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# Theoretical review of parton saturation

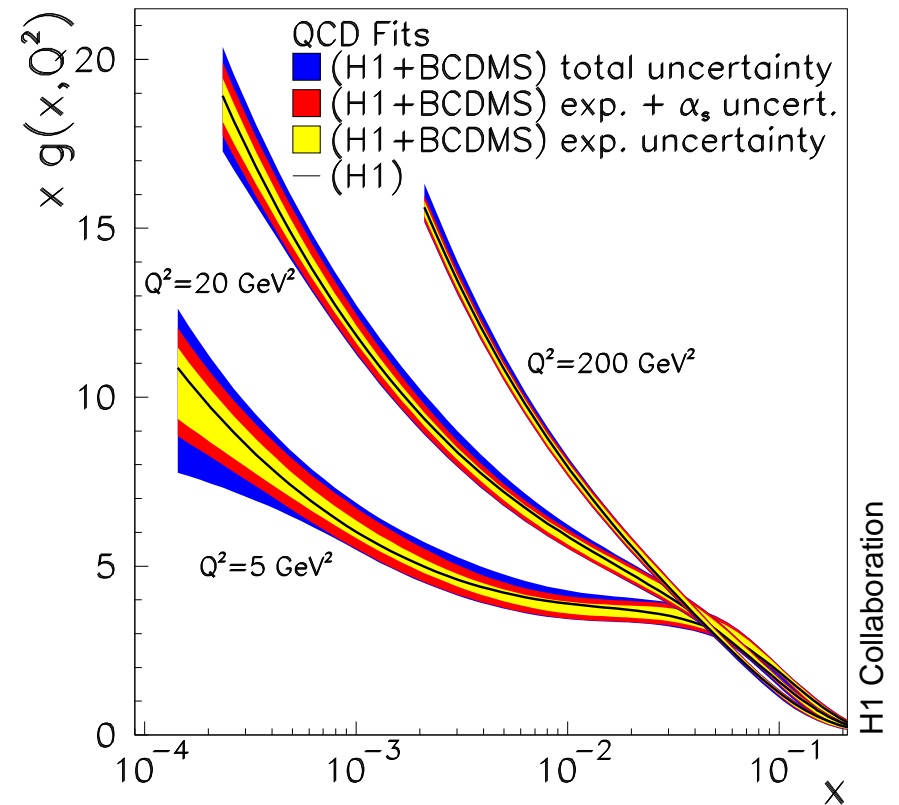
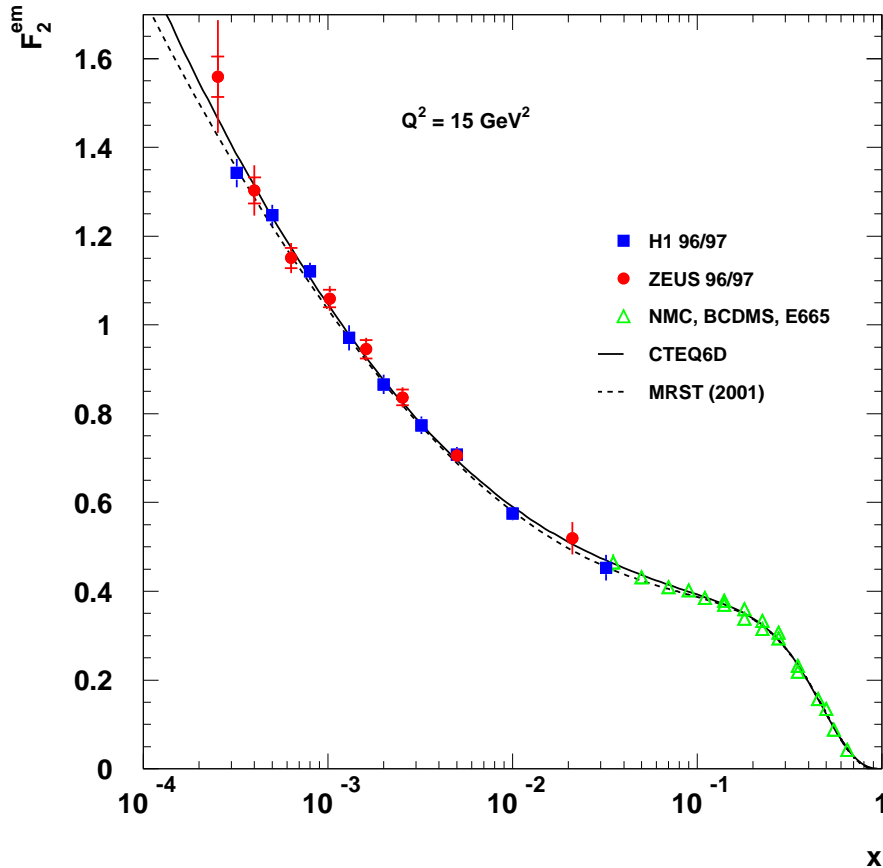
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# DIS results from HERA



