# HYDRODYNAMICS & PERFECT FLUIDS: Uniform description of soft observables in Au+Au collisions at RHIC

M. Chojnacki, W. F., W. Broniowski, A. Kisiel, **Phys. Rev. C78 (2008) 014905**

W. Broniowski, M. Chojnacki, W. F., and A. Kisiel, **Phys. Rev. Lett. 101 (2008) 022301**

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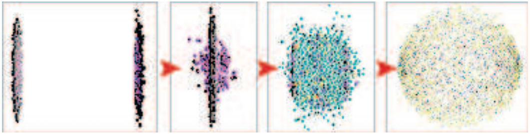
# Motivation

1) development of a hydrodynamical model describing most of the relevant aspects of soft-hadron production in the heavy-ion collisions at the RHIC energies

2) formulation of the predictions for the heavy-ion collisions at the LHC energies

our approach combines **well known elements** but each of those elements is **treated in quite sophisticated/modern way**

**main difference → different initial conditions → departure from the standard Glauber initial conditions → more explosive evolution → physical reason crucial for the success**



# Soft-observables in the midrapidity region

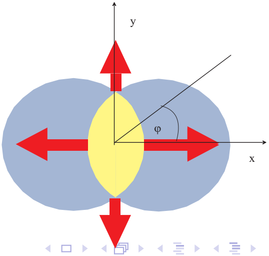
in our analysis we concentrate on the soft-hadronic observables, $y \approx 0$

- transverse-momentum spectra
- the elliptic flow coefficient $v_2$

$$\frac{dN}{dy d^2 p_T} = \frac{dN}{dy\, 2\pi p_T\, dp_T} \left(1 + 2 v_2(p_T) \cos(2\phi_p) + \ldots\right).$$

- the pion correlation functions described by the 3 HBT radii: $R_{\rm side}(k_T), R_{\rm out}(k_T), R_{\rm long}(k_T)$,
  pion pairs studied with fixed mean momentum $\vec{k} = \frac{1}{2}(\vec{p}_1 + \vec{p}_2)$,
  "Fourier transform" with respect to $\vec{q} = \vec{p}_1 - \vec{p}_2$ gives the HBT radii

formation of the elliptic flow at RHIC explained by the relativistic hydrodynamics of perfect fluid, spatial asymmetry transforms into asymmetry of the momentum distribution, indication of secondary interactions!



# RHIC puzzle(s)

1) at RHIC there is a well known difficulty of simultaneous good description of those observables, the problem known as the RHIC HBT puzzle

this problem is connected with the application of relativistic hydrodynamics, very much successful in describing the spectra and $v_2$

2) description of large $v_2$ with early thermalization time used in hydro without viscosity leads to the conjuncture that at RHIC we deal with perfect fluid

early thermalization time remains the second RHIC puzzle (**possible solution proposed by A. Bialas et al., Phys. Lett. B661 (2008) 325, 2D hydro gives large $v_2$ consistently with the spectra**)

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development of the hydrodynamic approach by: Heinz, Huovinen, Kolb, Heiselberg, Wiedemann, Hirano, Shuryak, Hung, Teaney, Bass, Nonaka, Hama, Kodama, Grassi, Ruuskanen, Eskola, Rasanen, Tuominen, .......

**our approach is based on the 2+1 (3D boost-invariant hydrodynamics of perfect fluid)**

# Outline

2. Hydrodynamic approach
3. Results for RHIC
4. Conclusions

# 2.1 Energy-momentum conservation laws

The relativistic hydrodynamic equations of perfect fluid follow from the assumption of local equilibrium and the conservation laws for the energy and momentum, which yield the relativistic Euler equations (3 eqs.) and the entropy conservation (1 eq.)

$$T^{\mu\nu} = (\varepsilon + P)\, u^\mu u^\nu - P g^{\mu\nu}, \qquad u^\mu \partial_\mu (T u^\nu) = \partial^\nu T,$$
$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu (s u^\mu) = 0$$

$\varepsilon$ - energy density, $P$ - pressure, $T$ - temperature,
$u^\mu$ - four-velocity of the fluid element, $s$ - entropy density,
$\varepsilon + P = Ts, d\varepsilon = Tds, dP = sdT, c_s^2 = dP/d\varepsilon = (s/T)dT/ds$

