Bose-Einstein study of position-momentum correlations of charged pions in hadronic Z^0 decays

Claudia Ciocca

University of Bologna
OPAL Collaboration

ISMD 2008
XXXVIII International Symposium on Multiparticle Dynamics
DESY, Hamburg, 19 September 2008

How to use BEC to retrive informations on interaction size

The space-time structure and evolution of a particle emitting source created in high energy collisions can be probed using intensity interferometry.

Formalism to be used for the analysis of the measured correlation function gradually realized during the last years mainly in the context of relativistic heavy ion collisions. BECs today of basical interest for the study of parton-hadron phase transition (QGP).

Significant theoretical and experimental improvement in the last years: Static source and 1-dim analysis → dynamic source and multi-dim BEC (also allowed by increased statistics) → new and detailed theoretical descriptions: useful parametrizations, models, simulations → guidance for detailed experimental studies

In the case of a dynamic source, the expansion leads to correlations between particle emission points and 4-momenta (position-momentum correlations).

The correlation function depends on both q and K:

$$C(p_1,p_2) = C(q,K) \qquad K = (p_1+p_2)/2 \quad q = (p_1 - p_2)$$

Ortogonality relation: $q \cdot K = 0$. Due the on-shell constraint, four components of q related by: $q^0 = \beta \cdot \mathbf{q}$ with $\beta = \frac{\mathbf{K}}{K^0} \approx \frac{\mathbf{K}}{\sqrt{m^2+K^2}}$

Correlation Function

Measured radii correspond to regions of homogeneity in K (effective source elements of pairs with momentum K) from which pions are emitted with momenta similar enough to interfere and contribute to the correlation function.

radius of the emitting source = dimension of source elements = correlation length

$\mathrm{e}^{+}\mathrm{e}^{-}$ collisions: longitudinal position-momentum correlations arise in string model as a consequence of string fragmentation. Model based on Bjorken-Gottfried hypothesis predict radii decreasing with transverse mass.

String model: source element or local patch = center of string fragment
Cluster description: source element or local patch = elementary color-neutral cluster

2-particle correlation function usually well approximated with a Gaussian in relative momentum q:

$$C(\mathbf{q},\mathbf{K}) = 1 + \exp\left[-q^{\mu} q^{\nu} \langle \tilde{x}_{\mu} \tilde{x}_{\nu} \rangle(\mathbf{K})\right] \qquad \tilde{x}_{\mu} = x_{\mu} - \langle x_{\mu} \rangle$$

The mass shell constraint can be used to eliminate redundant q-components.

Bertsch-Pratt parametrization

Using $q^0 = \beta_\perp q_o + \beta_l q_l$ (for azimuthally symmetric source):

$$C'(Q_\ell, Q_{t_{\mathrm{side}}}, Q_{t_{\mathrm{out}}}) = N[1 + \lambda e^{-(Q_\ell^2 R_{\mathrm{long}}^2 + Q_{t_{\mathrm{side}}}^2 R_{t_{\mathrm{side}}}^2 + Q_{t_{\mathrm{out}}}^2 R_{t_{\mathrm{out}}}^2 + 2 Q_\ell Q_{t_{\mathrm{out}}} R_{\mathrm{long},t_{\mathrm{out}}}^2)}] F(Q_\ell, Q_{t_{\mathrm{side}}}, Q_{t_{\mathrm{out}}})$$

N = normalization factor
λ = degree of incoherence of the pion sources (fraction of pairs that interfere)
$F(Q_\ell, Q_{t_{\mathrm{side}}}, Q_{t_{\mathrm{out}}}) = (1 + \epsilon_{\mathrm{long}} Q_\ell + \epsilon_{t_{\mathrm{side}}} Q_{t_{\mathrm{side}}} + \epsilon_{t_{\mathrm{out}}} Q_{t_{\mathrm{out}}})$ = long-range 2-particle correlations
ϵ_{i} = free parameters