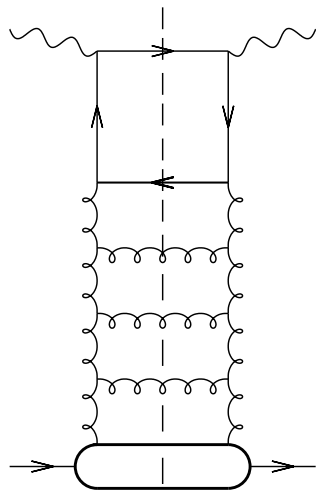
- Power-like rise:  $F_2 \sim x^{-\lambda}$  when  $x = Q^2/s \rightarrow 0$  and  $Q^2$  fixed.
- Rise driven by strong growth of the gluon distribution.

# Collinear factorization approach

- Convolution in longitudinal momenta

$$F_2(x, Q^2) = \underbrace{\sum_{i=q,g} C_i(x, Q^2) \otimes f_i(x, Q^2)}_{\text{leading twist}} + \underbrace{\sum_{n=1} \frac{\Lambda_n(x)}{Q^{2n}}}_{\text{higher twists}}$$

- Universal parton distribution functions  $f_i = \{q, \bar{q}, G\}$  evolved in  $Q^2$  by DGLAP evolution equations.

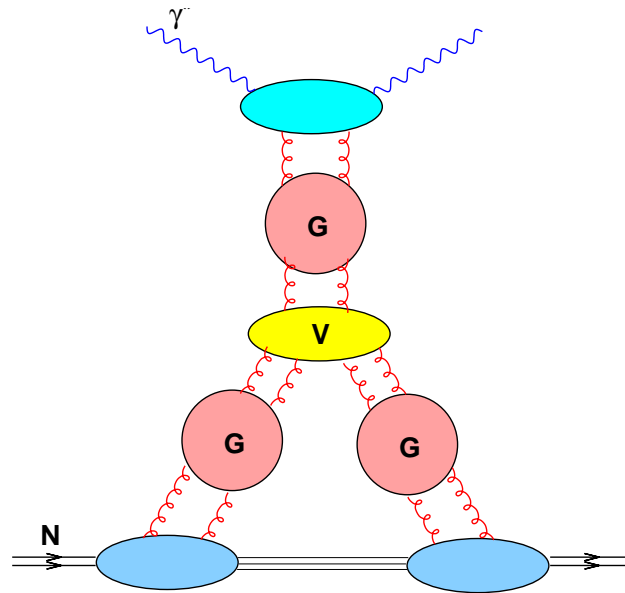


$$\frac{\partial G}{\partial \ln Q^2 \ln(1/x)} = \bar{\alpha}_s G$$

$$G \sim \exp 2\sqrt{\bar{\alpha}_s \ln Q^2 \ln(1/x)}$$

- Higher twist terms usually small.