4 equations for 5 unknown functions, equation of state is needed, $s(T)$

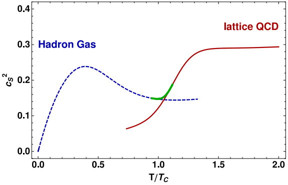
we consider a boost-invariant hydro model – reasonable approximation for midrapidity region at RHIC, where we may also assume that $\mu_B \approx 0$

**much more difficult situation at lower energies (SPS) - finite $\mu_B \approx 0$, central region mixed with the fragmentation region**

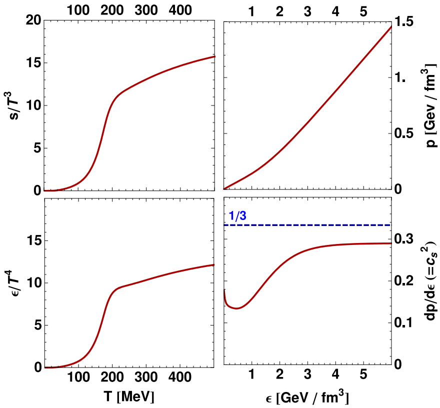
# 2.2 Equation of state

we use a formalism by Baym et al. where EOS is encoded in the temperature dependent sound velocity function $c_s(T)$

at low temperatures we use the hadron gas model, at high temperatures we use the lattice QCD results (**Fodor et al.**), in the transition region we make a simple interpolation, **stiffer than usual EOS! (Pratt and Vredevoogd, arXiv:0809.0516 (nucl-th))**




the function $c_s(T)$ determines uniquely other thermodynamic functions

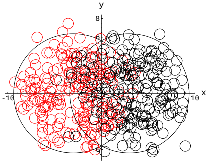


# 2.3 Initial conditions

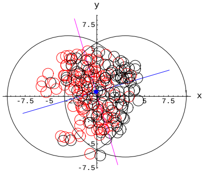
most of the hydrodynamic calculations uses the scaling for the initial energy/entropy density

$$\varepsilon\left(\vec{x}_T\right) \propto \rho\left(\vec{x}_T\right) = \frac{1-\kappa}{2}\,\rho_{\rm W}\left(\vec{x}_T\right) + \kappa\rho_{\rm bin}\left(\vec{x}_T\right)$$

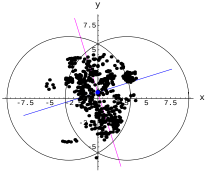
$\kappa = 0$ corresponds to the wounded-nucleon model, Bialas & Czyz, 1976, $\kappa = 1$ would include the binary collisions only, the PHOBOS analysis of the particle multiplicities yields $\kappa = 0.14$ at $\sqrt{s_{NN}} = 200$ GeV



all nucleons



wounded nucleons



binary collisions

our hydrodynamic approach is based on **GLISSANDO** - Monte-Carlo version of the Glauber model - **Broniowski, Rybczynski, Bozek** to be published in Computer Phys. Comm.

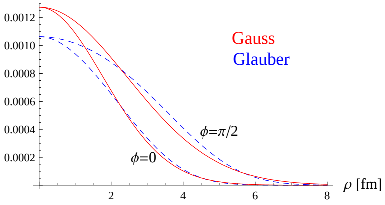
we use the initial condition for hydrodynamics of the Gaussian form

$$\varepsilon(x,y) \sim \exp\left(-\frac{x^2}{2a^2} - \frac{y^2}{2b^2}\right),$$

where $x$ and $y$ denote the transverse coordinates, the width parameters $a$ and $b$ depend on centrality

| $c$ (%) | 0-5 | 0-20 | 20-30 | 20-40 |
|---|---|---|---|---|
| $a$ (fm) | 2.65 | 2.41 | 1.94 | 1.78 |
| $b$ (fm) | 2.90 | 2.78 | 2.52 | 2.45 |

