Equation of state, initial conditions and final state in the exact solutions of perfect fluid hydrodynamics

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Outline

- The role of hydrodynamics in heavy ion physics
- Exact vs. numerical hydrodynamical modeling
- Solutions to the hydrodynamical equations
    - Nonrelativistic solutions
    - Review of well-known solutions
    - Recent results in relativistic perfect fluid hydrodynamics
- Phenomenological applications
- Outlook: where we are
- Outlook: where do we go

Motivation

Goal of heavy-ion physics: Understand the phase structure of the strongly interacting matter (hard task)

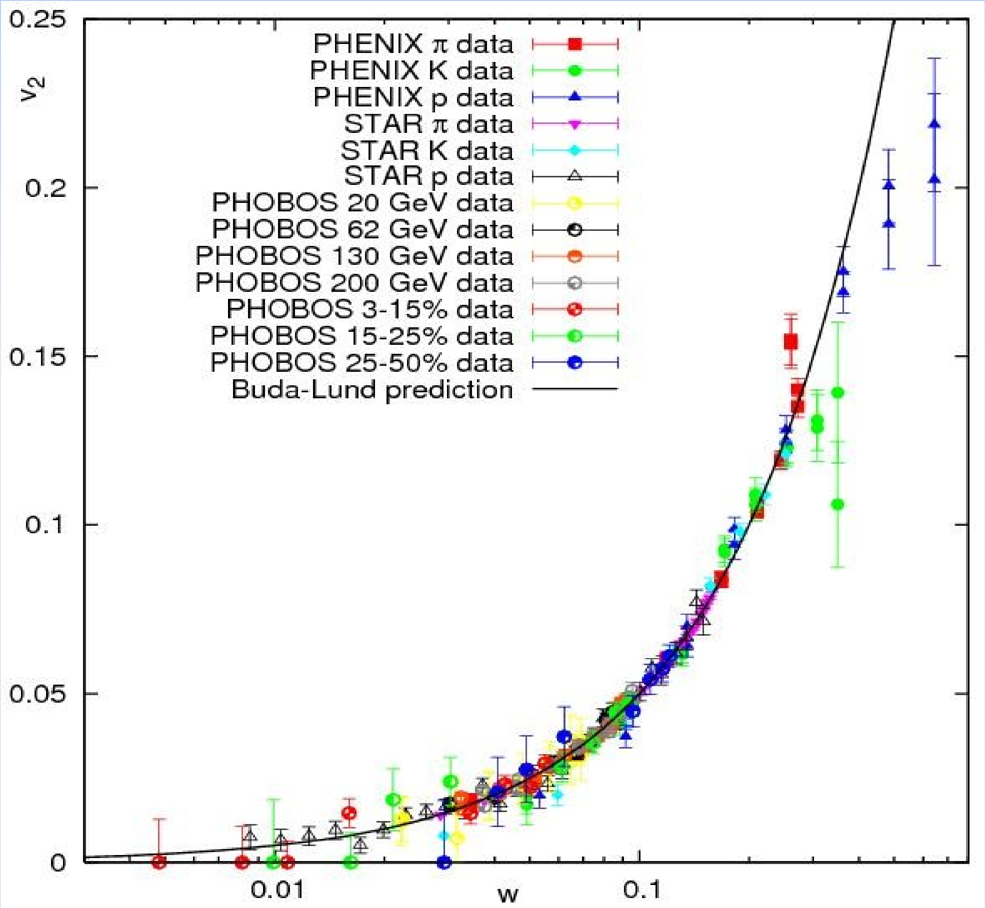
- Experimentally: the matter created in heavy-ion collisions is an almost perfect fluid
  - Hydrodynamics is the only way which dynamically connects the initial state, the final state, and the equation of state (EoS)
  - To explore the EoS is perhaps the final aim: but we can get to it only by knowing properly the space-time picture of the collisions
  - Lots of models failed to describe the observations correctly
- Hydrodynamical modelling can tell something about the dynamics of the matter, thus finally on the equation of state