# Taming the growth for $x \rightarrow 0$



- Gribov-Levin-Ryskin evolution equation for the gluon distribution

$$\frac{\partial G}{\partial \ln Q^2 \ln(1/x)} = \bar{\alpha}_s G - \frac{\bar{\alpha}_s^2}{Q^2 R_H^2} G^2$$

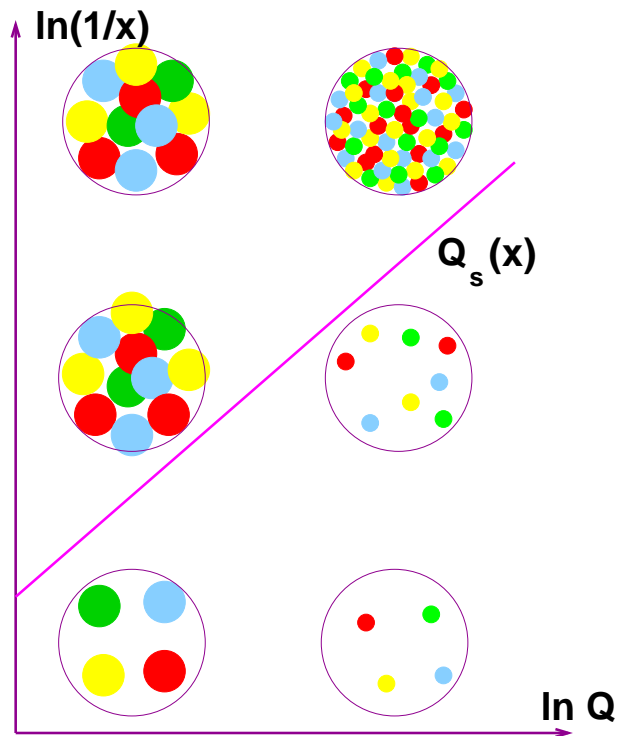
- **Still** leading twist description.

# Saturation scale

- Saturation of gluon density when **nonlinear term**  $\approx$  **linear term**

$$\frac{\alpha_s(Q_s^2)}{Q_s^2} G(x, Q_s^2) \approx \pi R_H^2$$

- $x$ -dependent saturation scale  $Q_s(x)$  emerges



## $k_{\perp}$ -factorization

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- At small  $x$  **higher twists** become very important (large  $\ln(1/x)$  terms).
- Partial resummation by exact treatment of quark transverse momentum  $p_{\perp}$  in the quark box (in DGLAP:  $Q \gg p_{\perp} \gg k_{\perp}$ )

$$F_2(x, Q^2) = \int d^2 k_{\perp} \phi_{q\bar{q}}(k_{\perp}/Q) f(x, k_{\perp})$$

- **Unintegrated** gluon density  $f(x, k_{\perp})$  obeys BFKL equation which gives power-like growth:

$$f(x, k_{\perp}) \sim x^{-4 \ln 2 \alpha_s}$$

- Still a problem!  $F_2$  rises too fast.

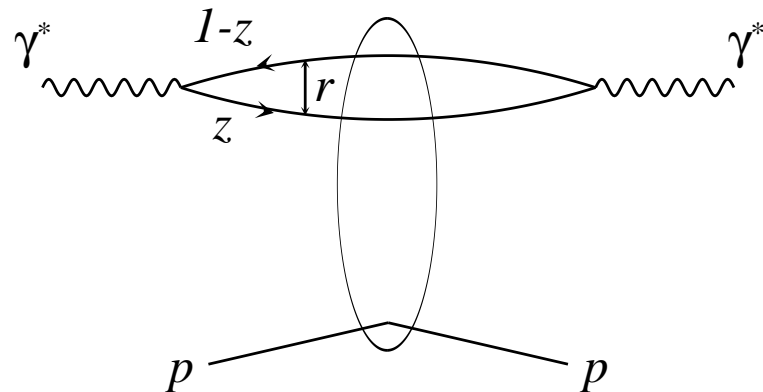
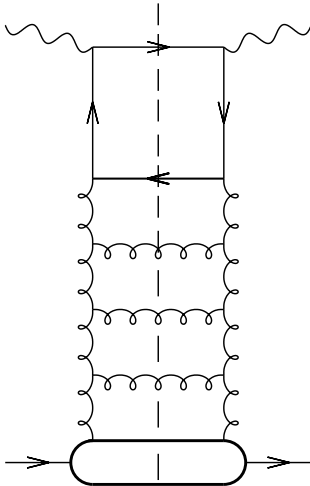
# Dipole models

- Fourier transform quark transverse momentum:  $p_{\perp} \leftrightarrow r$

$$\sigma_{T,L}(x, Q^2) = \int d^2r \int_0^1 dz |\Psi_{T,L}(z, r, Q^2)|^2 \sigma_{q\bar{q}}(x, r)$$