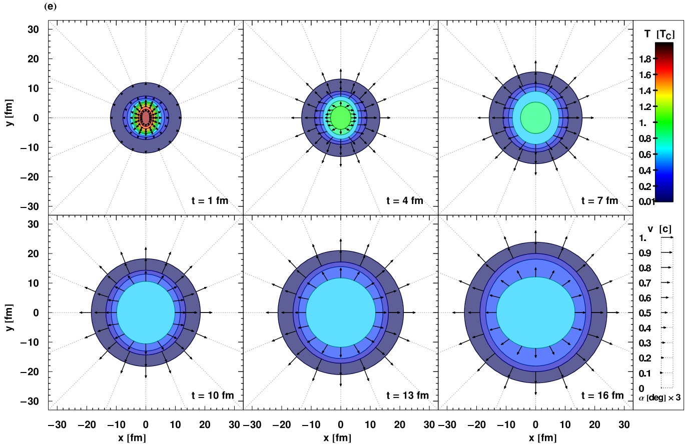
we run the GLISSANDO Glauber Monte Carlo simulations which include the eccentricity fluctuations (**Hama et al.**), then we match $a^2$ and $b^2$ to reproduce the values $\langle x^2 \rangle$ and $\langle y^2 \rangle$ from the GLISSANDO profiles




**"more compact" source**
**faster initial expansion**

# 2.4 Time evolution

typical hydro evolution in the transverse plane (videos by M. Chojnacki avaialable on the web, calculations done with MATHEMATICA, total entropy conserved at the level $10^{-4}$ or better)



# 2.5 Freeze-out with THERMINATOR

the freeze-out hypersurface $\Sigma^\mu$ is specified by the value of the final temperature $T = T_f$, the Cooper-Frye formula is then used to obtain the particle spectra

$$\frac{dN}{dy d^2 p_T} = \int d\Sigma^\mu p_\mu f_{\rm eq}\,(p \cdot u)$$

this formula is used in the Monte-Carlo thermal generator THERMINATOR
A. Kisiel, T. Tałuć, W. Broniowski, WF, Computer Physics Communications **174** (2006)

all hadrons (including all known resonances) are produced on the hypersurface, the resonances decay in cascades leading to final stable hadrons such as pions, kaons and nucleons, single-freeze-out approximation assumed, for $y = 0$

$$\frac{dN}{dy d^2 p_T} = \frac{dN}{dy\, 2\pi p_T\, dp_T}\left(1 + 2 v_2(p_T) \cos(2\phi_p) + ...\right)$$

# 2.5 Correlation functions at freeze-out

The correlation function for identical pions is obtained with the two-particle Monte-Carlo method, in this approach the evaluation of the correlation function is reduced to the calculation of the following expression

$$C(\vec{q},\vec{k}) = \frac{\sum\limits_{i}\sum\limits_{j\neq i}\delta_\Delta(\vec{q}-\vec{p}_i+\vec{p}_j)\delta_\Delta(\vec{k}-\frac{1}{2}(\vec{p}_i+\vec{p}_j))|\Psi(\vec{k}^*,\vec{r}^*)|^2}{\sum\limits_{i}\sum\limits_{j\neq i}\delta_\Delta(\vec{q}-\vec{p}_i+\vec{p}_j)\delta_\Delta(k-\frac{1}{2}(\vec{p}_i+\vec{p}_j))},$$

where $\delta_\Delta$ denotes the box function with $\Delta = 5\,\mathrm{MeV}$. The correlation function is expressed with the help of the Bertsch-Pratt coordinates $k_T, q_{\rm out}, q_{\rm side}, q_{\rm long}$ and approximated by the **Bowler-Sinyukov** formula

$$C = (1-\lambda) + \lambda K_{\rm coul}(q_{\rm inv})\left[1+\exp\left(-{R_{\rm out}}^2 q_{\rm out}^2 - {R_{\rm side}}^2 q_{\rm side}^2 - {R_{\rm long}}^2 q_{\rm long}^2\right)\right],$$

where $K_{\rm coul}(q_{\rm inv})$ with $q_{\rm inv} = 2k^*$ is the squared Coulomb wave function integrated over a static gaussian source.

# 3.1 RHIC spectra

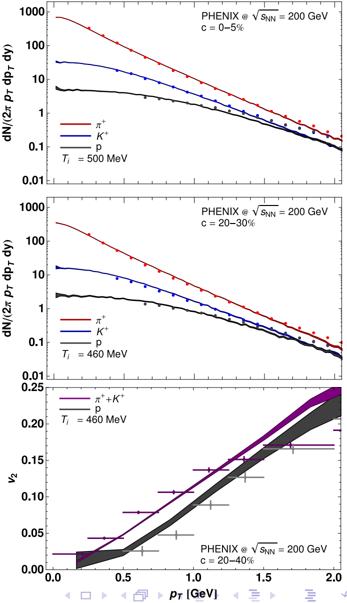
centrality classes 0-5%
initial central temperature $T_i$ = 500 MeV

