High-energy heavy-ion collisions: from CGC to Glasma

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Abstract
I present an overview of recent theoretical developments towards "first principle" description of heavy-ion collisions at high energies.

1 Introduction

Relativistic heavy-ion collisions are multi-step phenomena which necessarily entail transition from high to low energy densities, or, equivalently from perturbative to non-perturbative kinematical regions (see Fig. 1). Thus, it is quite difficult to describe *all* the steps within the first-principle (i.e., QCD-based) calculations even though the collision energy is taken to be high enough. Nevertheless, we believe that at least the first two steps (the initial condition and the earliest stage well before thermalization) allow a firm QCD-based description because, as I will explain later, these two essentially occur around a large semi-hard momentum scale. From the viewpoint of high energy QCD, the initial condition and the earliest stage after the collision are respectively described by the Color Glass Condensate (CGC) [1] and the Glasma [2]. In this talk, I overview the recent developments towards understanding the dynamics of CGC and Glasma, and discuss a possible scenario of the heavy-ion collisions at high energy. In particular, unstable dynamics of the Glasma provides a novel mechanism for early thermalization.

2 Initial conditions : CGC

2.1 What is the CGC?

Consider one nucleus that is moving very fast in the z direction. When the scattering energy is quite high, what we measure is not a simple valence structure of each nucleon, but a state with a huge number of gluons that are emitted either directly from the valence partons or successively from (already emitted) gluons. Such a highly dense gluonic state is now called the CGC, and is indeed observed experimentally through the electron deep inelastic scattering off a proton. We

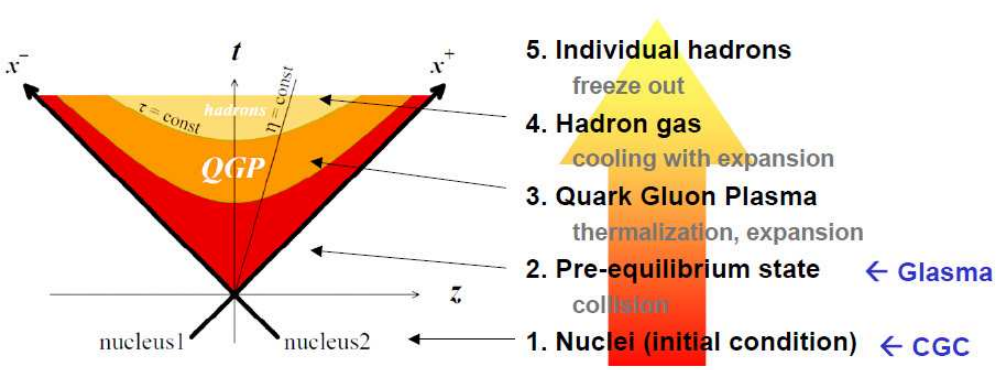


Fig. 1: Relativistic heavy-ion collision in the high energy limit

describe the CGC by separating the whole degrees of freedom into large and small x partons (x is the fraction of momentum carried by a parton). Large x partons are distributed on a Lorentz contracted nucleus and their motion is very slow compared to the time scale of the collision. Thus we treat them altogether as a *static* color source $\rho^a(x_\perp)$. We also assume that ρ^a can be taken as random reflecting the unpredictable configuration of partons at the moment of collision. Small x partons (mostly gluons) are then regarded as a coherent radiation field created by the color source (large x partons). Hence, we investigate the following stochastic Yang-Mills equation:

$$(D_\nu F^{\nu\mu})^a = \delta^{\mu +}\rho^a(x_\perp)\,. \tag{1}$$

We further introduce a weight function $W[\rho]$ (that is however a priori unknown) to control the randomness of the source $\rho^a(x_\perp)$. These are the basic strategies of CGC (see Ref. [1] for details).