- $R_{t_{\mathrm{side}}}^2 = \langle \tilde{y}^2 \rangle$ = transverse radius
- $R_{\mathrm{long}} = \langle (\tilde{z} - \beta_l \tilde{t})^2 \rangle = \langle \tilde{z}^2 \rangle$ = longitudinal radius (in LCMS)
- $R_{t_{\mathrm{out}}} = \langle (\tilde{x} - \beta_\perp \tilde{t})^2 \rangle$ = combination of both spatial and temporal extension
- $R_{\mathrm{long},t_{\mathrm{out}}} = \langle (\tilde{x} - \beta_\perp \tilde{t})(\tilde{z} - \beta_l \tilde{t}) \rangle = \langle \tilde{z}(\tilde{x} - \beta_\perp \tilde{t}) \rangle$ = combination of both spatial and temporal extension (in LCMS)
- $R_{t_{\mathrm{out}}}^2 - R_{t_{\mathrm{side}}}^2 \approx \beta_\perp^2 \langle \tilde{t}^2 \rangle \propto$ duration of the particle emission process

Yano-Koonin parametrization

Using $q_\perp = \sqrt{q_o^2 + q_s^2}$ (for azimuthally symmetric source):

$$C'(q_\mathrm{t}, q_\ell, q_0) = N\{1 + \lambda \mathrm{e}^{-[q_\mathrm{t}^2 R_\mathrm{t}^2 + \gamma^2 (q_\ell - v q_0)^2 R_\ell^2 + \gamma^2 (q_0 - v q_\ell)^2 R_0^2]}\} F(q_\mathrm{t}, q_\ell, q_0)$$

$$\gamma^2 = \frac{1}{1 - v^2} \qquad c = 1$$

N = normalization factor
λ = degree of incoherence of the pion sources (fraction of pairs that interfere)
$F(q_\mathrm{t}, q_\ell, q_0)$ = $(1 + \delta_\mathrm{t} q_\mathrm{t} + \delta_\ell q_\ell + \delta_0 q_0)$ = long-range 2-particle correlations

ϵ_i = free parameters

- v = longitudinal velocity of the source element (in CMS)
- $R_0 \approx \langle \tilde{t}^2 \rangle$ = duration of the particle emission process
- R_t = $\langle \tilde{y}^2 \rangle$ = transverse radius
- $R_\ell \approx \langle \tilde{z}^2 \rangle$ = longitudinal radius

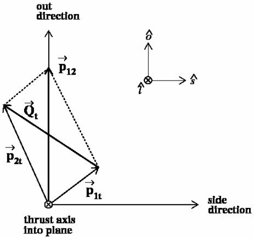
Separation between spatial and temporal aspects of the source and parameters do not depend on the frame in which the correlation function is measured, since evaluated in the rest frame of the source element (YK frame defined by $v = 0$).

System of coordinates

Measurement of three-dimensional Bose-Einstein correlation functions, analyzed in order to measure their dependence on K and investigate potential dynamical features of the pion-emitting source created after an $\mathrm{e}^{+}\mathrm{e}^{-}$ annihilation at a centre-of-mass energy of about 91 GeV. Results based on the high statistics data obtained with the OPAL detector at LEP (about 4.3M multihadronic events).

Correlations measured as functions of two sets of variables, components of q, in two different frames (in order to be fitted by the two parametrizations of interest):

- $(Q_{\ell}, Q_{\mathrm{t_{side}}}, Q_{\mathrm{t_{out}}})$ evualuated in the LCMS frame. In this system the components $Q_{\mathrm{t_{side}}}$ and Q_{ℓ} reflect only the difference in emission space while $Q_{\mathrm{t_{out}}}$ depends on the difference in emission time as well. LCMS coincide with longitudinal rest frame of the pair. In LCMS $\beta_l = Y = 0$
- $(q_{\mathrm{t}}, q_{\ell}, q_0)$ evaluated in the CMS frame. $q_0 = (E_1 - E_2) \geq 0$



LCMS

Experimental correlation function and dependence on K

Experimental 3-dimensional correlation function:

$$C = \frac{N_{\pi^+\pi^+} + N_{\pi^-\pi^-}}{N_{\pi^+\pi^-}} = \frac{N_{\mathrm{like}}}{N_{\mathrm{unlike}}}$$

