High-energy heavy ion collisions

from CGC to Glasma

K. Itakura

KEK, Japan

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Aim & Plan

Aim :

to overview recent theoretical developments towards "first-principle" description of (the very early stages of) heavy-ion collisions at high energies

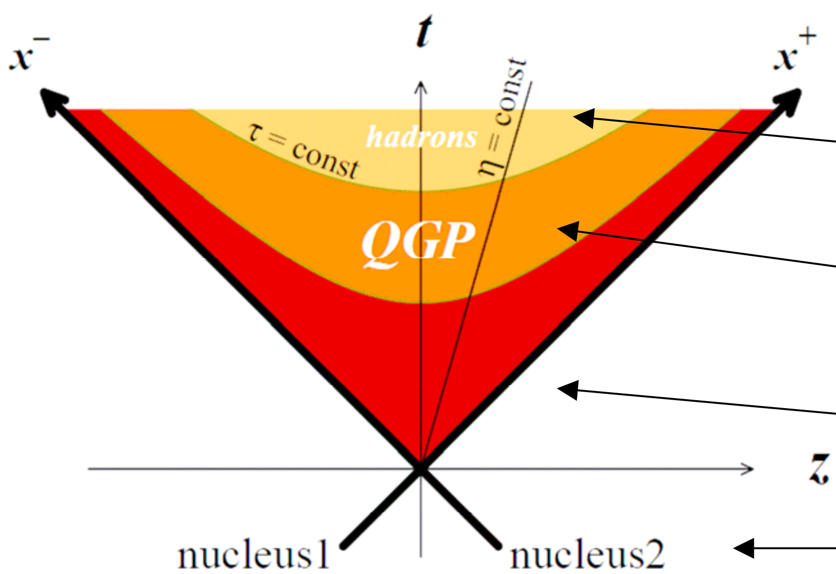
Plan :

1. Introduction
2. Initial conditions : CGC
3. Pre-equilibrium states : Glasma

   Stable and unstable dynamics
4. Summary

Introduction

Relativistic Heavy Ion Collisions in High Energy Limit



5. Individual hadrons

freeze out

4. Hadron gas

cooling with expansion

3. Quark Gluon Plasma

thermalization, expansion

2. Pre-equilibrium state ← **Glasma**

collision

1. Nuclei (initial condition) ← **CGC**

Not possible to describe *all* the steps within 1st principle (QCD-based) calculation (transition from *high* energy density to *low* energy density)

→ Focus on the first two steps (1 & 2) that may allow QCD-based calculation

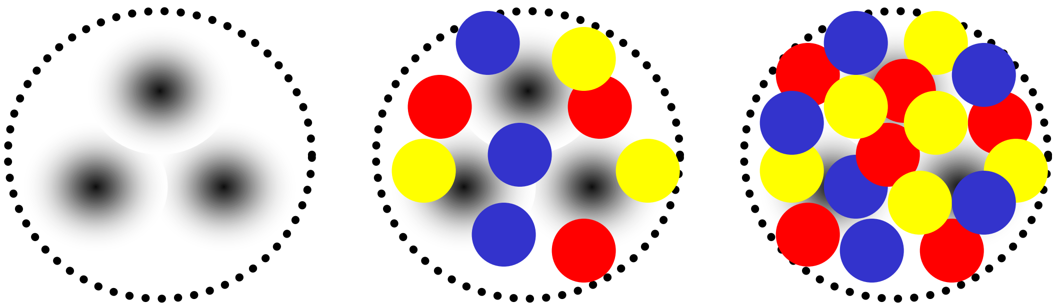
Q: How to describe the initial condition / transition towards QGP ?

At very high energies → Color Glass Condensate & Glasma

Initial Conditions: CGC

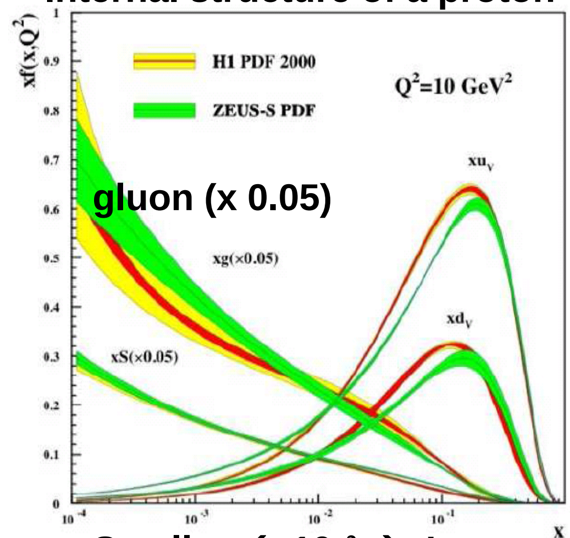
Hadrons at Very High Energies

Higher energies (smaller x → 0)



Valence partons **gluon cascade** **dense gluon state = CGC**

Internal structure of a proton



Small x ← 10^{-2} → Large x

Color Glass Condensate

Saturation scale $Q_s >> \Lambda_{QCD}$

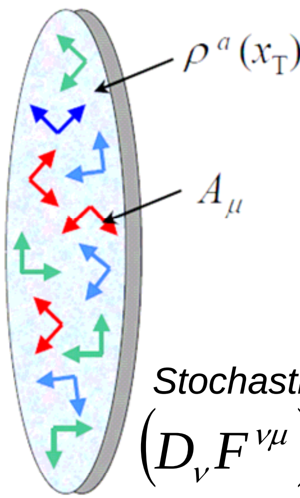
typical transverse size ~ $1/Q_s$

$$Q_s^{\,2}(x, A) \sim A^{1/3} x^{-0.3}$$

→ **weak coupling $\alpha_s(Q_s) << 1$ at high energy**

Strong gauge field $A \sim Q_s/g, \quad E, B \sim Q_s^2/g$

CGC is a weakly-coupled many body system with high non-linearity !



Valence partons as static random color source

Small x gluons as radiation field created by $\rho(x)$.