Most of gluons in CGC have relatively large transverse momentum called the saturation momentum, $Q_s \gg \Lambda_{\rm QCD}$ whose inverse corresponds to a typical transverse 'size' of gluons when they fill up the transverse disk and start to interact with each other. One can compute in QCD the energy (or x) and atomic mass number A dependences of Q_s as

$$Q_s^2(x,A) \propto A^{1/3}(1/x)^\lambda\,, \quad \lambda \simeq 0.3\,, \tag{2}$$

which is surprisingly consistent with the scale Q_s^2 determined from experimental data through the geometric scaling. Since $Q_s(x,A)$ grows with increasing energy $(x \sim \ln 1/s \to 0)$, the weak-coupling treatment becomes better and better with increasing energy, where $\alpha_s(Q_s) \ll 1$.

Another important feature of CGC is that, as a result of the large number of gluons, it has a strong gauge field $A \sim Q_s/g$ and thus strong color electric and magnetic fields $E, B \sim Q_s^2/g$. This is the region where we cannot ignore the nonlinear terms in the interaction. Therefore, CGC is a weakly-coupled many body system of gluons which shows coherent and nonlinear behavior.

2.2 CGC as the initial condition of heavy-ion collision

Let us now consider the collision of two nuclei in the center of mass frame where both nuclei can be equally treated as CGCs [3] (see Fig. 1). In this case, the right-hand-side in eq. (1) is replaced by $J^\mu = \delta^{\mu+}\delta(x^-)\rho_1(x_\perp) + \delta^{\mu-}\delta(x^+)\rho_2(x_\perp)$ with ρ_1 (ρ_2) being a color source of the right (left) moving nucleus 1 (2). Before the collision, classical gauge fields belonging to each nucleus are created by these color sources. What is truly nontrivial occurs in the forward light cone ($x^\pm > 0$), where we expect real gluon emissions and non-equilibrium transition towards QGP. We describe the very early stage of time evolution by solving *source free* Yang-Mills equations in the forward light cone, with the initial condition specified by the CGC fields of each nucleus.

Note also that the created matter which locates in between two (passed) nuclei will expand in the longitudinal direction almost at the speed of light, and we expect that it is a good approximation to describe the solution to the Yang-Mills equation by a boost invariant field. Namely, we consider the solution in the following form:

$$A^\pm = \pm x^\pm \alpha(\tau, x_\perp), \quad A^i = \alpha_3^i(\tau, x_\perp)\,, \tag{3}$$

where $\tau = \sqrt{2x^+x^-} > 0$ is the proper time. Indeed, this expression gives a solution independent of *rapidity* $\eta = \frac{1}{2}\ln(x^+/x^-)$ which can be easily seen if one defines vector fields in the (τ, η)

coordinates: $A_\eta = x^+A^- - x^-A^+ = -\tau^2\alpha(\tau, x_\perp)$. The initial condition for the fields α and α_3^i is specified at $\tau = 0+$ by using the CGC fields of each nucleus, α_1 and α_2:

$$\alpha|_{\tau=0} = \frac{ig}{2}[\alpha_1^i, \alpha_2^i], \quad \alpha_3^i|_{\tau=0} = \alpha_1^i + \alpha_2^i, \tag{4}$$

and for the time derivatives

$$\partial_\tau \alpha|_{\tau=0} = \partial_\tau \alpha_3^i|_{\tau=0} = 0. \tag{5}$$

Obviously, *the initial condition is completely determined by the CGC fields of each nucleus* which depend only on transverse coordinates, $\alpha_{1,2}^i(x_\perp)$.

3 Pre-equilibrium stages : Glasma

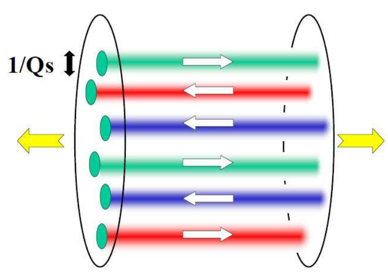
Unlike the CGC, the gluonic matter created after the collision shows strong time dependence of the field as a result of rapid expansion in the longitudinal direction (recall that the CGC is static, i.e., x^+-independent). Thus, to identify such a unique nature of the created matter, we now use a new name "Glasma" meaning the transitional state between 'glass' and 'plasma' [2]. Glasma is a rapidly expanding and interacting gluon field. Immediately after the collision, it will still remember the properties of CGC, and most of the gluons will have transverse momenta of the order of Q_s. Namely, the Glasma can still be treated as a weak coupling system.