C' introduced in order to reduce distorsions due to long range correlations and pions from resonance decays:

$$\boxed{C' = \frac{C^{\mathrm{DATA}}}{C^{\mathrm{MC}}} = \frac{N^{\mathrm{DATA}}_{\mathrm{like}} / N^{\mathrm{DATA}}_{\mathrm{unlike}}}{N^{\mathrm{MC}}_{\mathrm{like}} / N^{\mathrm{MC}}_{\mathrm{unlike}}}}$$

MC = 7.2M Jetset events without BEC

Dependence of the correlation functions on the pair average 4-momentum K analyzed by selecting pions in different intervals of two components of K: the pair rapidity

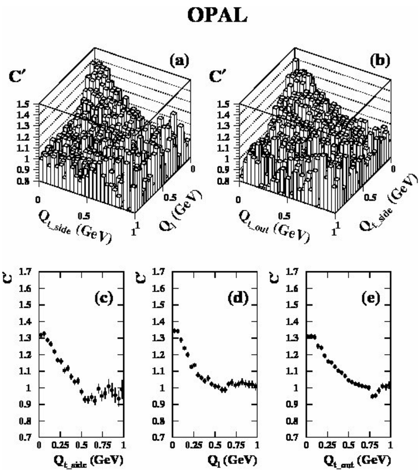
$$|Y| = \frac{1}{2} \ln \left[\frac{(E_1+E_2)+(p_{\ell,1}+p_{\ell,2})}{(E_1+E_2)-(p_{\ell,1}+p_{\ell,2})} \right]$$

and the pair average transverse momentum with respect to the event thrust direction

$$k_{\mathrm{t}} = \frac{1}{2} \left| (\mathbf{p}_{\mathrm{t},1} + \mathbf{p}_{\mathrm{t},2}) \right|$$

Correlation functions studied in 3 bins of $|Y|$ and 5 bins of k_{t}.

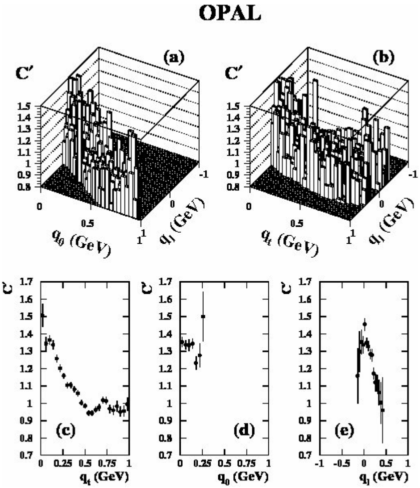
Correlation function $C'(Q_\ell, Q_{\mathrm{t_{side}}}, Q_{\mathrm{t_{out}}})$



2- and 1-dimensional projections of $C'(Q_\ell, Q_{\mathrm{t_{side}}}, Q_{\mathrm{t_{out}}})$ for $0.8 \leq |Y| < 1.6$ and 0.3 GeV $\leq k_\mathrm{t} < 0.4$ GeV (the missing components is limited to 0.2 GeV).

Bose-Einstein correlation peaks are visible at low $Q_\ell, Q_{\mathrm{t_{side}}}, Q_{\mathrm{t_{out}}}$.

Correlation function $C'(q_{\mathrm{t}}, q_{\ell}, q_0)$

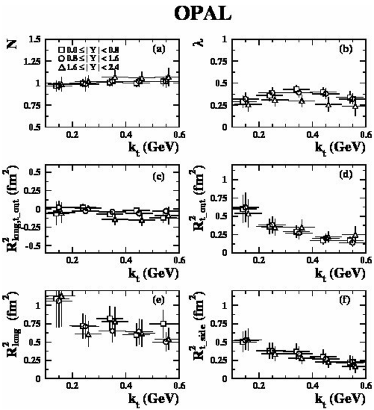


2- and 1-dimensional projections of $C'(q_{\mathrm{t}}, q_{\ell}, q_0)$ for $0.8 \leq |Y| < 1.6$ and 0.3 GeV $\leq k_{\mathrm{t}} < 0.4$ GeV (the missing components are limited to 0.2 GeV).