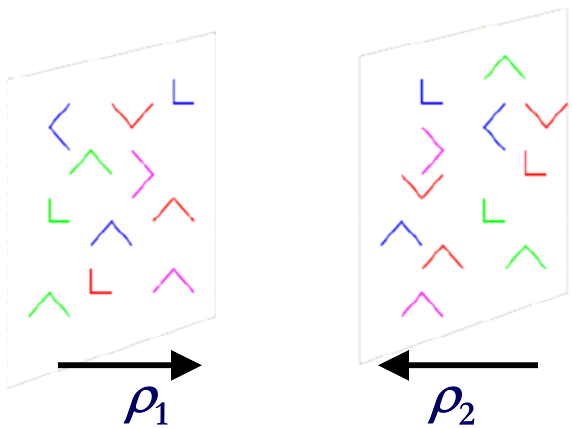
Stochastic Yang-Mills equation

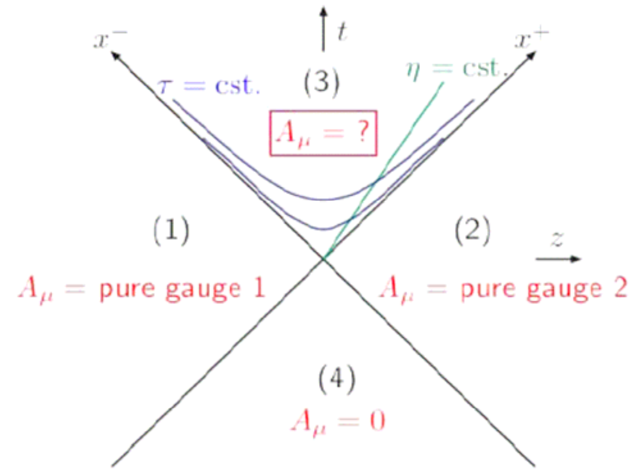
$$\left(D_\nu F^{\nu\mu}\right)^a = \delta^{\mu+}\delta(x^-)\rho^a(x_\perp)$$

Initial Conditions: CGC

High Energy HIC = collision of two CGC's

[Kovner, Weigert, McLerran, et al.]





Before the collision, each source creates the gluon field for each nucleus.

$$J^{\mu} = \delta^{\mu +}\delta(x^{-})\rho_1(x_{\perp}) + \delta^{\mu -}\delta(x^{+})\rho_2(x_{\perp})$$

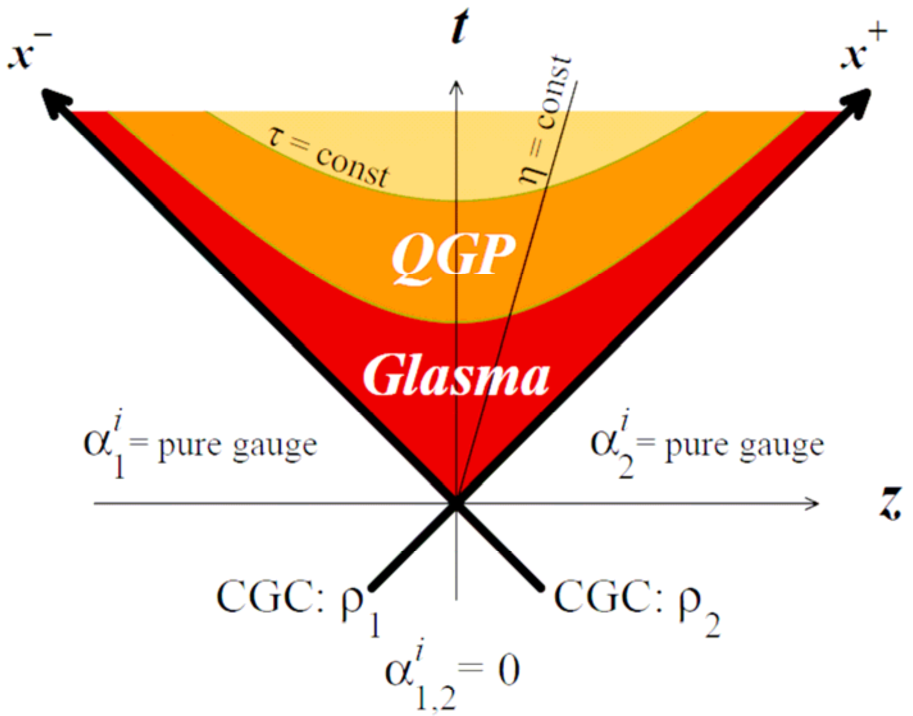
$$-D_i \alpha_I^i = \rho_I(x_{\perp}) \qquad \alpha_1 , \ \alpha_2 : \text{ gluon fields of nuclei}$$

After the collision, and at $\tau = 0+$, the gauge field is determined by α_1 and α_2

$$\begin{cases} A^i = \alpha_3^i(\tau, x_{\perp}) \\ A^{\pm} = \pm x^{\pm}\alpha(\tau, x_{\perp}) \end{cases} \qquad \begin{cases} \alpha_3^i |_{\tau=0} = \alpha_1^i + \alpha_2^i \\ \alpha |_{\tau=0} = \dfrac{ig}{2}[\alpha_1^i, \alpha_2^i] \end{cases}$$

Pre-equilibrium states: Glasma

Definition:
Non-equilibrium state between Color Glass Condensate and Quark Gluon Plasma which is created in high-energy heavy-ion collisions.



Solve the *source free* Yang Mills eq.

$$[D_\mu, F^{\mu\nu}] = 0$$

in *expanding geometry* with the CGC initial condition

Formulate in τ–η coordinates

$$\tau = \sqrt{t^2 - z^2} = \sqrt{2x^+x^-}, \quad \eta = \frac{1}{2}\ln\frac{x^+}{x^-}$$

proper time **rapidity**

Aim: to understand ***TIME EVOLUTION*** of an expanding system of strong Yang-Mills fields

Stable dynamics: Boost-invariant Glasma

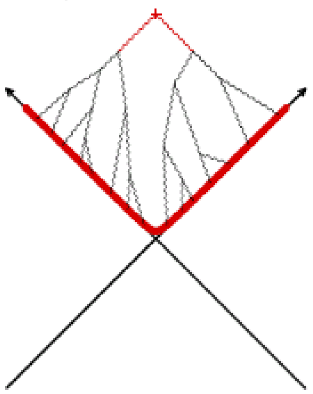
Numerical calculation of classical boost-invariant Glasma