Perfect fluid hydrodynamics

Perfectness of the fluid: important question

- In the relativistic case, even the equations are not well-defined for viscous fluids
- In nonrelativistic case, equations are easier to solve
- Surprisingly, they lead to a good description of the observables
- Elliptic flow: scaling behaviour predicted

Hydrodynamics naturally leads to scaling



Relativistic hydrodynamics

Only assumption: local thermal equilibrium & local energy-momentum conservation (no internal scale -> scaling laws)

Stress-energy-momentum tensor: $T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu}$

- Equations follow from that its four-divergence vanishes $\partial_\nu T^{\mu\nu} = 0$

Equations for perfect fluid:

- Euler equation: $w u^\nu \partial_\nu u^\mu = (g^{\mu\rho} - u^\mu u^\rho)\, \partial_\rho p.$
- Energy conservation: $w \partial_\mu u^\mu = -u^\mu \partial_\mu \varepsilon.$
- Entropy conservation: $\partial_\mu (\sigma u^\mu) = 0.$

Hydrodynamical modelling

Now how to specify a hydrodynamical model?

There are different types:

- Hydrodynamics inspired parametrization: describes just the final state (at freeze-out), time evolution not considered
- Numerical models: solve the equations numerically
- Parametric solutions: exact solutions with fit parameters
- Models based on exact solutions: either use a particular solution directly, or a parametrization that simplifies to known exact solutions

A hydrodynamical model is a combination of

initial conditions + dynamical equations + freeze-out condition

- Common: calculate observables from thermal distributions

Exact vs. numerical solutions

Numerical solutions:

- Take a (more or less) arbitrary initial condition, and see where it evolves. If agrees with data, justifies the specific EoS and initial condition

Exact solutions:

- Drawbacks:
  - Obvious: an exact solution can be only approximately realistic (but experimental results also have systematic uncertainty)
  - Needs imagination, a hard theoretical challenge
- Advantages:
  - No approximation, could be used for code-testing
  - Usually parametric ones: can map a manifold of initial conditions

Scaling solutions arise naturally

Landau-Khalatnikov solution

Implicit, exact, accelerating 1+1D solution

- "Father" of relativistic hydrodynamics
- Used in the description of elementary particle reactions (p+p in cosmic rays)
- Nowadays: it seems that hydrodynamics rally "begins" in p+p (not in e^++e^-) reactions

Initial condition, final state:

- Initially: a finite slab of matter, constant temperature
- Final observables: approximately Gaussian rapidity distribution
- Formulas are very complicated

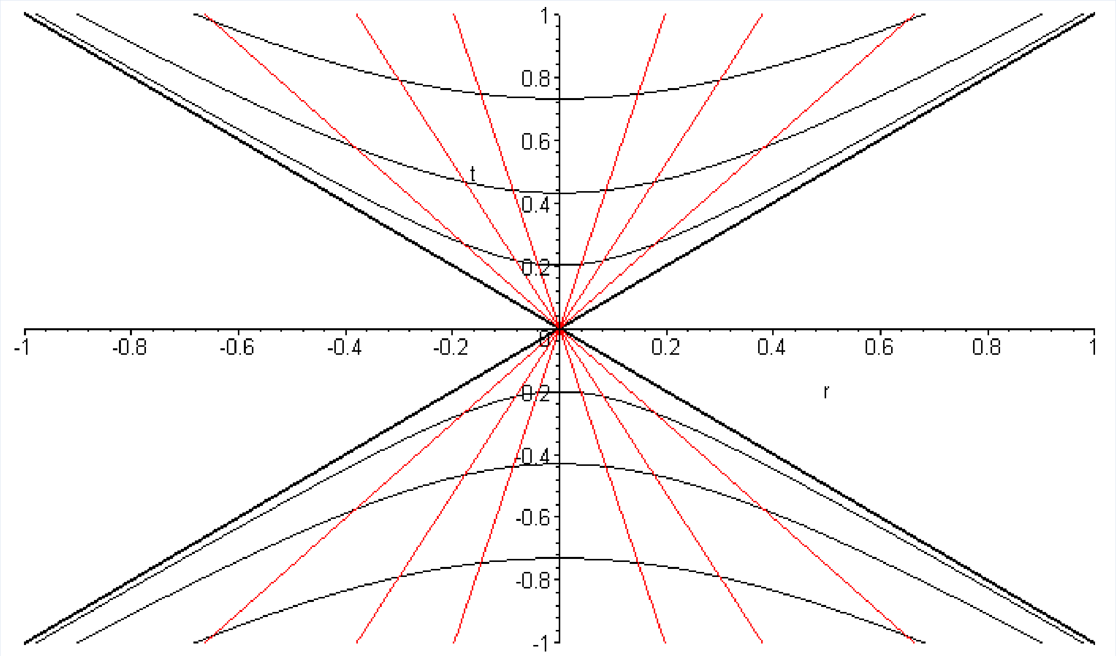
L. D. Landau, Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953)