centrality classes 20-30% and 20-40%
initial central temperature $T_i$ = 460 MeV

final freeze-out temperature $T_f$ = 145 MeV

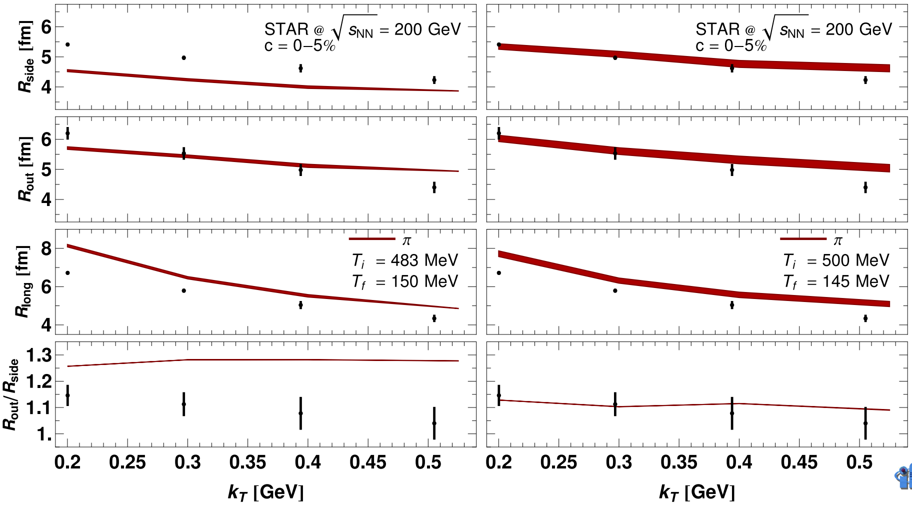
initial time $\tau_0 = 0.25$ fm
single-freeze-out approximation
neglecting the final-state hadron rescattering
(**different from Bass, effects mainly for protons** )

**altogether only 5 parameters:** $T_i, T_f, a, b, \tau_0$



# 3.2 RHIC correlations

## HBT radii for RHIC, Glauber (left) vs. Gauss (right)



# 3.2 RHIC correlations

Azimuthally-sensitive femtoscopy from RHIC to LHC, arXiv:0808.3363 (nucl-th)

$$C(q_{\rm out}, q_{\rm side}, q_{\rm long}) = 1 + \lambda \exp(-R^2_{\rm out} q^2_{\rm out} - R^2_{\rm side} q^2_{\rm side} - R^2_{\rm long} q^2_{\rm long} - 2R_{\rm out-side} q_{\rm out} q_{\rm side} - 2R_{\rm out-long} q_{\rm out} q_{\rm long} - 2R_{\rm side-long} q_{\rm side} q_{\rm long})$$

$$\begin{aligned}
R^2_{\rm out}(\phi) &= R^2_{\rm out,0} + 2R^2_{\rm out,2}\cos(2\phi), \\
R^2_{\rm side}(\phi) &= R^2_{\rm side,0} + 2R^2_{\rm side,2}\cos(2\phi), \\
R^2_{\rm long}(\phi) &= R^2_{\rm long,0} + 2R^2_{\rm long,2}\cos(2\phi), \\
R_{\rm out-side}(\phi) &= 2R_{\rm out-side,2}\sin(2\phi), \\
R_{\rm out-long}(\phi) &= 2R_{\rm out-long,2}\cos(2\phi), \\
R_{\rm side-long}(\phi) &= 2R_{\rm side-long,2}\sin(2\phi)
\end{aligned}$$