[Krasnitz-Nara-Venugopalan, Lappi]

LO Glasma is boost-invariant (rapidity independent)

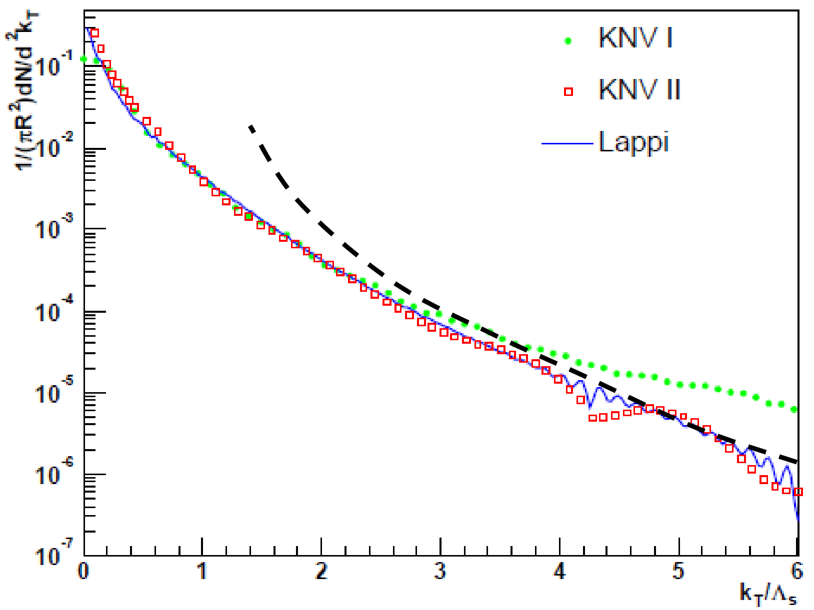
→ 2+1 dim. real time (τ) simulation on the transverse lattice (*a la* Kogut-Susskind)

LO* diagrams



* O($1/g^2$) and all orders in $(g\,\rho)^n$
(figure by F.Gelis)

Gluon transverse momentum distribution



Softening at small k_t (saturation)

pQCD power tail $\sim 1/k_t^4$ (dashed)

<u>Energy density</u> (LO)

$\varepsilon \sim 20 - 40$ GeV/fm^3 at $\tau \sim 0.3$ fm/c for $Q_s \sim 1 - 1.2$ GeV

Stable dynamics: Boost-invariant Glasma

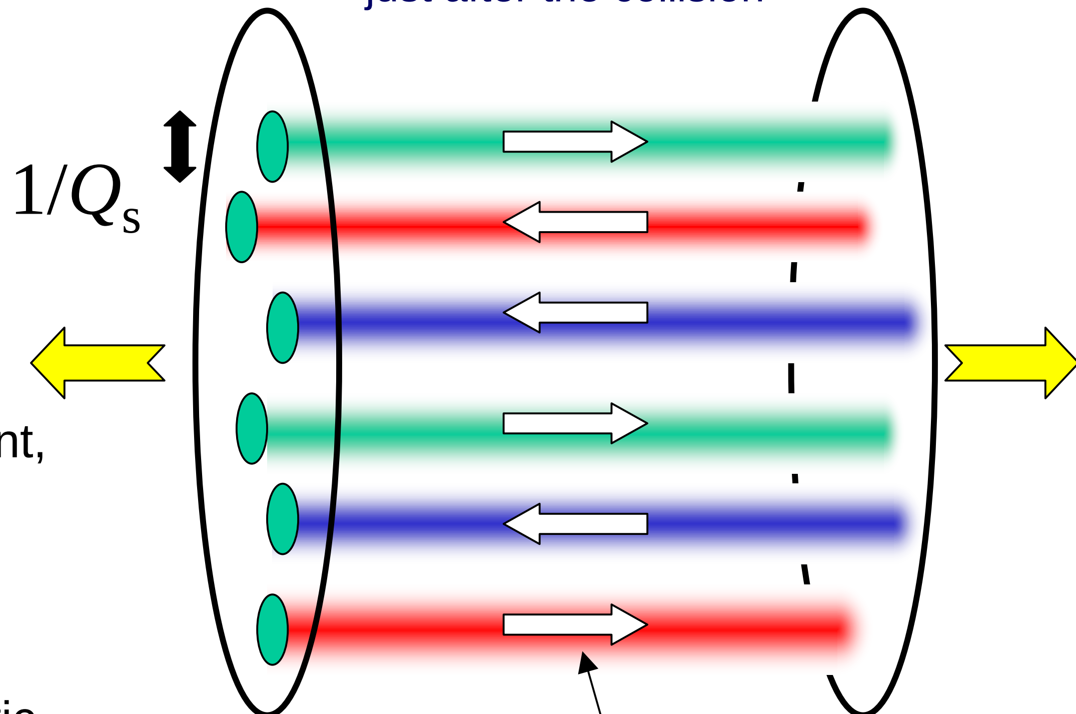
[Fries, Kapusta, Li, Lappi, McLerran]

There appears a flux tube structure !!

Longitudinal fields are generated at $\tau = 0_+$

$$E^z = ig[\alpha_1^i, \alpha_2^i]$$
$$B^z = ig\epsilon^{ij}[\alpha_1^i, \alpha_2^j].$$

Typical configuration of a single event just after the collision



E or B, or E&B

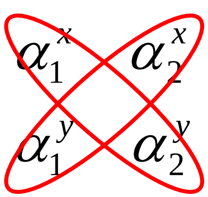
Similar to Lund string models but

** transverse correlation $1/Q_s$*

** magnetic flux tube possible*

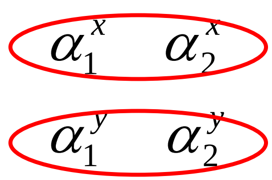
In general both E^z and B^z are present,

but



purely electric

$E^z \neq 0, B^z = 0$



purely magnetic

$E^z = 0, B^z \neq 0$

Nonzero $\boldsymbol{E}\cdot\boldsymbol{B}$ → generation of nonzero topological charge [Kharzeev,Krasnitz,Venugopalan]