The combination $[(q_{\mathrm{t}}^2 + q_{\ell}^2) - q_0^2]$ is an invariant ≥ 0. This condition and the bound on the pair rapidity constrain the correlation function to be different from zero only in a limited region of the (q_{ℓ}, q_0) plane.

BEC enhancements are clearly seen in both q_{t} and q_{ℓ} projections while the range available to the variable q_0 is quite restricted, and that no BEC peak can be observed.

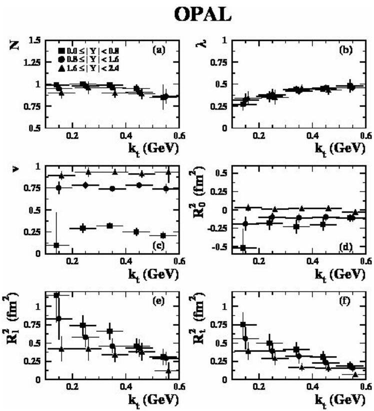
Bertsch-Pratt parameters



- $R^2_{t_{\mathrm{side}}}$, $R^2_{t_{\mathrm{out}}}$ and R^2_{long} decrease with increasing k_t. Presence of correlations between particle production points and momenta indicates that pion source expands during the particle emission process
- $R^2_{\mathrm{long}} > R^2_{t_{\mathrm{side}}}$ in agreement with a pion source elongated in the direction of the event thrust axis (OPAL Coll., G. Abbiendi *et al.*, Eur. Phys. J. C16 (2000) 423, L3 Coll., M. Acciarri *et al.*, Phys. Lett. B458 (1999) 517)
- $R^2_{\mathrm{long},t_{\mathrm{out}}}$ compatible with zero. This result may be explained assuming that source velocity, measured with respect to the rest frame of the pair, is close to 0
- $(R^2_{t_{\mathrm{out}}} - R^2_{t_{\mathrm{side}}})$ for $|Y| < 1.6$ positive at low k_t, then decreases and becomes negative for $k_t \geq 0.3$ GeV. In the highest rapidity interval compatible with zero for all k_t. As a consequence, it is not possible to estimate the particle emission time from $(R^2_{t_{\mathrm{out}}} - R^2_{t_{\mathrm{side}}})$
- λ between 0.25 and 0.4. Significantly anti-correlated with N: -0.35

Best-fit parameters of the Bertsch-Pratt parameterization to the correlation function $C'(Q_\ell, Q_{t_{\mathrm{side}}}, Q_{t_{\mathrm{out}}})$, as a function of k_t, for different intervals of rapidity $|Y|$. Bin size = 40 MeV. Fit range = [0.04,1.0] GeV. Horizontal bars represent bin widths and vertical bars include both statistical and systematic errors. Dependence of some parameters on k_t, minor dependence on $|Y|$.

Yano-Koonin parameters



- R_t^2 and R_ℓ^2 decrease with increasing k_t and $|Y|$
- $R_\ell^2 > R_t^2$
- R_0^2 compatible with zero at high rapidities and negative for $|Y| < 1.6$. This excludes an interpretation of R_0 in terms of the time duration of the particle emission process. Difficulties in achieving reliable results for R_0^2 due to the limited phase-space available in $\gamma^2(q_0 - vq_\ell)^2$
- v does not depend on k_t, strongly correlated with the pair rapidity
- λ increases with k_t. Significantly anti-correlated with N: -0.60

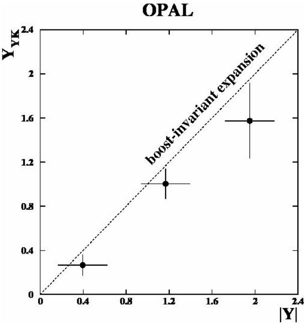
Best-fit parameters of the Yano-Koonin parameterization to the correlation function $C'(q_t, q_\ell, q_0)$, as a function of k_t, for different intervals of rapidity $|Y|$. Bin size = 40 MeV. Fit range = [0.04,1.0] GeV. Horizontal bars represent bin widths and vertical bars include both statistical and systematic errors. Dependence of some parameters on k_t and $|Y|$.