- $F_2 = Q^2(\sigma_T + \sigma_L)$

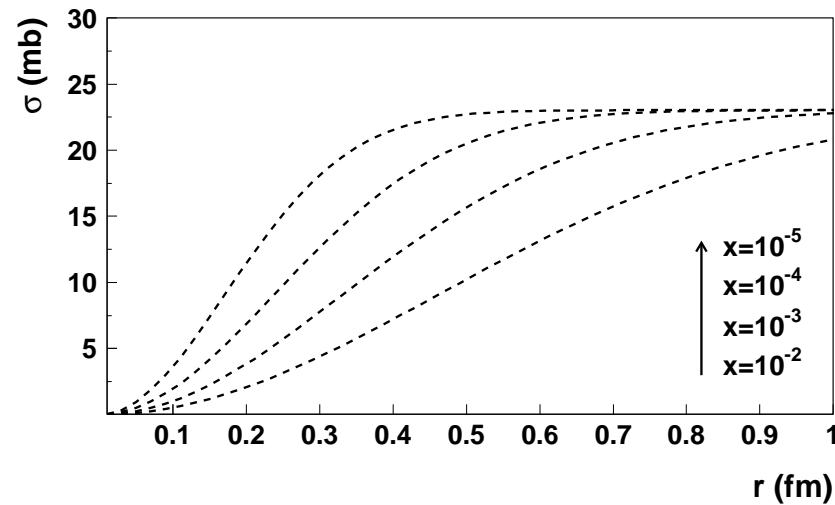
- $q\bar{q}$  dipole probes the proton structure (in the frame where photon  $q^+$  momentum is large)



# Dipole cross section

- Gluonic interactions contained in the dipole cross section

$$\sigma_{q\bar{q}}(x, r) = 2 \int d^2b N(x, r, b)$$

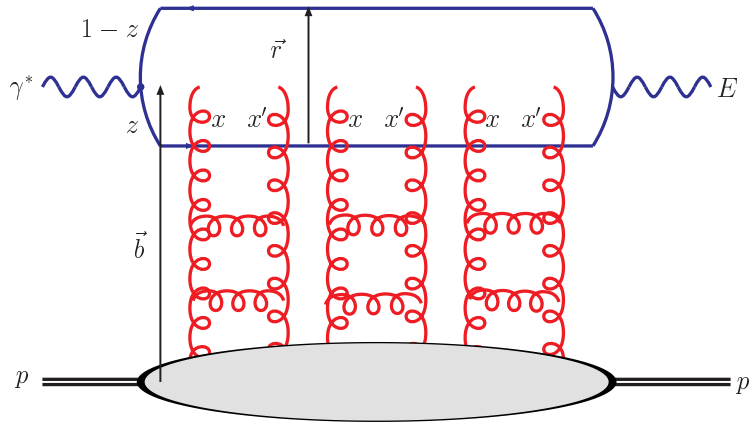


- Dipole scattering amplitude  $N$  obeys:
  - color transparency:  $N \sim r^2$  for  $r \rightarrow 0$
  - unitarity constraint:  $N \leq 1$  always



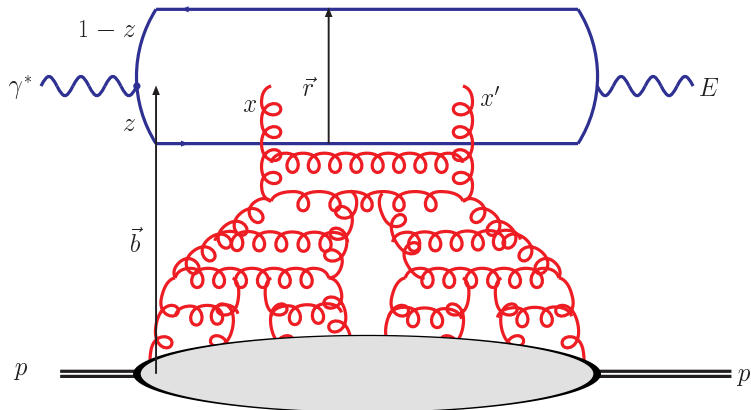
# QCD models of dipole scattering amplitude

