Introducing Saturation Effects into Event Generators

Emil Avsar
Institut de Physique Théorique de Saclay
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Abstract
We present the results of a numerical study on applying an absorptive boundary on the BFKL equation mimicking the full BK equation solution for $k_\perp$ above the saturation momentum. It is explained how this strategy can be used to introduce saturation effects into event generators based on the linear small x dynamics.

During the past years there has been much progress in our understanding of the high energy, or small-x, region of QCD. The QCD analysis of the dynamics in this region suggest that one reaches a new state of matter referred to as the Color Glass Condensate (CGC) [1] in which the energy, or x, evolution of the relevant physical processes is highly nonlinear. Although theoretically well motivated, it is not really clear that the physics of the CGC has been observed at present collider experiments. There are a few hints at saturation at HERA, but the problem is that one has been looking for saturation effects mostly in inclusive observables such as F_2, for which the expected signatures of saturation, such as geometric scaling, can be mimicked by the linear evolution as well. Besides, analytical estimates often involve many approximations and large uncertainties. It should be emphasized that the question of whether or not saturation effects are already important at colliders is not an academic one. If in fact the nonlinear physics is important then the extraction of our PDFs, based on the linear collinear factorization, is wrong and since the PDFs will be used for almost all the physics analyses in LHC, the problem could potentially be very severe. It is therefore very important to estimate the size of the expected nonlinear corrections.

Although the theory of the nonlinear dynamics is generally understood, detailed calculations are often very difficult. The small-x evolution of a hadron wavefunction in the CGC formalism is governed by the JIMWLK equation which can be rewritten as a Langevin equation which generates an infinite hierarchy of evolution equations for the scattering amplitudes T^k. Having a Langevin formulation, the JIMWLK evolution equation is amenable for a numerical study. Although a numerical simulation of the JIMWLK equation exists, one is still quite far from building an event generator from which properties of exclusive final states as observed in experiments could be studied. At the present we therefore have to conclude that we are far from building an event generator based on the full nonlinear dynamics[1]

In this talk we describe how one can introduce saturation effects into event generators based on the linear evolution without knowing the full details of the nonlinear evolution. Although the analytic ideas apply naturally to the BFKL and the nonlinear BK [5, 6] equations, we shall ultimately be interested in modifying the CASCADE [7] event generator which is based

[1] A possibility could be to use the much simpler dipole model. Investigations on this possibility have been reported in a series of papers [2–4], but there is yet no final result.

on the CCFM [8] evolution equation. We will study the effects of saturation for $k_\perp$ above the saturation momentum $Q_s(x)$, where we can rely on $k_\perp$-factorization on which both the CCFM and BFKL evolutions are based. At first sight it might sound strange to look for the signatures of saturation above Q_s, since Q_s is supposed to mark the border between the nonlinear and the linear evolutions. It is, however, not true that saturation effects abruptly set in below Q_s while being negligible above it. The nonlinear evolution which certainly dominates the physics below Q_s can still modify what happens at $k_\perp$ above[2] Q_s, where one would naively think that the linear evolution would be valid.

In discussing saturation, it is convenient to work with the unintegrated gluon density $\phi(x, k_\perp)$, which in light-cone gauge can be defined as the expectation value of the Fock space number operator $\langle a_k^\dagger a_k \rangle$. This gluon density is related to the scattering amplitude $T(x, r_\perp)$ via the relation

$$T(x, r_\perp) = r_\perp^2 \int \frac{d^2k_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \phi(x, k_\perp). \tag{1}$$

Although the unintegrated gluon density which enters the $k_\perp$-factorization is a different quantity, the BFKL equation is identical in both definitions. For BK, the nonlinear term will look different whether one uses ϕ or the $k_\perp$-factorizable gluon density. However, our analysis will not give a correct treatment of the $k_\perp \leq Q_s(x)$ region anyhow, so it does therefore not matter which quantity we choose. As the nonlinear term written for ϕ (see below) is much simpler, we shall use ϕ as our unintegrated gluon density in what follows, which thus is not the $k_\perp$-factorizable density.

