Suitable Factorization Scheme for NLO Monte Carlo Event Generators

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International Symposium on Multiparticle Dynamics,
DESY, 18 September 2008

Outline

1. Monte Carlo event generators
2. Initial parton showers
3. The $P^{(1)} = 0$ factorization scheme

Monte Carlo Event Generators

Monte Carlo simulations of elementary particle collisions consist of three basic stages:

1. Hard process
   - hard collision described by cross-sections at parton level
   - fully calculable within the framework of perturbative theory
2. Parton showers
   - partons in initial and final states can radiate other partons
   - inclusion of dominant part of higher order contributions of pQCD — soft and collinear emissions
3. Hadronization
   - conversion of partons created in parton showers into observable hadrons
   - pQCD cannot be used — only phenomenological models

When simulating hadron collisions, we need to know parton distribution functions describing the partonic structure of the colliding hadrons.

Monte Carlo Simulations of Initial Parton Showers

Initial parton showers induce the scale dependence of parton distribution functions, which is described by the evolution equations (DGLAP).

Monte Carlo simulations of initial parton showers have some important advantages in comparison with the analytical calculations (DGLAP):

- generation of transverse momenta
- color description
- partons radiated in initial parton showers are involved in hadronization

Accuracy of Monte Carlo Event Generators

The standard Monte Carlo event generators have the LO accuracy:

- LO cross-sections for hard process
- LL parton showers (LO splitting functions)

Many QCD cross-sections at parton level are known at NLO accuracy and algorithms for their incorporation in Monte Carlo event generators have been developed. However, these algorithms match the NLO cross-sections with parton showers only at LL accuracy, because generating NLL parton showers is very complicated in the standard $\overline{\mathrm{MS}}$ factorization scheme. Why?

- The NLO splitting functions no longer correspond to basic QCD vertices.
- The NLO splitting functions are expressed by much more complicated formulae than the LO ones.
- The NLO splitting functions are negative for some x, which prevents us from using straightforward probabilistic interpretation.

NLL Initial Parton Showers

NLO splitting functions, contrary to the LO ones, are not universal — depend on the chosen factorization scheme. Hence, a suitable choice of the factorization scheme could eliminate the difficulties with generating NLL initial parton showers that occur in the standard $\overline{\text{MS}}$ factorization scheme.

The suitable factorization scheme is that in which all NLO splitting functions vanish. Such a factorization scheme really exists — $P^{(1)} = 0$ factorization scheme.

In the $P^{(1)} = 0$ factorization scheme, NLL initial parton showers are formally identical to the LL ones and therefore the existing algorithms for parton showering and for matching NLO cross-sections with parton showers need not be changed. The only thing which is necessary to do is transforming cross-sections of hard process and parton distribution functions from the standard $\overline{\text{MS}}$ factorization scheme to the $P^{(1)} = 0$ factorization scheme.

Factorization Schemes

The factorization scheme specifies the way in which the so called collinear singularities, which are contained in cross-sections at parton level, are absorbed into the dressed parton distribution functions. Within the framework of dimensional regularization, the relation between the dressed and bare distribution functions is given by the formula:

$$D_i(x, \mathrm{FS}, M) = \sum_j \int_x^1 \frac{\mathrm{d}y}{y} D_j^{(0)}\left(\frac{x}{y}\right) \left[\delta_{ij}\delta(1-y) + a(M)\left(\frac{1}{\epsilon}A_{ij}^{(11)}(y) + A_{ij}^{(10)}(y)\right)\right.$$
$$\left. + a^2(M)\left(\frac{1}{\epsilon^2}A_{ij}^{(22)}(y) + \frac{1}{\epsilon}A_{ij}^{(21)}(y) + A_{ij}^{(20)}(y)\right) + \cdots\right].$$

The matrices $A^{(k0)}(x)$ can be chosen arbitrarily and their choice fully specifies the factorization scheme. It can be proven that the factorization scheme can also be specified by higher orders of the corresponding splitting functions.

The standard $\overline{\mathrm{MS}}$ factorization scheme is defined by setting the matrices $A^{(k0)}(x)$ equal to zero.

Change of the Factorization Scheme and Scale

In the case of a hadron collision, a cross-section $\sigma(P)$ (in general differential) depending on observables P is given by the formula

$$\sigma(P) = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \, \mathrm{d}x_2 \, D_{i/A}(x_1, \mathrm{FS}, M) D_{j/B}(x_2, \mathrm{FS}, M) \sigma_{ij}(x_1, x_2; P; \mathrm{FS}, M)$$

where

$$\sigma_{ij}(x_1, x_2; P; \mathrm{FS}, M) = \sigma_{ij}^{(0)}(x_1, x_2; P) + a(\mu)\sigma_{ij}^{(1)}(x_1, x_2; P; \mathrm{FS}, M) + \mathcal{O}\big(a^2(\mu)\big).$$

The formula for changing the factorization scale and scheme at NLO:

$$\sigma_{ij}^{(1)}(x_1, x_2; P; \mathrm{FS}, M) = \sigma_{ij}^{(1)}(x_1, x_2; P; \overline{\mathrm{MS}}, M_0) + \sum_k \int_0^1 \mathrm{d}y \Bigg[\sigma_{ik}^{(0)}(x_1, yx_2; P) \times$$
$$\times \left(P_{kj}^{(0)}(y) \ln \frac{M_0}{M} + T_{kj}^{(1)}(y) \right) + \sigma_{kj}^{(0)}(yx_1, x_2; P) \left(P_{ki}^{(0)}(y) \ln \frac{M_0}{M} + T_{ki}^{(1)}(y) \right) \Bigg].$$