## ● Glauber-Mueller eikonal rescattering



$$N = 1 - \exp\{-\kappa r^2 G(x, 1/r^2) T(b)\}$$

## ● BFKL pomeron fan diagram resummation



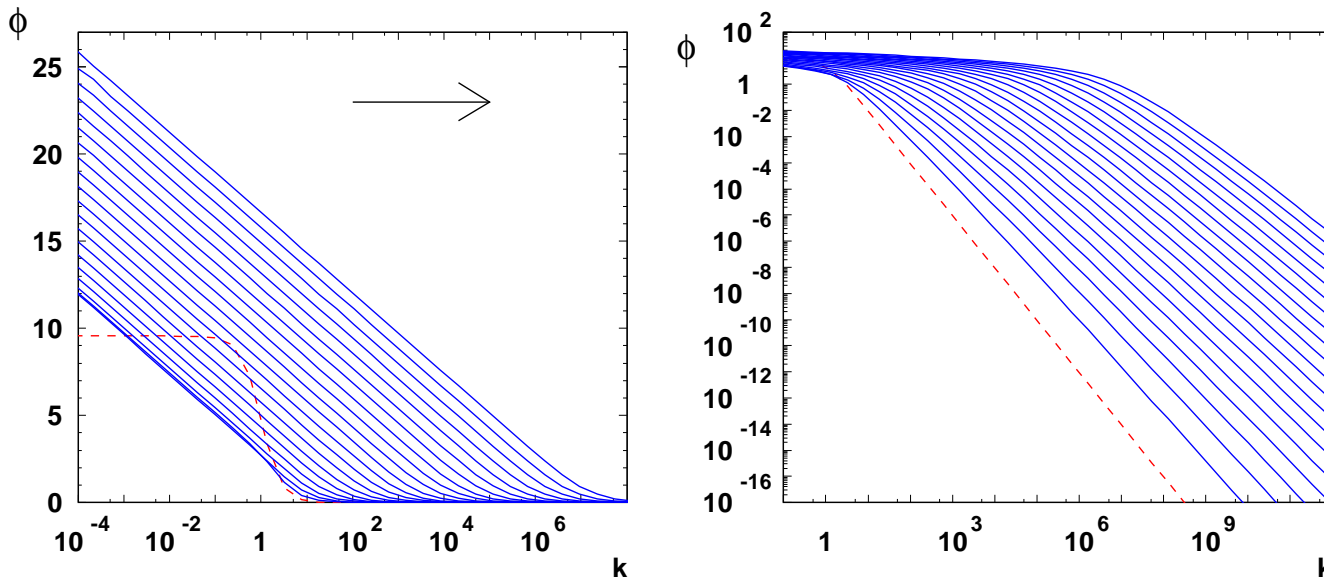
Balitsky-Kovchegov equation

$$\partial_Y N = K_{BFKL} \otimes (N - N^2)$$

# Saturation scale and geometric scaling

- Gluon density saturation from the BK equation solution:

$$\phi(x, k_{\perp}) = \frac{dN_g}{dY dk_{\perp}^2} = \int d^2r \frac{\sigma(x, r)}{r^2} e^{ik_{\perp} \cdot r}$$