The BK equation is written in terms of ϕ as

$$\partial_Y \phi(Y,k) = \int \frac{dk'^2}{k'^2} \bar{\alpha}_s(max(k^2, k'^2)) \left\{ \frac{k'^2\phi(Y,k') - k^2\phi(Y,k)}{|k^2 - k'^2|} + \frac{k^2\phi(Y,k)}{\sqrt{4k'^4 + k^4}} \right\} - \\ -\bar{\alpha}_s(k^2)\phi^2(Y,k) \tag{2}$$

where as usual $Y \equiv \ln 1/x$ and $\bar{\alpha}_s \equiv \frac{N_c \alpha_s}{\pi}$. Here we have introduced a running α_s which should be seen as a phenomenological modification of the leading order equation for which α_s is fixed. The linear part of this equation is the BFKL equation. What we shall do below is to solve the BFKL and BK equations numerically. We will also solve the BFKL equation in the presence of an absorptive boundary which mimics the full BK equation above Q_s. We now describe this procedure.

A few years ago it was suggested by Mueller and Triantafyllopoulos [9] that one could obtain the correct Y dependence of Q_s, and also the correct form for $T(Y, r)$, above Q_s (for r this means $r \leq Q_s^{-1}$) by simply studying the linear evolution in the presence of an absorptive boundary. The fact that the essential information of the nonlinear evolution can be obtained without knowing the details of it is suggested (for fixed α_s) by a correspondence between small-x QCD and statistical physics. However, one should also be aware that this formal correspondence is of limited relevance for phenomenology since the usual scales for Y and $k_\perp$ involved are

[2] How far up in $k_\perp$ the effects of saturation are visible for a given x is of course not entirely clear. The numerical analysis is therefore important.

typically much beyond what is studied at colliders. We shall discuss phenomenological issues more below. The idea of the absorptive boundary can be outlined as follows.

To control the approach towards the saturation region one can in BFKL follow the evolution along lines of constant amplitude[3] T. In particular when T is close to, but strictly below unity, the line of constant amplitude can be identifed with Q_s. A saddle point approximation then determines the anomalous dimension which determines the behaviour of T near the saturation boundary. However, even though one follows lines of constant amplitude with T strictly below unity, one has to be careful since the diffusive nature of the BFKL solution means that there may be "paths" contributing to the solution and which pass through the saturation region. For such paths the BFKL equation does not give the correct treatment. The idea is therefore to endow BFKL with an absorptive boundary such that all those paths are cut out from the solution. As one is throwing out some of the contributions to the BFKL saddle point solution, the definition of the lines of constant amplitude are also modified. It then turns out that Q_s behaves as (for fixed α_s)

$$\ln Q_s^2 = C + cY - \frac{3}{2\gamma_s}\ln Y \tag{3}$$

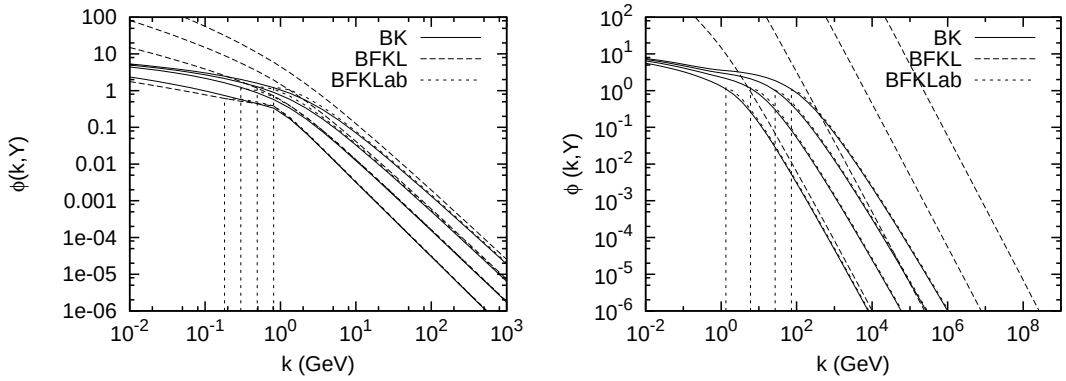
where C is some constant (depending on α_s) and $c \approx 4.9$ and $\gamma_s \approx 0.63$. If one had just studied the lines of constant amplitude for the original BFKL solution one would instead of the term $\frac{3}{2\gamma_s}\ln Y$ get $\frac{1}{2\gamma_s}\ln Y$. The difference between these two terms represents the modification due to the nonlinear physics.