I.M. Khalatnikov, Zhur. Eksp. Teor. Fiz. 27, 529 (1954)

Hwa-Bjorken solution

Exact explicit 1+1D, expanding, boost-invariant solution

- Almost trivial one, but phenomenologically important
- Estimation of initial energy density: used for decades
- Final rapidity distribution: constant in η



R. C. Hwa, Phys. Rev. D 10, 2260 (1974) *J. D. Bjorken, Phys. Rev. D 27, 140 (1983)*

Nonrelativistic solutions

In the NR limit, there are many 1+3D realistic exact solutions:

- First exact Hubble-like solution in a broad class (with finite density):

J. Bondorf, S. Garpman, J. Zimányi, Nucl. Phys. A 296 320 B 37 483 (1978)

- Directional Hubble velocity field, general temperature profile:

T. Csörgő, Acta Phys. Polon. B 37 483 (2006)

- Directional Hubble velocity field, Gaussian density profile, for arbitrary temperature-dependent speed of sound:

T. Csörgő, S. V. Akkelin, Y. Hama, B. Lukács, Y. M. Sinyukov, PRC 67 034904 (2003)

To my best knowledge, this is the only exact solution where QCD EoS can be directly utilized (at μ_B=0)

- The Buda-Lund model is based on these solutions, explained scaling of elliptic flow, RHIC HBT scaling, etc ...

Relativistic solutions in 3D

Many interesting solutions for homogeneous expansion:

- Spherical expansion & ellipsoidally symmetric pressure:

$$u^\mu = \frac{x^\mu}{\tau} \qquad s = \frac{r_x^2}{\dot{X}_0^2 t^2} + \frac{r_y^2}{\dot{Y}_0^2 t^2} + \frac{r_z^2}{\dot{Z}_0^2 t^2}$$

T. Csörgő, F. Grassi, Y. Hama, T. Kodama, Phys. Lett. B 565, 107 (2003)

T. Csörgő, L. P. Csernai, Y. Hama, T. Kodama, Heavy Ion Phys. A 21, 73 (2004)

- General ellipsoidal solutions:
  - Velocity field can have different principal axes