Stable dynamics: Boost-invariant Glasma

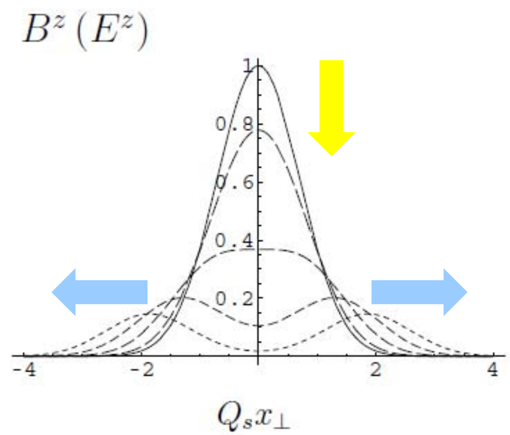
[Fujii, Itakura]

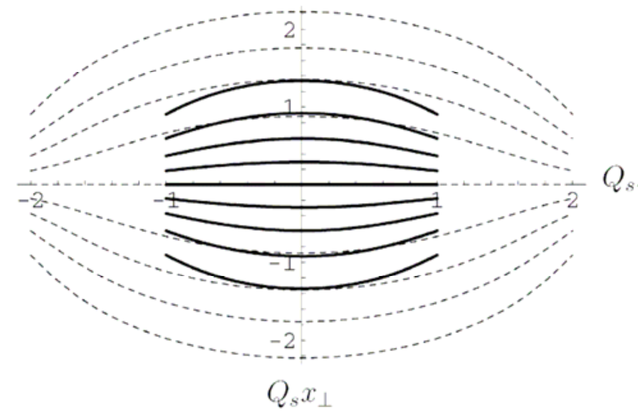
Expanding flux tube

Inside $x_t < 1/Q_s$: strong but homogeneous gauge field

Outside : weaker field → Can be approximately described by *Abelian* field

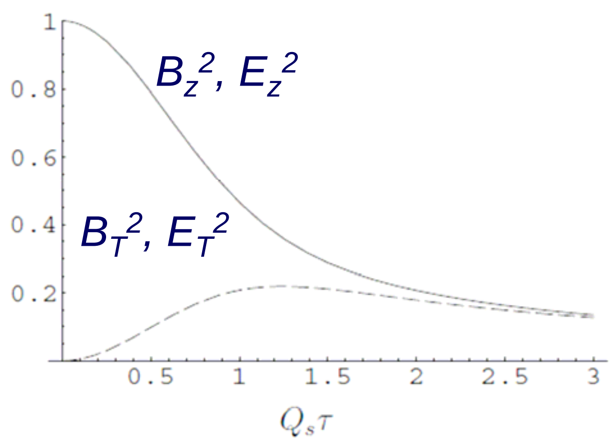
Transverse profile of a Gaussian flux tube at $Q_s\tau$ =0, 0.5, … 2 (left) and $Q_s t$ = 1, 2 (right).

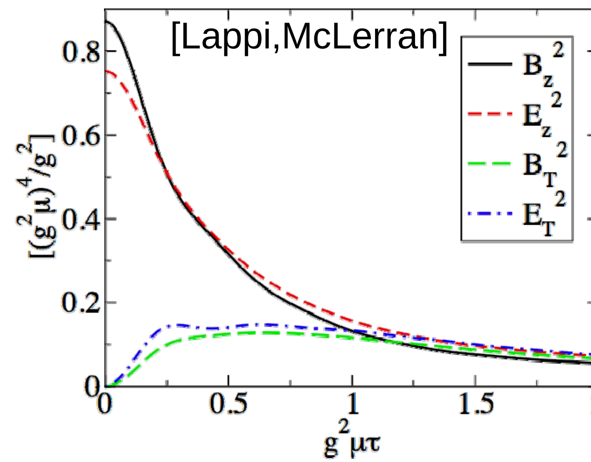




***τ* dependence of field strength from a single flux tube** (averaged over transverse space)

compared with the result of classical numerical simulation of boost-invariant Glasma





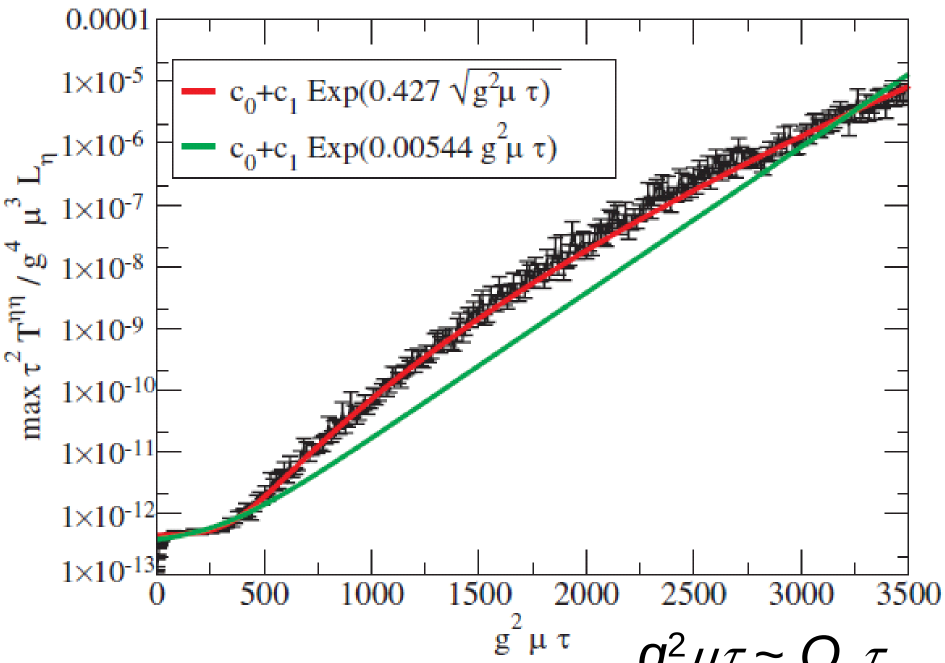
Nonlinear effects are to be included in the stability analysis

Unstable dynamics: Boost-noninvariant Glasma

Boost invariant Glasma (without rapidity dependence) cannot thermalize
→Need to violate the boost invariance !!!
origin : quantum fluctuation [Fukushima,Gelis,McLerran]

P. Romatschke & R. Venugopalan, 2006
Small *rapidity dependent* *fluctuations* can grow exponentially and generate longitudinal pressure .