YK rapidity versus pair rapidity

Dependence of v on $|Y|$ presented introducing the Yano-Koonin rapidity:

$$Y_{\mathrm{YK}} = \frac{1}{2} \ln\left(\frac{1+v}{1-v}\right)$$

Y_{YK} measures the rapidity of the source element with respect to the CMS frame: a non-expanding source would correspond to $Y_{\mathrm{YK}} \approx 0$ for any $|Y|$, for a (longitudinally) boost-invariant source the strict correlation $Y_{\mathrm{YK}} = |Y|$ is expected.



The Yano-Koonin rapidity Y_{YK} as a function of the pion pair rapidity $|Y|$. A clear positive correlation between Y_{YK} and $|Y|$ is observed, even if $Y_{\mathrm{YK}} < |Y|$ at the largest pair rapidities. This is in agreement with a pion source which is emitting particles in a nearly boost-invariant way.

Each $|Y|$ computed as the weighted average of the corresponding bin. Y_{YK} values computed using the average value of v over all k_{t} in that $|Y|$ bin (since v almost independent on k_{t} in a given Y interval). Horizontal bars are deviations from the average. Vertical bars include both statistical and systematic errors.

BP and YK parameters relations

Relations between the parameters (correlation lengths) of the BP and YK parametrizations measured in the LCMS and CMS frames respectively:

$$\begin{aligned} R^2_{\mathrm{t_{side}}} &= R^2_{\mathrm{t}} \\ R^2_{\mathrm{long}} &= \gamma^2_{\mathrm{LCMS}}(R^2_\ell + \beta^2_{\mathrm{LCMS}} R^2_0) \\ (R^2_{\mathrm{t_{out}}} - R^2_{\mathrm{t_{side}}}) &= \beta^2_{\mathrm{t}}\gamma^2_{\mathrm{LCMS}}(R^2_0 + \beta^2_{\mathrm{LCMS}} R^2_\ell) \end{aligned}$$

β_{LCMS} = velocity of the source element measured in LCMS (i.e. with respect to pair rest frame)

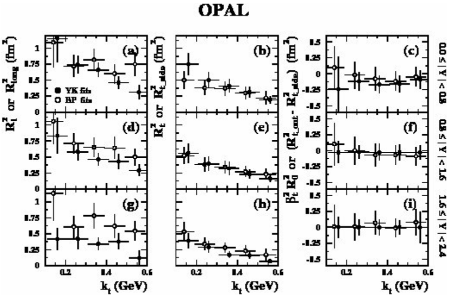
$\gamma_{\mathrm{LCMS}} = 1/\sqrt{1-\beta^2_{\mathrm{LCMS}}}$

$\beta^2_{\mathrm{t}} = \left\langle \frac{2k_{\mathrm{t}}}{E_1+E_2} \right\rangle^2$ (brackets stand for average over all pairs in the given Y and k_{t} range)

For a boost-invariant source, $\beta_{\mathrm{LCMS}} = 0$ and equations reduce to:

$$\begin{aligned} R^2_{\mathrm{long}} &= R^2_\ell \\ (R^2_{\mathrm{t_{out}}} - R^2_{\mathrm{t_{side}}}) &= \beta^2_{\mathrm{t}} R^2_0 \end{aligned}$$

BP and YK parameters comparison



- $R^2_{\text{long}} > R^2_\ell$ in all rapidity intervals. This corresponds to $\beta_{\text{LCMS}} > 0$, in agreement with a pion source whose expansion is not completely boost-invariant.
- $R^2_{\text{t}_{\text{side}}} = R^2_{\text{t}}$ within errors, with possible deviations at low k_{t}
- negative values of R^2_0 and $(R^2_{\text{t}_{\text{out}}} - R^2_{\text{t}_{\text{side}}})$ appearing in the two first rapidity intervals prevent an interpretation in terms of the time duration of the particle emission process.