3.1 Stable dynamics : boost-invariant Glasma

The first attempts towards understanding nonlinear dynamics of the Glasma were numerically done in real-time simulations of classical Yang-Mills fields on the lattice. Most of the simulations were performed in the boost-invariant case. Obtained physical quantities such as the gluon transverse momentum spectra and the energy density were found to be reasonable enough. More recently, such numerical results have driven people to think of the analytic aspects of the Glasma. The most important recognition is the emergence of a *flux tube structure* (Fig. 2, left). Before the collision, each CGC has purely transverse $\mathbf{E}$ and $\mathbf{B}$ that are orthogonal to each other $\mathbf{E}\cdot\mathbf{B} = E_\perp \cdot B_\perp = 0$. However, just after the collision, the field strength instantaneously becomes purely *longitudinal*. Indeed, the z-components at $\tau = 0+$ are explicitly given by

$$E^z|_{\tau=0^+} = -ig[\alpha_1^i, \alpha_2^i], \qquad B^z|_{\tau=0^+} = ig\epsilon_{ij}[\alpha_1^i, \alpha_2^j], \tag{6}$$

with $\alpha_{1,2}$ being the CGC fields, while all the transverse components are vanishing. Such longitudinally extended fields in between two receding nuclei remind us of the Lund string model, but



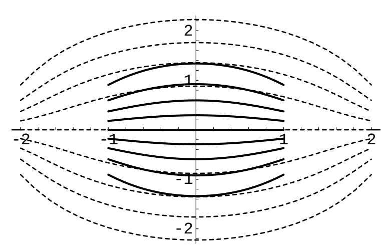


Fig. 2: Flux tube structure of the Glasma (left) and how the flux tube expands in time (right)

there are two significant differences. First, reflecting the CGC structures of the colliding nuclei, transverse coherence length of the flux tubes should be of order $1/Q_s$, instead of $1/\Lambda_{\rm QCD}$ as in the Lund model. This is so because the Glasma flux tube is a perturbative object while the Lund model simulates nonperturbative dynamics of string breaking. Second, the Glasma flux tube can have a magnetic field in it while the Lund model treats only electric flux tubes. In fact, even a purely magnetic flux tube is possible if one takes the same color structure for the same spatial components α_1^i and α_2^i ($i = x, y$) but different for different components.

Dynamics of an isolated flux tube can be reasonably understood within Abelian approximation [4]. If one looks deeply inside of the flux tube, the field strengths may be large, but are regular and homogeneous. Thus one can gauge-transform the field so that it is directed to the third color component. On the other hand, if one looks well outside of the flux tube, the field is weak enough, and one can ignore nonlinear effects. Hence, if the field profile is not singular in the tube and decays rapidly outside the tube, Abelian approximation is expected to be reasonable enough. In this approximation, one can easily solve the Yang-Mills equation even in expanding geometry, and can compute the time dependence of *E* and *B* for a simple profile such as a Gaussian. The right panel of Fig. 2 shows how a single flux tube evolves in the actual time t (not in τ). Since the Glasma flux tube is essentially 'perturbative', it expands outwards and the strength in the tube decays rapidly in time (in contrast, a nonperturbative flux tube does not expand in the transverse direction and the strength inside the tube does not change). Lastly, we note that the τ dependence of each component of the field strength computed in this simple picture is remarkably consistent with the numerical result reported in Ref. [2].

