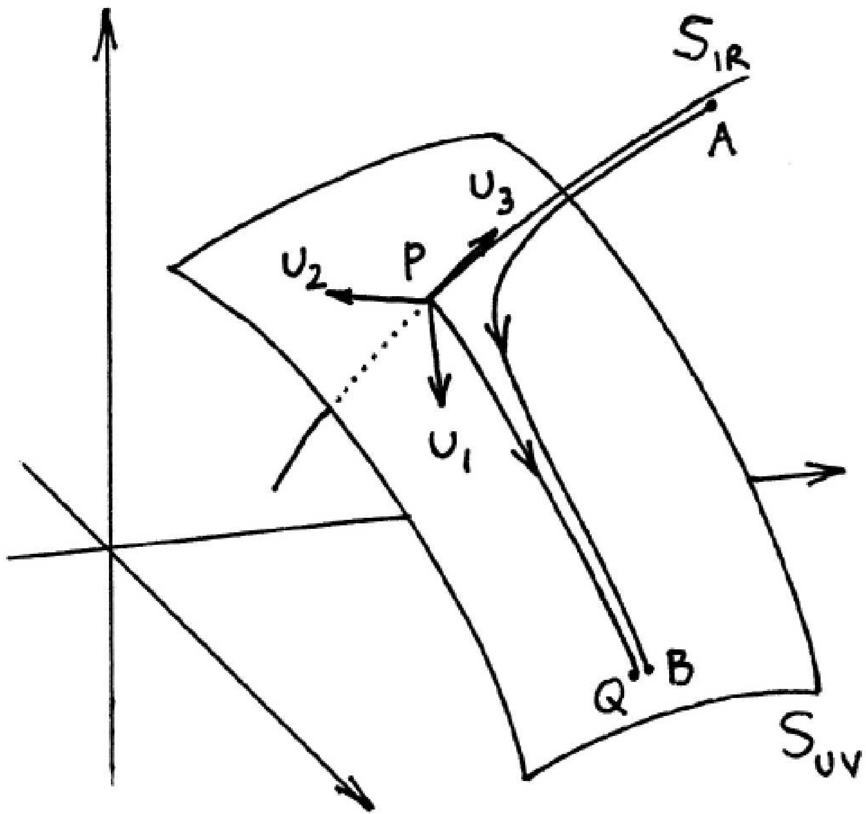
$$\left. -\frac{2}{6}\kappa^2 RX(\nabla\phi)^2 - \frac{1}{2}\kappa^2 m^2(\nabla\phi)^2\right]$$

$$X = \log\left(\frac{-\Box}{k_0^2}\right)$$

General picture

- 3-point vertex ⇒ 3 order in generalized HK curvatures
- Mass resummation problem
- Ambiguity with gravitational formfactors
- Next order observables sensitive to precise RG flow
- Possible improvement by including UV FP of RG for irrelevant operators in IR like R^3 terms

General picture of RG flow



Summary

- GR is valid EFT below Planck scale
- The leading quantum corrections
- Perturbative calculations using Feynman diagrams give quick answer for gravitational formfactors
- Advantages of using HK techniques
- Necessity of including nonlocality in the effective action
- Possible interpretations in AS scenario for QG
- Third order computation work is ongoing!

Thank you
for attention!