    Yu. M. Sinyukov and I. A. Karpenko, nucl-th/0506002

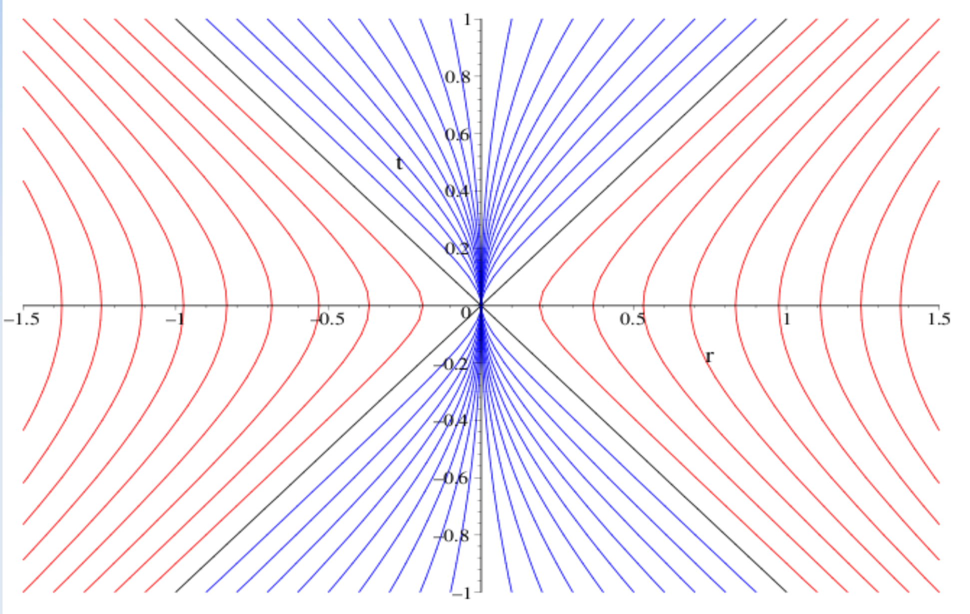
- Other important solution:

  T. S. Biró, Phys. Lett. B 487, 133 (2000)

  S. Pratt, Phys. Rev. C 75 024907 (2007)

Accelerating relativistic solutions

A simple exact accelerating solution:



$$v = \frac{2tr}{t^2 + r^2}$$

- First accelerating explicit solution
- Constant acceleration in rest frame
- Framework for Unruh-effect

Extension (parameters constrained):

$$v = \tanh \lambda\eta,$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda d \frac{\kappa+1}{\kappa}} \left(\cosh \frac{\eta}{2}\right)^{-(d-1)\phi_\lambda}$$

M. Nagy, T. Csörgő, M. Csanád, Phys. Rev. C 77 024908 (2008)

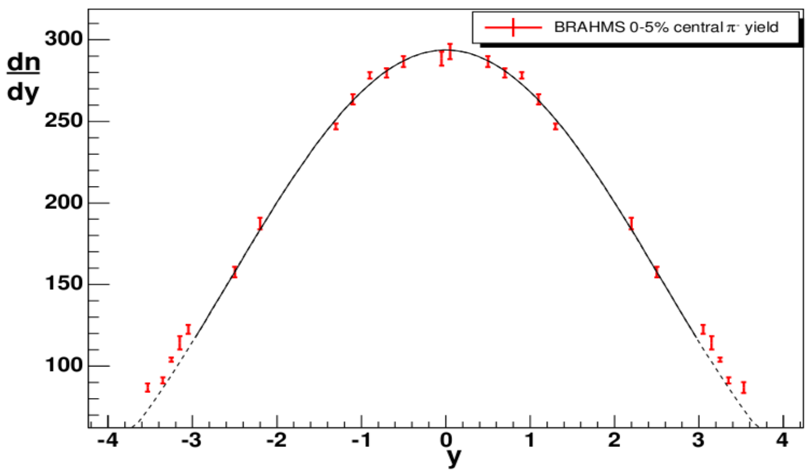
Applications

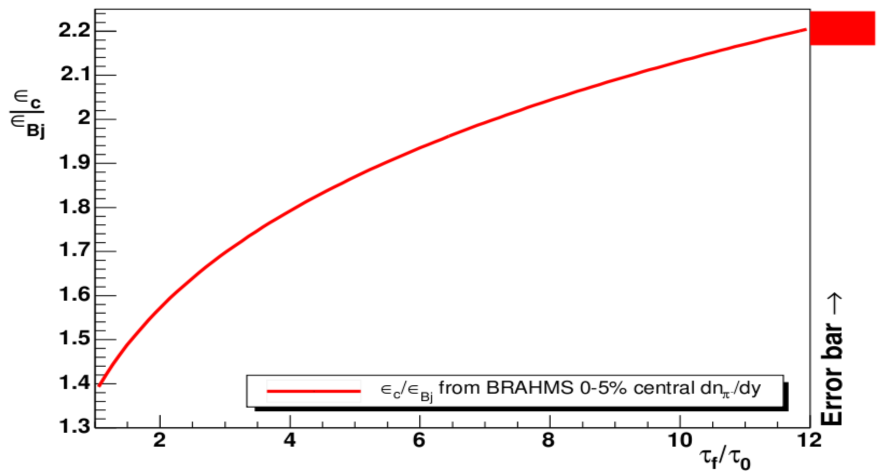
Calculation of the rapidity distribution leads to finite widths