Best-fit BP parameters R^2_{long}, $R^2_{\text{t}_{\text{side}}}$ and $(R^2_{\text{t}_{\text{out}}} - R^2_{\text{t}_{\text{side}}})$ compared respectively with the YK parameters R^2_ℓ, R^2_{t} and $\beta^2_{\text{t}}\text{R}^2_0$. Errors on the parameters include both statistical and systematic uncertainties.

Interpretations

Physical interpretation of the R_0^2 parameter:

- source opacity (surface, dominated emission) U. Heinz *et al*., Heiselberg and Vischer
  $R_0^2 = \langle \tilde{t}^2 \rangle - \frac{2}{\beta_\perp} \langle \tilde{x}\tilde{t} \rangle + \frac{1}{\beta_\perp^2} \langle \tilde{x}^2 - \tilde{y}^2 \rangle$
  leading geometrical contribution: $\langle \tilde{x}^2 - \tilde{y}^2 \rangle < 0$
  $R_{\mathrm{t_{out}}}^2 - R_{\mathrm{t_{side}}}^2 = \beta_\perp^2 \langle \tilde{t}^2 \rangle - 2\beta_\perp \langle \tilde{x}\tilde{t} \rangle + \langle \tilde{x}^2 - \tilde{y}^2 \rangle$
  [Heavy ions based on hydrodynamical models - fireball decouple (freeze-out) outside-inwards]
- inside-outside cascade particle formation - hadron production on a surface of costant proper time (Bialas *et al*.)
- τ model: $x(p) = a\tau p, a = 1/m_t, \tau = \sqrt{t^2} - z^2$ expansion ring in transverse direction (Csorgo *et al.*and L3 Coll.) based on Bjorken-Gottfried condition $x = dp$
- freez-out hypersurface (hyperbola in t-z space-time diagram of the fireball evolution), Tomasik
- models considering Gaussian deviation

Results from other experiments

Previous results:

- Heavy ions collisions: AGS (E877), SPS (NA49, NA44, WA98), GIBS, RHIC (STAR, PHENIX, PHOBOS)
- e^+e^- collisions: Delphi (transverse mass dependence)

Negative R_0^2 parameter:

- Heavy ions collisions: RHIC, STAR and PHENIX "The RHIC Puzzle"
- e^+e^- collisions: OPAL, L3 and Delphi (BP parametrization)

Development and solutions:

- Heavy ions collisions: HBT puzzle@RHIC now solved: faster initial evolution, see Florkowski talk
- e^+e^- collisions: L3 analysis based on τ model, see Meztger and Csorgo talks

Conclusions

Analysis of Bose-Einstein correlations in e^+e^- annihilation events at the Z^0 peak performed in bins of the average 4-momentum of the pair, K, presented for the first time. Dynamic features of the pion emitting source investigated. Previous BEC analyses, not differential in K, not sensitive to these features.

Using the Yano-Koonin and the Bertsch-Pratt formalisms, correlation functions studied in intervals of two components of K: the pion pair rapidity $|Y|$ and the mean transverse momentum k_t. Transverse and longitudinal radii of the pion sources decrease for increasing k_t, indicating the presence of correlations between the particle production points and their momenta. The Yano-Koonin rapidity scales approximately with the pair rapidity, in agreement with a nearly boost-invariant expansion of the source of pions. Limitations in the available phase space did not allow measurement of the duration of the particle emission process.

Similar results observed in more complex systems, such as the pion sources created in pp and heavy-ion collisions, now complemented with such measurements in the simpler hadronic system formed in e^+e^- annihilations (OPAL and L3 @ LEP).

Great interest and development on hadronic source properties. Several theoretical guidance and experimental studies in progress.

Backup slides

Systematic uncertainties

Study of systematic uncertainties of the fit parameters and stability of the results concerning the dependence of the transverse and longitudinal radii on k_t:

- Gamow factor correction for Coulomb interactions
- modified selection
- modified fit range
- bin size = 60 MeV
- Edgeworth expansion of the Gaussian
- correlation function *C*
- kurtosis
- 2-jets event sample