The $T^{(1)}$ for the $P^{(1)} = 0$ Factorization Scheme

The Mellin moments $T^{(1)}(n)$ corresponding to the $P^{(1)} = 0$ factorization scheme are determined by the equation

$$\left[\mathbf{T}^{(1)}(n), \mathbf{P}^{(0)}(n)\right] - b\mathbf{T}^{(1)}(n) = \mathbf{P}^{(1)}(n, \overline{\mathrm{MS}})\,.$$

- The solution can be expressed in analytic form.
- The Mellin inversion to x-space has to be calculated numerically.

The obtained results are very surprising!

- For $x \lesssim 0.1$:

$$\begin{aligned} T^{(1)}(x) &\propto x^{-4.63} && \text{for} \quad n_\mathrm{f} = 3 \\ T^{(1)}(x) &\propto x^{-3.85} && \text{for} \quad n_\mathrm{f} = 4 \end{aligned}$$

- $\left|T^{(1)}(x)\right| \geq 10^7 \quad \text{for} \quad x \lesssim 10^{-2}$

Applicability of the $P^{(1)} = 0$ Factorization Scheme

It is very probable that the region where the $P^{(1)} = 0$ factorization scheme gives reasonable predictions at NLO approximation is smaller than the region where the evolution equations (DGLAP) provide a good description of experimental data.

The limits of applicability of the $P^{(1)} = 0$ factorization scheme can be obtained by comparing theoretical predictions for some quantity (e.g. $F_2(x, Q^2)$) in the $P^{(1)} = 0$ factorization scheme with the predictions for the same quantity in the standard $\overline{\mathrm{MS}}$ factorization scheme.

It is interesting that in the non-singlet case there are no problems with applicability of the $P^{(1)} = 0$ factorization scheme — the singularities contained in the matrix $T^{(1)}(x)$ mutually cancel out and the result behaves for small x as the non-singlet splitting functions.

Almost $P^{(1)} = 0$ Factorization Schemes

It seems that the $P^{(1)} = 0$ factorization scheme should be replaced by some almost $P^{(1)} = 0$ factorization scheme which is sufficiently close to the $P^{(1)} = 0$ factorization scheme and in which the NLO cross-sections have proper behaviour for small x. However, there is a question whether such a factorization scheme can be found.

I have already started searching for a suitable almost $P^{(1)} = 0$ factorization scheme and determining the region of applicability of the $P^{(1)} = 0$ factorization scheme in the case of the structure function $F_2(x, Q^2)$, but I am still at the beginning.

Another Use of the $P^{(1)} = 0$ Factorization Scheme

Although the choice of the factorization scheme is as important as the choice of the factorization scale, the freedom in the choice of the factorization scheme is totally unexplored — the investigation of the dependence on a factorization scheme is much more complicated than that in the case of the factorization scale.

Except the standard $\overline{\mathrm{MS}}$ factorization scheme, which is convenient for theoretical calculations, the so called DIS factorization scheme is sometimes used. The DIS factorization scheme:

- The structure function $F_2(x, Q^2)$ is expressed in the same way as in the parton model (at LO).
- All higher order corrections to the corresponding coefficient function are set to zero.
- All NLO corrections to the structure function $F_2(x, Q^2)$ are included in the NLO splitting functions and are thus exponentiated by the evolution equations.

Another Use of the $P^{(1)} = 0$ Factorization Scheme

The $P^{(1)} = 0$ factorization scheme is in some sense opposite to the DIS factorization scheme — in the $P^{(1)} = 0$ factorization scheme all NLO splitting functions are set to zero. Contrary to the DIS factorization scheme, the definition of the $P^{(1)} = 0$ factorization scheme does not prefer any process or any quantity.

In the $P^{(1)} = 0$ factorization scheme all NLO corrections are included in cross-sections of hard process and only LO contributions are exponentiated by the evolution equations. In other factorization schemes the evolution equations always exponentiate at least a part of NLO corrections. The $P^{(1)} = 0$ factorization scheme is therefore in some sense a prominent factorization scheme. It is also interesting that for some quantities (e.g. Mellin moments of non-singlet structure functions) the $P^{(1)} = 0$ factorization scheme is close to that determined from the principle of minimal sensitivity.

These preceding facts represent a motivation for using the $P^{(1)} = 0$ factorization scheme outside the framework of Monte Carlo simulations. The $P^{(1)} = 0$ factorization scheme used together with the $\overline{\text{MS}}$ and DIS factorization schemes can also improve the estimation of theoretical uncertainties.

Summary

- The optimal factorization scheme for generating NLL initial parton showers is the $P^{(1)} = 0$ factorization scheme. However, this scheme seems to have a very limited range of applicability at NLO approximation.
- It is probably necessary to replace it by some almost $P^{(1)} = 0$ factorization scheme.
- A suitable almost $P^{(1)} = 0$ factorization scheme is being searched.
- The almost $P^{(1)} = 0$ factorization scheme can also improve the estimation of theoretical uncertainties.