- Acceleration parameter can be fitted to measurements (BRAHMS)

Develops the estimate of the initial energy density & lifetime

- Life-time: shows a ~20% increase of previous estimates
- Energy density: a ~120% increase compared to the Bjorken-estimate!
- With softer EoS, conjectured a further ~50% increase

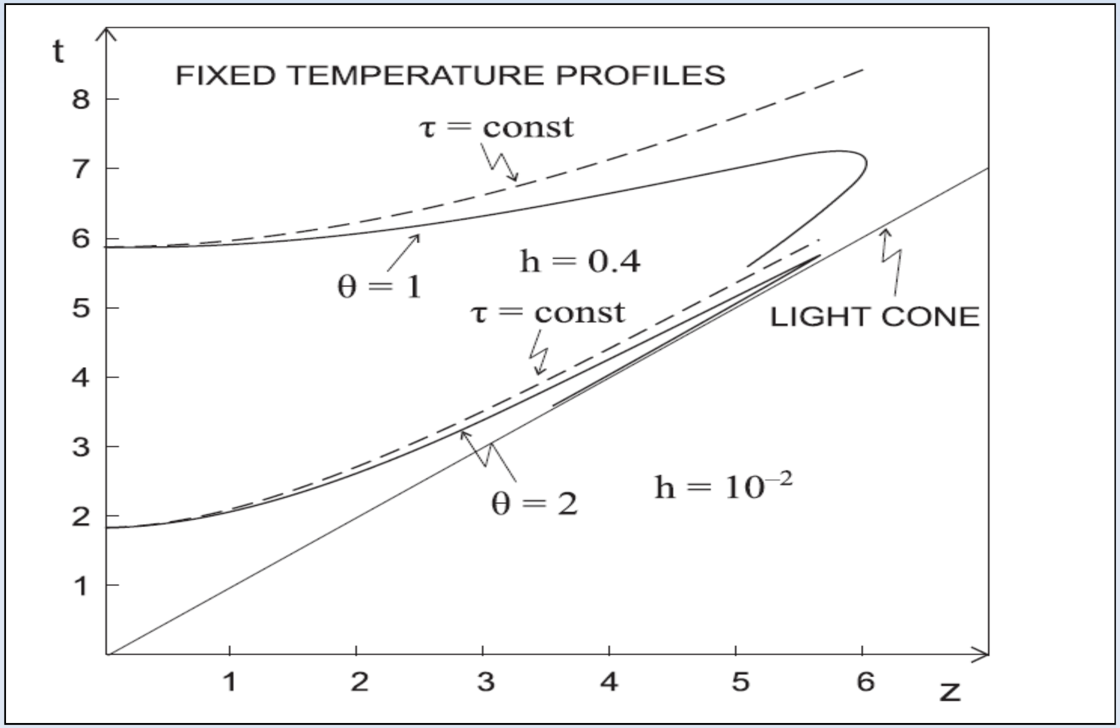




New 1D solutions

Recent result: new accelerating expanding solution in 1D

- Valid for arbitrary speed of sound
- interpolates between the Landau and the Bjorken solutions



A. Bialas, R. Janik, R. Peschanski, Phys. Rev. C 76 054901 (2007)

General solutions for special EoS

For a very stiff EoS (κ=1), the equations of relativistic hydrodynamics simplify

- Expressing them in terms of a potential function, we obtain a wave-equation for the potential
- Possible to have <u>exact</u> solution for <u>any initial condition</u>
  - 1D solution based on this idea:

    M. Nagy, T. Csörgő, M. Csanád, Phys. Rev. C 77 024908 (2008)
  - Spherical solutions in terms of spherical harmonics:

    M. Borshch, V. Zhdanov, SIGMA 3, 16 (2007)
- Proof of stability of such solutions is at hand
  - Since general solution is available, these are naturally stable
- Generalization to softer EoS is yet to be found (not straightforward)

Summary: where are we now

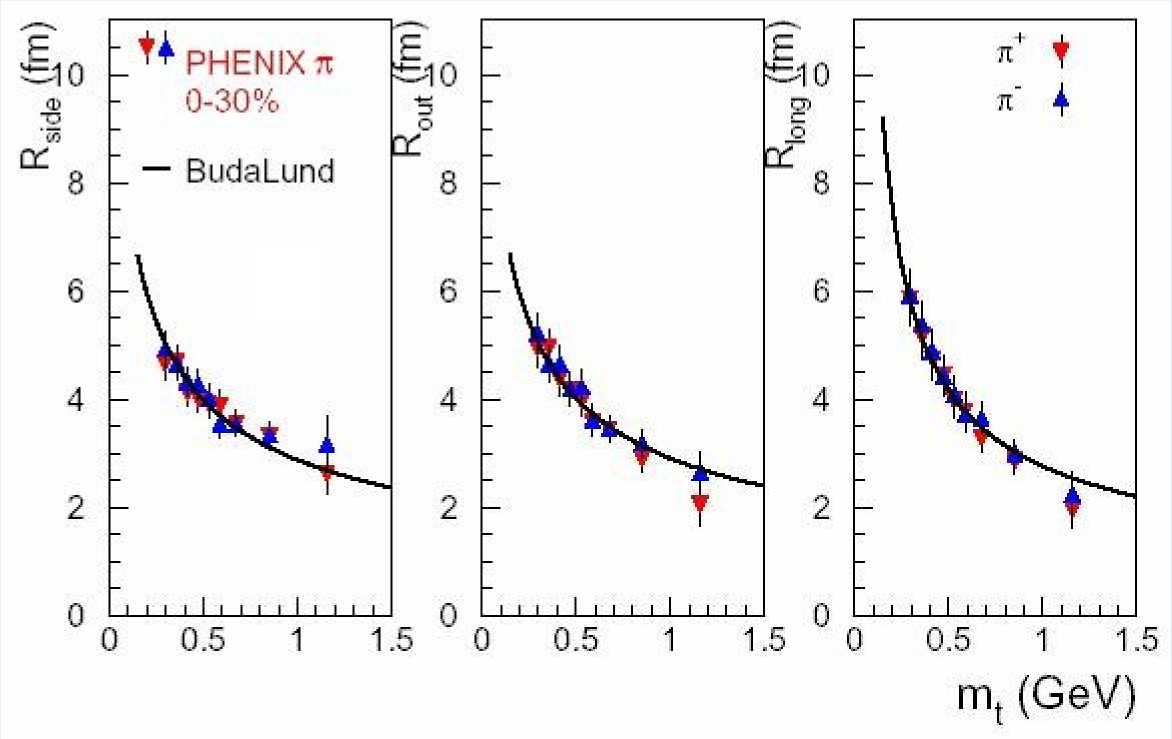
There is a revival of interest in exact hydrodynamical solutions

Exact (parametric) solutions are useful for

- Follow the time evolution (Constraints on initial conditions provided we know the final state)
- If initial state is known, constrains EoS

Models based on exact solutions describe well collective behaviour

- Scaling of elliptic flow and HBT radii predicted & successfully tested



Summary: where to go

It is natural to develop the models with new exact solutions

- The quest for these solutions is a hard task, and a good challenge

Now: there are many exact solutions, some of them are:

- Realistic in EoS (works for any speed of sound)
- Realistic in geometry (3D expansion, ellipsoidal symmetry)
- Realistic in flow profile: asymptotics (Hubble), or acceleration

But we don't have solutions, which have each of these properties simultaneously!

- Any result would be an essential step in understanding the dynamics of heavy-ion reactions, understanding the dynamics will lead to understanding the properties of the matter, (EoS, initial conditions, viscosity), thus is of great importance

Thank you for your attention!