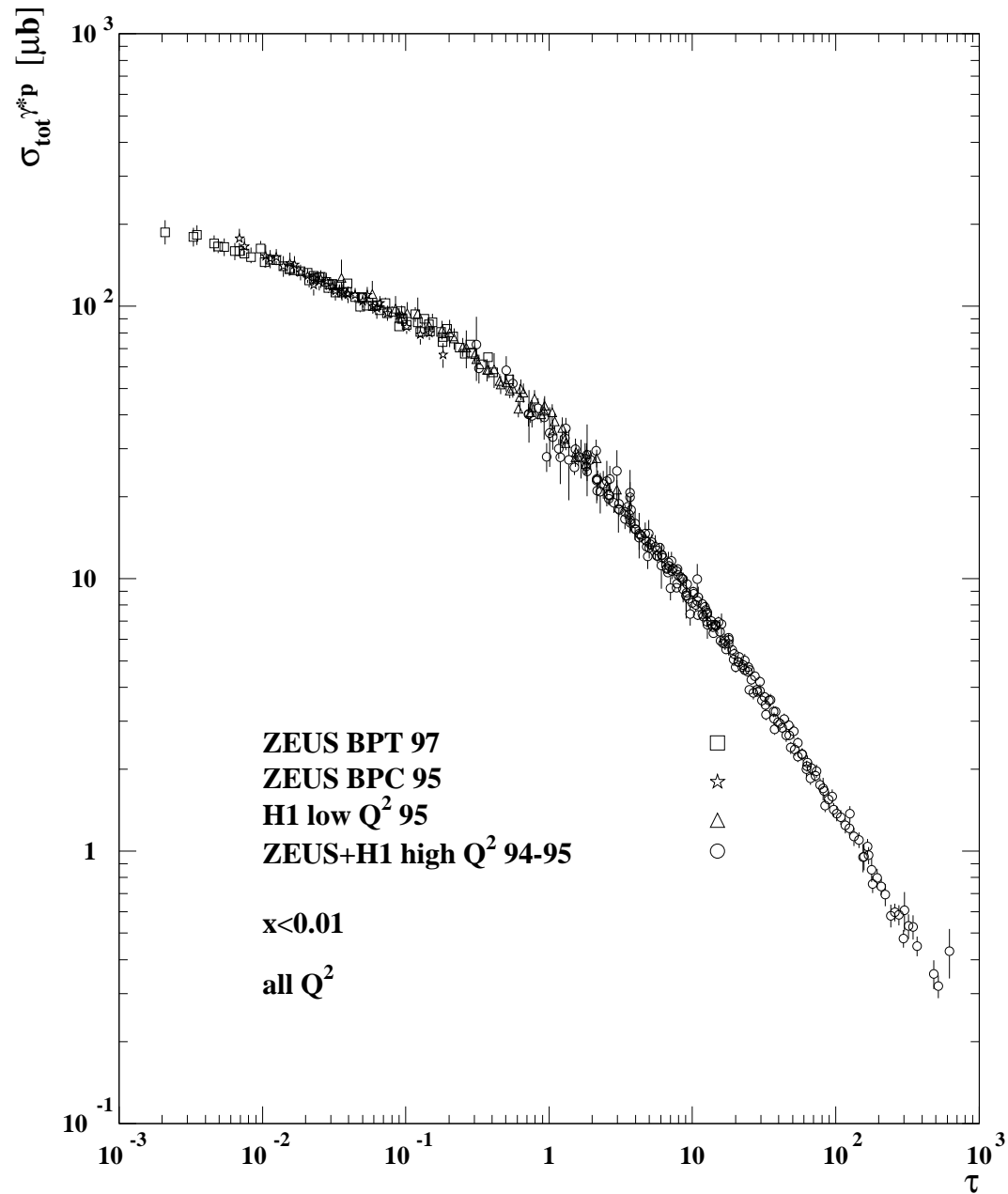


- Geometric scaling in gluon density:  $\phi(x, k_{\perp}) = \phi(k_{\perp}/Q_s(x))$

- Geometric scaling in observable:

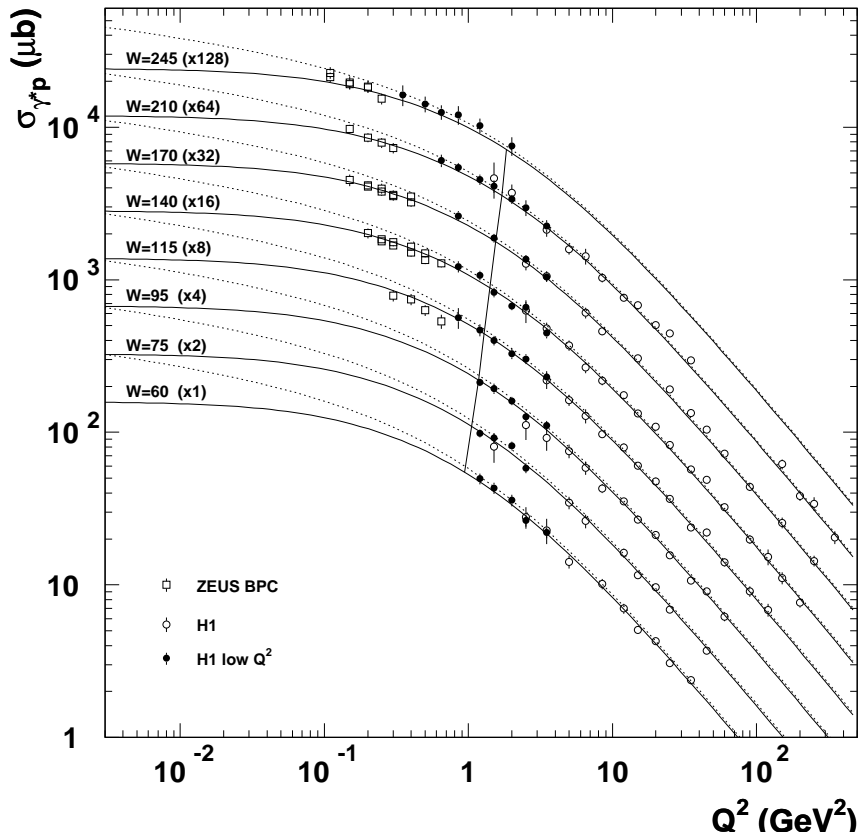
$$\sigma^{\gamma^* p}(x, Q^2) = F_2/Q^2 = \sigma\left(\underbrace{Q^2/Q_s^2(x)}_{\tau}\right)$$