longitudinal pressure



$g^2\mu\tau \sim Q_s\tau$

3+1D numerical simulation

$$P_L \sim \exp\left(C\sqrt{g^2\mu\tau}\right)$$

Similar to Weibel instability in expanding plasma
[Romatschke, Rebhan]

Unstable dynamics: Boost-noninvariant Glasma

[Fujii, Itakura,Iwazaki]

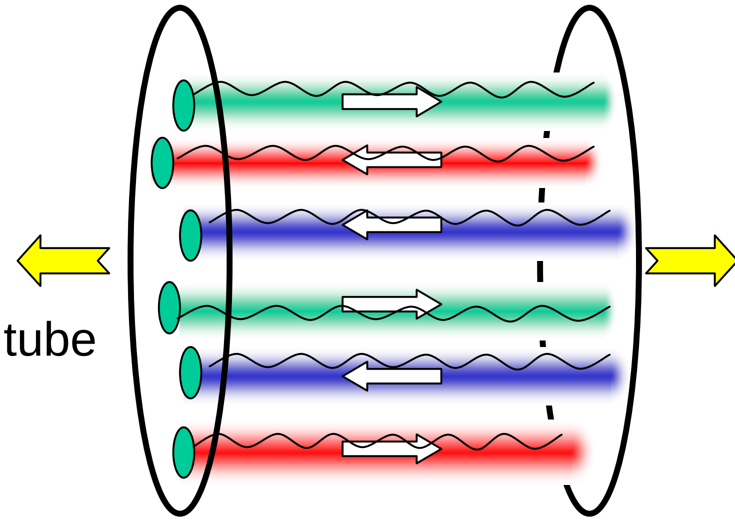
Analytic study of instability

→ Investigate the effects of fluctuation on a single flux tube

Rapidity dependent fluctuation

$$A_i = \mathcal{A}_i + a_i(\tau, \eta, x_\perp)\,, \quad A_\eta = \mathcal{A}_\eta + a_\eta(\tau, \eta, x_\perp)\,,$$

Background field = boost invariant Glasma
→ constant magnetic and/or electric field in a flux tube



Consider SU(2) for simplicity

Linearize the equations of motion wrt fluctuations

Unstable dynamics: Boost-noninvariant Glasma

[Fujii, Itakura, Iwazaki]

Equations for fluctuation

SU(2), constant *B* and *E* directed to 3rd color and *z* direction

$$\frac{1}{\tau}\partial_\tau\left(\tau\,\tilde{a}_+^{(\pm)}\right)+\left\{\frac{1}{\tau^2}\left(\nu\pm\frac{gE}{2}\tau^2\right)^2+(2n+|m|+1\mp m\pm 2)gB\right\}\tilde{a}_+^{(\pm)}=0$$

ν: conjugate to rapidity η

$$\tilde{a}_+^{(\pm)}=e^{i\nu\eta}(a_+^{\,1}\pm i a_+^{\,2})$$

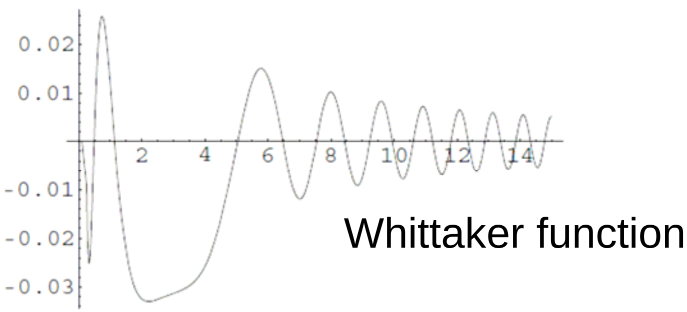
$$a_+=a_x+ia_y$$

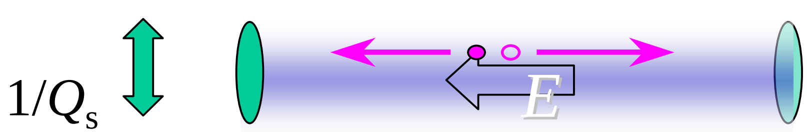
B = 0

E = 0

Schwinger mechanism

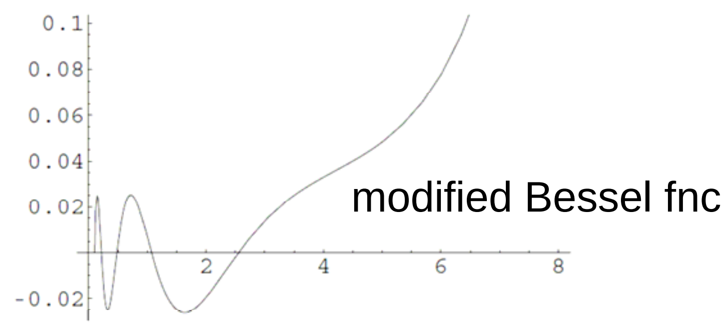
Infinite acceleration of massless charged fluctuations.
No amplification of the field

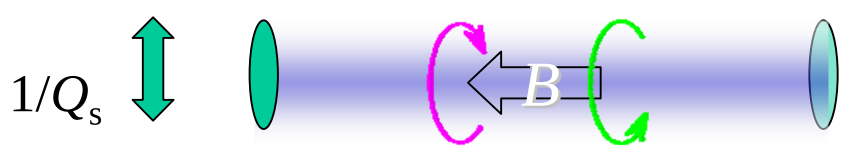




Nielsen-Olesen instability

Lowest Landau level (n=0) gets unstable due to ***anomalous magnetic moment -2gB***
(not Weibel instability)

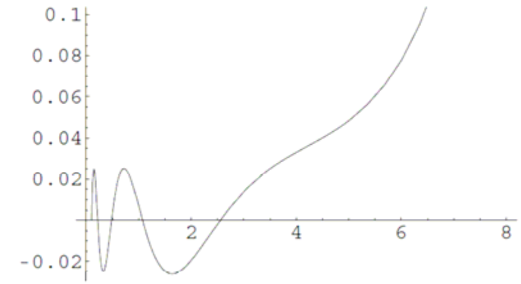




Unstable dynamics: Boost-noninvariant Glasma

[Fujii, Itakura]