3.2 Unstable dynamics : boost-noninvariant Glasma

It should be noticed that the boost-invariant Glasma cannot say anything about thermalization because boost invariance means eternal absence of nontrivial p_z dependence. Therefore, even isotropization (a necessary condition for thermalization) never occurs with boost invariant solutions. Of course this is a serious problem in the CGC-glasma description of the heavy-ion collision, and people have been investigating this both numerically [5] and analytically [4, 6, 7]. Below, I explain one of the recent findings of analytic approaches that *the rapidity-dependent fluctuation undergoes Nielsen-Olesen instability and can grow exponentially* [4].

We perform a stability analysis of the system against rapidity-dependent perturbations $a_{i,\eta}$:

$$A_i = \mathcal{A}_i(\tau, x_\perp) + a_i(\tau, \eta, x_\perp)\,, \quad A_\eta = \mathcal{A}_\eta(\tau, x_\perp) + a_\eta(\tau, \eta, x_\perp)\,, \tag{7}$$

where $\mathcal{A}_i$ and $\mathcal{A}_\eta$ are boost-invariant background fields given in eq. (3). Coupling between $\mathcal{A}_{i,\eta}$ and $a_{i,\eta}$ is present due to the nonlinear interaction in the non-Abelian gauge theory. For simplicity, we replace the background fields by τ-independent and spatially constant electric/magnetic fields, and consider the SU(2) group.[1] The first simplification was done because we expect that the time scale of instability is much shorter than that of the background field, and because we consider the region deep inside of the flux tube. In Ref. [4], the cases with either electric or magnetic field were explicitly shown, but one can similarly discuss the case where both are present [9].

[1]Generalization to SU(3) should be straightforward [9]. We have two constant background fields (directed to the 3rd and 8th color components).

When we have both electric and magnetic fields, the linearized equation for the fluctuation[2] $\tilde{a}_+^{(\pm)}$ which is the Fourier component having the third color charge $(\pm)$ and positive spin $+$ is given by [9]

$$\frac{1}{\tau}\partial_\tau(\tau\partial_\tau\tilde{a}_+^{(\pm)}) + \left\{\frac{1}{\tau^2}\left(\nu \pm \frac{gE}{2}\tau\right)^2 + (2n+|m|+1\mp m\pm 2)gB\right\}\tilde{a}_+^{(\pm)} = 0\,, \qquad (8)$$

where m and ν, respectively, are the orbital angular momentum and the momentum conjugate to the rapidity (a similar equation holds for negative spin $-$). Note that the term $\pm 2gB$ originates from the anomalous magnetic moment.

When we have only the electric field ($E \neq 0,\ B = 0$), the situation is similar to the Schwinger mechanism. Massless charged fluctuations are infinitely accelerated, but there is no amplification of the field (no instability). On the other hand, when we have only the magnetic field ($E = 0,\ B \neq 0$), the fluctuation forms Landau levels, and the lowest level ($n = 0$) becomes unstable. This is the *Nielsen-Olesen instability* which is known for non-expanding Yang-Mills systems [8]. Indeed, the explicit form of the solution is given by the modified Bessel function $I_{i\nu}(\sqrt{gB}\tau)$, which asymptotically shows divergent behavior:

$$I_{i\nu}(\sqrt{gB}\tau) \sim \mathrm{e}^{\sqrt{gB}\tau}\Big/\sqrt{2\pi\sqrt{gB}\tau}.$$

Note that the magnetic field given by the CGC can be strong $\sqrt{gB} \sim Q_s$. Therefore, we conclude that the mode with ν grows exponentially with the time scale given by $\tau_{\rm grow} = 1/Q_s$.

In relation to the early thermalization problem in RHIC, extensive investigation is performed for the plasma instability scenario. However, being formulated in a kinetic equation, it is applicable only after $\tau \sim 1/Q_s$ and thus cannot say anything about the very early stage of heavy-ion collisions $\tau < 1/Q_s$. This is the place where the *Glasma* instabilities play a unique important role. As we discussed, the characteristic time scale of the Glasma instabilities is $1/Q_s$. This implies that the system begins to show unstable behavior well before the kinetic description can be applicable.

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