# Geometric scaling in HERA data



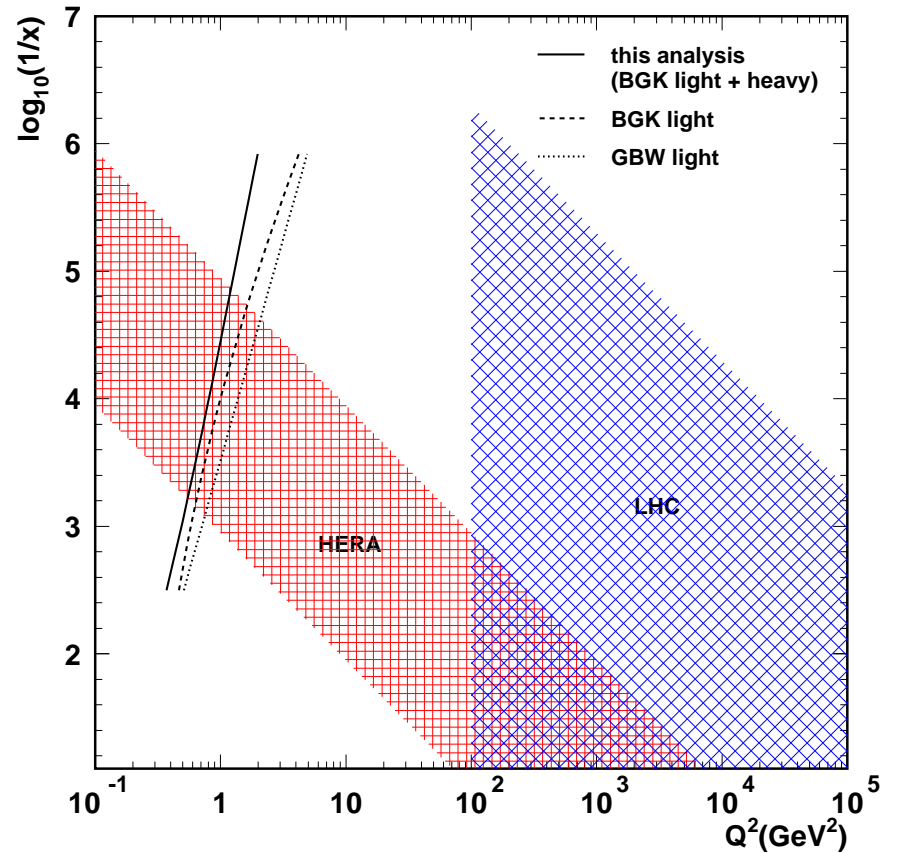
# Saturation at HERA

## Transition to low $Q^2$



$$\ln(Q_s^2(x)/Q^2) \rightarrow Q_s^2(x)/Q^2$$

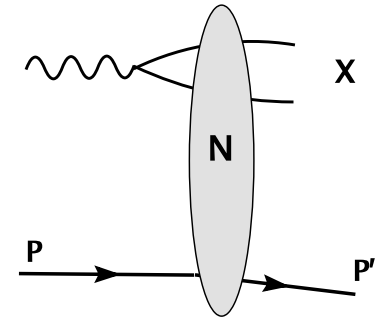
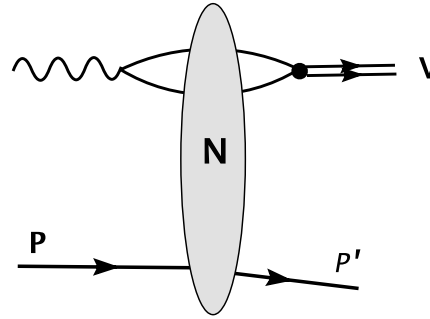
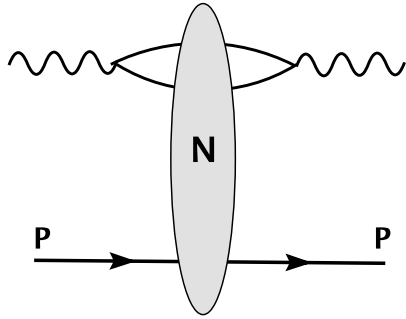
## Saturation line



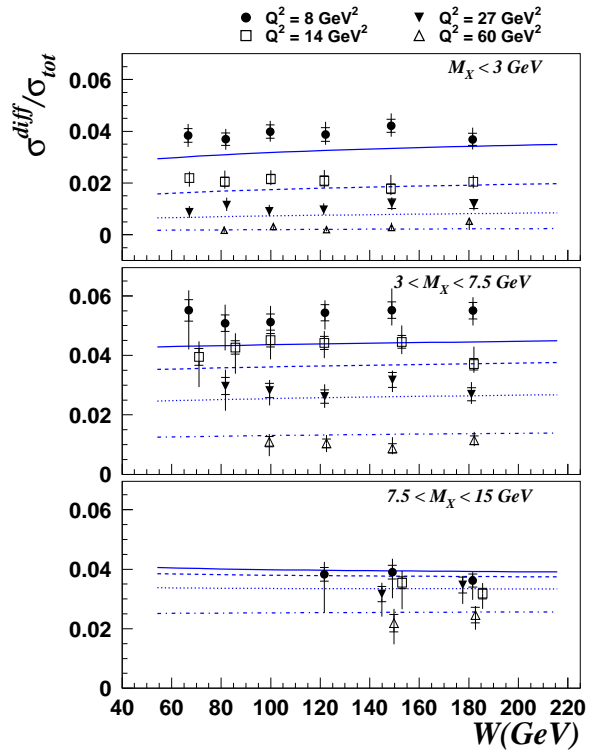
$$Q_s^2(x) \sim x^{-\lambda} \sim s^\lambda$$

# Saturation at diffraction at HERA

## Universal description



## Constant ratio for open diffraction in DIS



$$\frac{\sigma_{diff}}{\sigma_{tot}} \sim \frac{1}{\ln(Q^2/Q_s^2(x))}$$

## More on experimental verification of saturation

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- $ep$  collision at HERA
  - transition of  $F_2$  to small  $Q^2$
  - inclusive structure functions:  $F_2$ ,  $F_2^{c\bar{c}}$ ,  $F_L$
  - inclusive DIS diffraction:  $F_2^D$
  - diffractive vector meson production
  - DVCS
- $pp$  and  $AA$  collisions
  - Drell-Yan process
  - central diffractive production
  - particle multiplicities
  - peripheral collisions
  - limiting fragmentation
  - nuclear modification factor (Cronin effect)

# Conclusions

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- Parton saturation models are robust in the description of  $ep$  processes in high energy limit.
- $q\bar{q}$  dipole probe reveals the small- $x$  structure the proton which can be used in the description of  $pp$  and  $AA$  high energy collisions.
- Geometric scaling and saturation scale are key elements for parton saturation.
- LHC results will provide enough motivation for more studies in this direction.