Nielsen-Olesen instability in expanding geometry



Solution : modified Bessel function $I_\nu(z)$

$$\tilde{a}_+^{(-)},\ \tilde{a}_-^{(+)} \propto e^{im\theta}\, r^{|m|}\, e^{-\frac{gBr^2}{4}}\, I_{i\nu}\left(\sqrt{gB}\,\tau\right) \sim \frac{e^{\sqrt{gB}\tau}}{\sqrt{2\pi\sqrt{gB}\,\tau}}$$

- Growth time ~ $1/(gB)^{1/2}$ ~$1/Q_s$ → instability grows rapidly $\tau_{\rm grow} \sim \frac{1}{\sqrt{gB}} \sim \frac{1}{Q_s}$
  Important for *early thermalization*?
- Transverse size ~ $1/(gB)^{1/2}$ ~$1/Q_s$ for gB~ Q_s^2
  ← consistent with the homogeneity assumption
- Oscillation at early times is due to expansion (observed in numerical results)
- Time-dependent B possible → $\exp\left\{\#\sqrt{gB(\tau)}\ \tau\right\}$
- Consistent with the numerical results by Romatchke and Venugopalan

More results to appear soon … [Fujii, Itakura,Iwazaki, in preparation]

- In the presence of both E and B, instability exists if B is much stronger than E
- Can compute the generation/decay of topological charge density
- SU(3) : two background fields (3 and 8) → 3 unstable modes

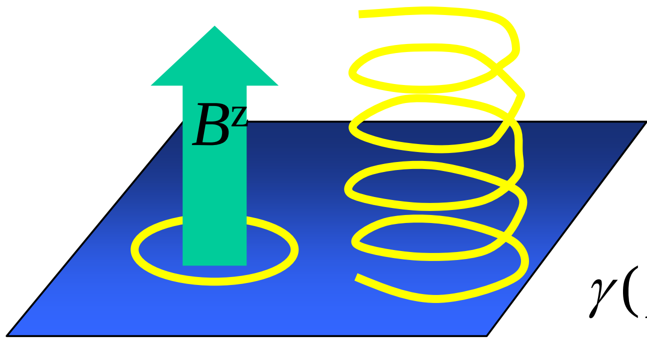
Unstable dynamics: *digression 1*

[Fujii, Itakura,Iwazaki, in preparation]

Glasma without expansion

Nielsen-Olesen instability in *t-z* coordinates (*B^z* in the 3rd color component)

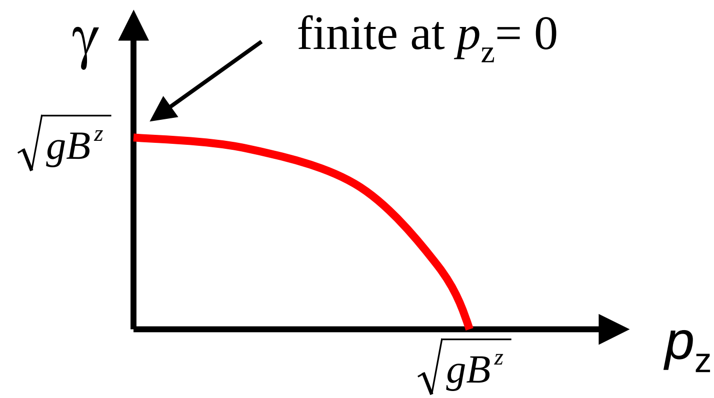
Nonzero p^z



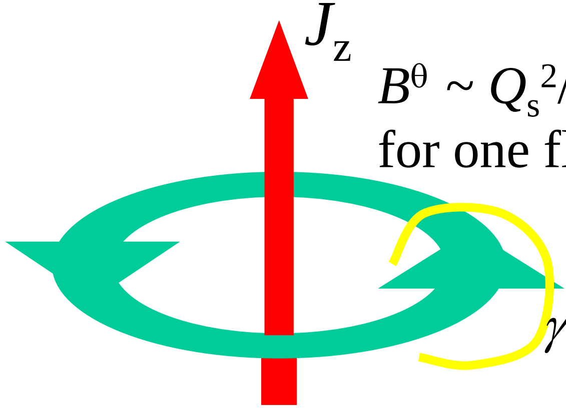
fluctuation

$$\sim e^{\gamma(p_z)t}$$

$$\gamma(p_z) \sim \sqrt{gB^z - p_z^2}$$

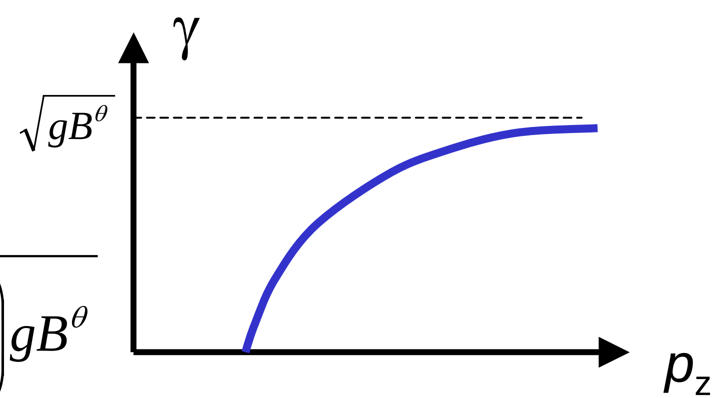


Instability current *J^z* generates *B^θ* → induces "*secondary*" N-O instability



$B^\theta \sim Q_s^2/g$
for one flux tube

$$\gamma(p_z) \sim \sqrt{\left(1 - \frac{5(gB^\theta)^2}{8p_z^4}\right) gB^\theta}$$