# 3.2 RHIC correlations

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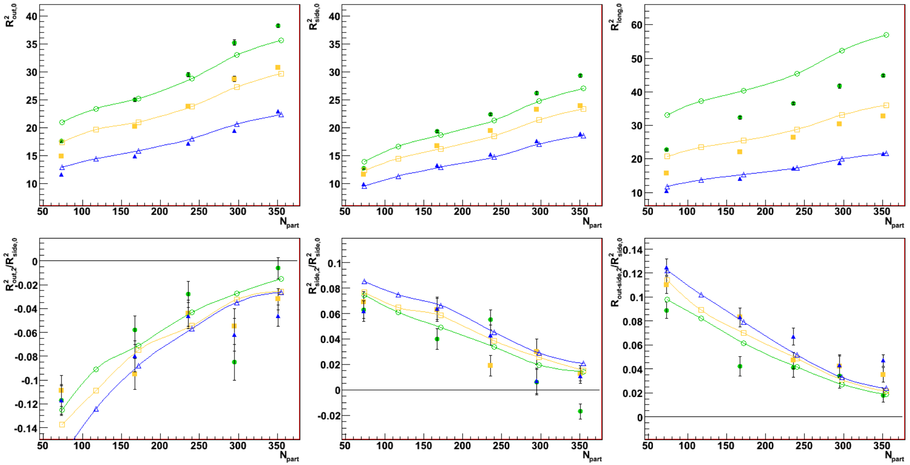
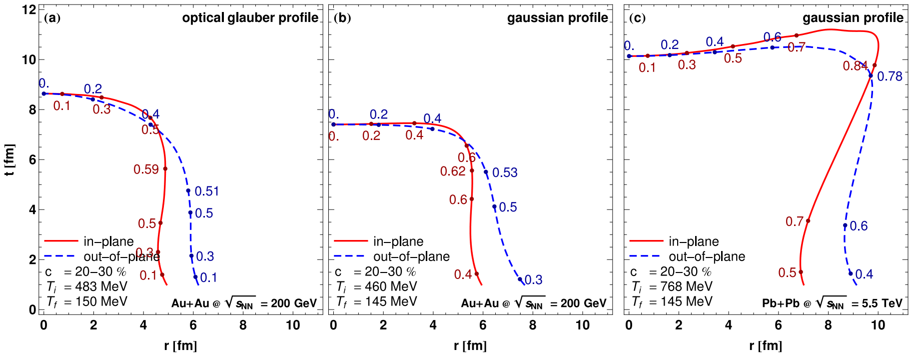



**non-identical pion-kaon correlations also well described, talk by Kisiel at WPCF2008**

# 3.2 Effects of the Shapes of Freeze-Out Hypersurfaces

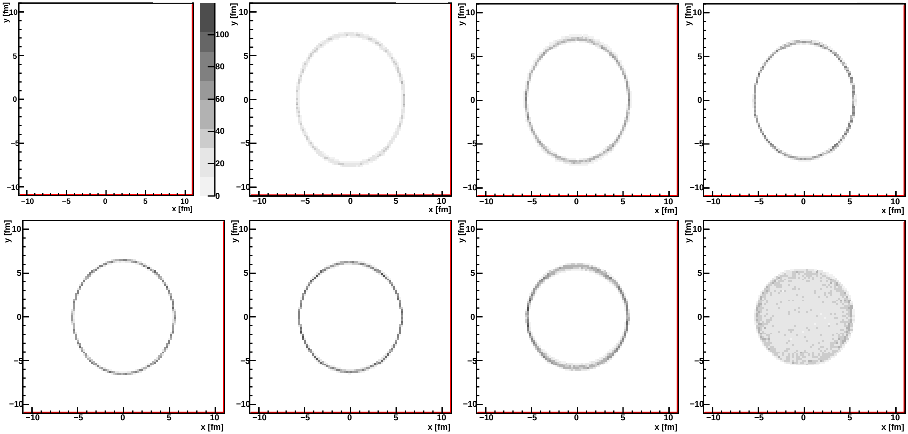
freeze-out hypersurfaces for different initial conditions




with gaussian initial conditions: $R_{\mathrm{out}}$ decreases while $R_{\mathrm{side}}$ increases

# 3.2 Spacetime positions of the emission points

emission points at different time intervals, $\Delta t$ = 1 fm




complicated structure, competition of the in- and out-of-plane emissions

# 4. Conclusions

1. We achieved successful uniform description of soft hadron production at RHIC – **with modified initial conditions leading to faster acceleration the HBT RHIC puzzle is solved!!!**
2. Extrapolation to higher energies indicates smooth changes of all studied quantities, our approach may be used as the event generator to test the detector identification capabilities at LHC (not discussed here)
3. What happens in the early partonic phase? Fast development of the flow required (Sinyukov, Pratt) – modified initial shapes due to more complicated nucleon-nucleon dynamics // initial 2D hydrodynamic expansion followed by 3D expansion (Bialas, Chojnacki, Ryblewski, WF) // viscous effects (Bozek, Pratt, Dusling) // color-field dynamics ...