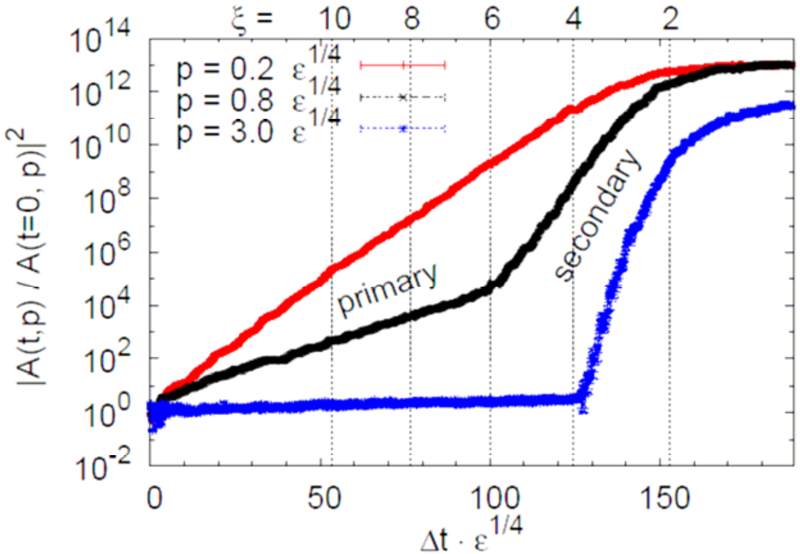
Unstable dynamics: digression 2

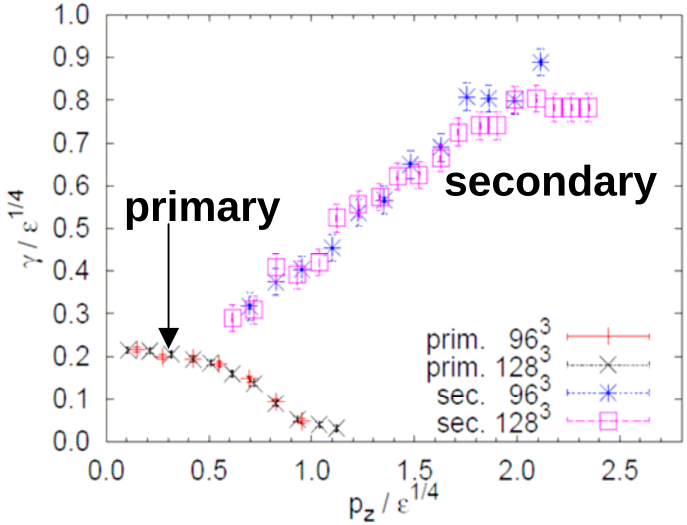
[Berges, Scheffler, Sexty]

Glasma without expansion: numerical results

Real time simulation of the stochastic Yang-Mills in *t-z* coordinates

Two different instabilities exist!!





Nielsen-Olesen instability can explain

* existence of two instabilities

* p_z dependence of each growth rate $\gamma(p_z)$

- finite at p_z=0 in primary instability

- increases with p_z and saturate in secondary instability

Summary

1. ***QCD-based descriptions of the initial conditions and the earliest dynamics of the high-energy heavy-ion collisions are now being understood within the framework of Color Glass Condensate and Glasma.***

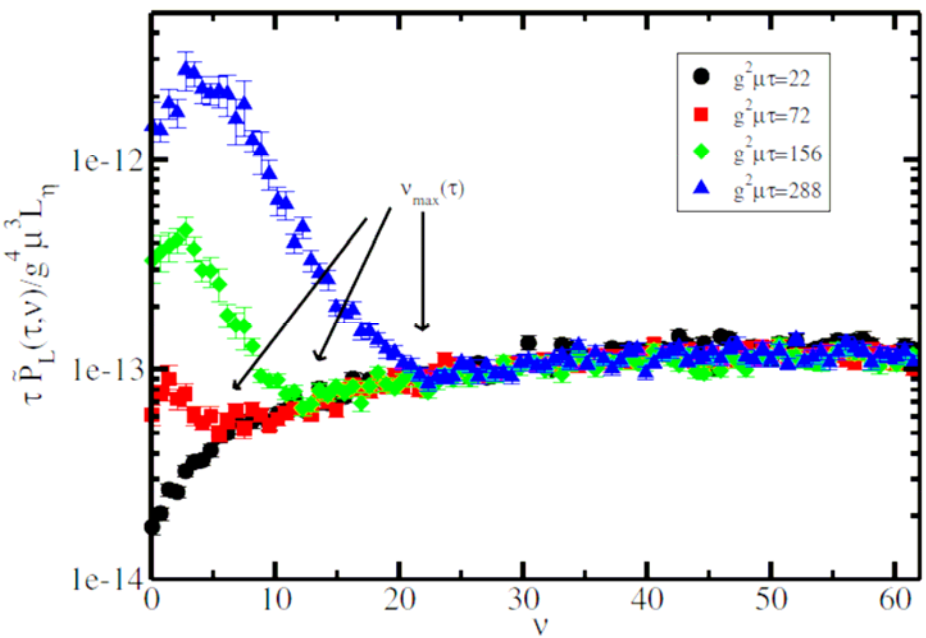
2. ***In particular, time evolution of the Glasma has rich interesting physics, such as flux tube formation, topological charge generation, and instability due to Nielsen-Olesen mechanism.***

Topics not covered in this talk:

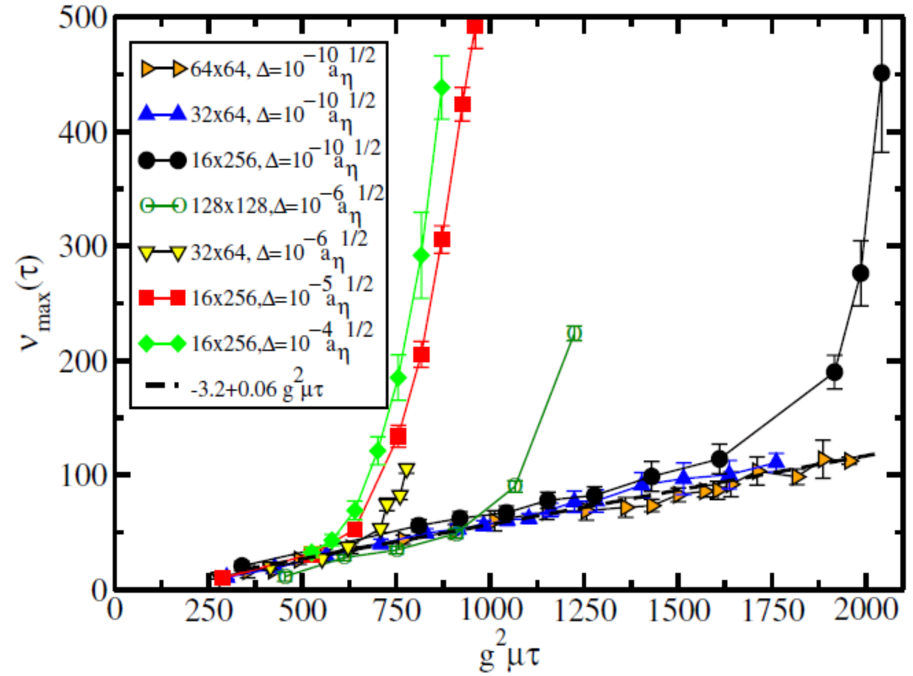
1. *Progress of theoretical framework of CGC:*
   NLO (running coupling)
   extension to dilute regime (Pomeron loop)
2. *Application of the Glasma to RHIC physics* → *talk by McLerran*
   Possible relation to the "Ridge"
   Long-range rapidity correlation
3. *Transition from Glasma to QGP / sQGP ?*

Unstable dynamics: Boost-noninvariant Glasma

$\nu_{max}(\tau)$: Largest ν participating instability increases linearly in τ



ν : conjugate to rapidity η

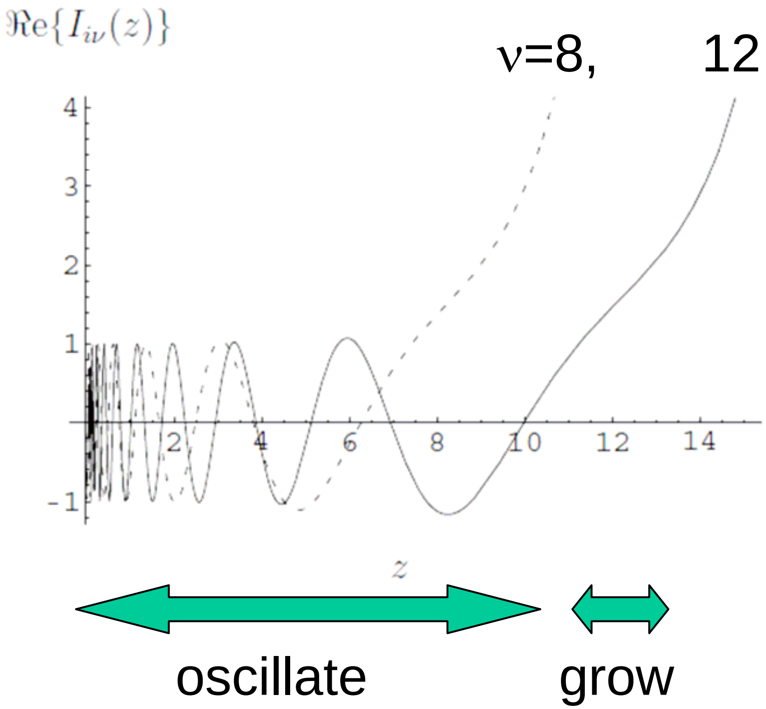


$\sim Q_s \tau$

Unstable dynamics: Boost-noninvariant Glasma

Modified Bessel function controls the instability $f \sim I_{i\nu}\left(\sqrt{gB}\,\tau\right)$

ν: conjugate to rapidity η



$$\partial_\tau^2 f + \frac{1}{\tau}\partial_\tau f + \left(-gB + \frac{\nu^2}{\tau^2}\right) f = 0\,.$$

$\left(-gB + \frac{\nu^2}{\tau^2}\right) > 0$ Stable oscillation

$\left(-gB + \frac{\nu^2}{\tau^2}\right) < 0$ Unstable

$$\tau_{\text{wait}} = \frac{\nu}{\sqrt{gB}} \sim \frac{\nu}{Qs}$$

The time for instability to become manifest

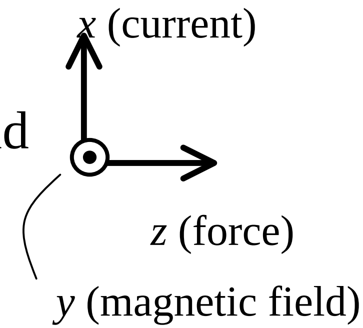
$$Q_s(\tau_{\text{wait}} + \tau_{\text{grow}}) \sim \nu + 1\,,$$

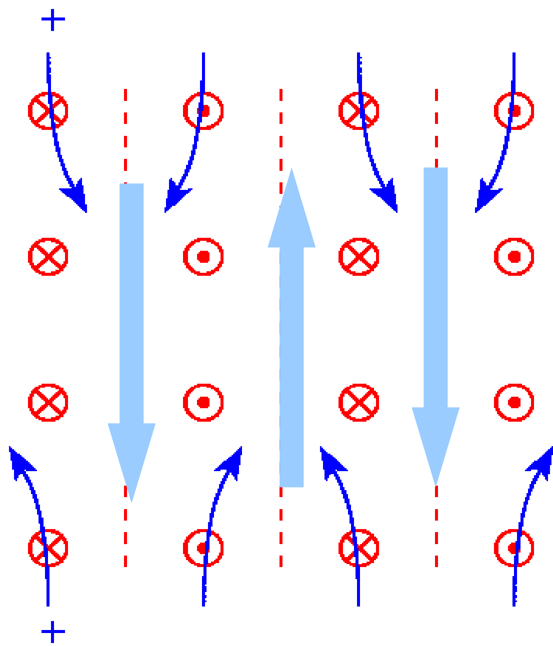
Modes with small ν grow fast ! For large ν → $\nu_{\max} \sim Q_s \tau\,.$

Unstable dynamics: Weibel vs Nielsen-Olesen

Weibel instability

- Two step process
- Motion of hard particles in the soft field additively generates soft gauge fields
- Impossible for homogeneous field
- Independent of statistics of charged particles





Nielsen-Olesen instability

* One step process

* Lowest Landau level in a strong magnetic field becomes unstable due to ***anomalous magnetic moment***

$$\omega^2 = 2(n+1/2)gB - \mathbf{2gB} < 0 \quad \text{for} \quad n=0$$

* Only in non-Abelian gauge field

vector field → spin 1

non-Abelian → coupling btw field and matter

* Possible even for homogeneous field