More specifically the absorptive boundary is applied as follows. Pick first a line of constant amplitude $Q_c(Y)$ so that $T(Y, Q_c^{-1}(Y)) = const$ where the constant can be any number much less than unity (actually Q_c is chosen such that T becomes a constant *after* the boundary has been applied). Then the BFKL saddle point solution is forced to vanish at some point $\rho \equiv \ln 1/(\Lambda r)^2 = \rho_c - \Delta$ where $\rho_c \equiv \ln(Q_c/\Lambda)^2$. The form of the BFKL solution is such that T will increase from the point $\rho = \rho_c$ down to some $\rho = \rho_s$ after which it will decrease to zero at $\rho = \rho_c - \Delta$. The point of the maximum, ρ_s, can then be identified with the saturation momentum[4] . The parameter Δ can in turn be determined by requiring the consistency constraint that $T(Y, \rho_s) = b$, for some $b < 1$. This procedure gives Q_s as written above.

In the numerical simulation we shall proceed in the same way. Thus we define some critical value c, corresponding to $\phi(Y, k_c) = c$ such that ϕ is forced to vanish for all $k^2 \leq k_c^2 \cdot \exp(-\Delta)$. The value $\phi(Y, k) = 0$ is, however, not a fixed point for the BFKL evolution which is nonlocal in k as can be seen from (2), and we therefore do not allow points where ϕ has been set to zero to evolve again. One should notice that neither the analytical nor the numerical procedure with the absorptive boundary gives the correct treatment of the dynamics below Q_s. The numerical simulation is important for a detailed analysis, and especially for phenomenology as lower Y values, which are the ones important for phenomenology, are not completely controlled by the analytical treatment.

[3] In [9] the analysis was done in coordinate space for T. However, the corresponding analysis in momentum space for ϕ is basicly the same so in the end we shall apply the boundary for the linear part of equation (2).

[4] This is just a convention. Any line of constant amplitude will give a valid definition. It turns out that $\rho_s - \rho_c$ is just a constant, which appears as the constant C in (3). The overall normalization of Q_s cannot be determined from theory.



In figure 1 we show the solutions to BK, BFKL and BFKL with absorptive boundary condition (BFKLab) respectively. In the left plot we have the values $Y = 2, 4, 6$ and 8 respectively while in the right plot we have $Y = 10, 20, 30$ and 40. The results for BFKLab have been obtained for a specific set of values of the parameters Δ and c. Generally we see that we have to choose the critical value c to be around 0.1-0.5 to match the full BK solution. It turns out that larger critical values match more smoothly with equations whose nonlinear term are cubic, quartic and so on, as opposed to BK which has a quadratic nonlinear term.

We thus see that the BFKLab solution mimics the full BK solution very well, and not only for high values for Y, but also for small values where the analytic arguments are much more uncertain. A very important consequence of the saturation mechanism, which has been known for some time, is that the evolution with a running α_s becomes much more stable and sensible. Note that for the completely linear case the solution is very unstable and we see that at around $Y = 20$ the linear curve is nowhere close to the nonlinear one, even at very high $k_\perp$. Here we have regulated the singularity in the running α_s by replacing the argument k^2 with $k^2 + k_0^2$. For the BFKL solution we used $k_0^2 = 2$ GeV^2 while for BK and BFKLab we have $k_0^2 = 0.5$ GeV^2. If for BFKL we choose this lower cutoff then the solution is even more unstable and deviates earlier from the nonlinear solutions. Thus the result depends very sensitively on the nonperturbative cutoff. For BFKLab the solution is completely stable and we have checked that there is essentially no dependence at all on k_0. This is in strong contrast to BFKL, and is an important consequence of the nonlinear physics. This problem appears also for CCFM which like BFKL shows a diffusive behaviour in $k_\perp$. In event generators based on the linear physics one has therefore a quite strong dependence on the soft cut.

We have just described how one can economically introduce saturation effects into the linear small-x evolution. This method is very suitable for use in a Monte Carlo (MC) event generator. The only issue we face now is to go from BFKL to CCFM as there are no event generators based on BFKL. The CCFM formalism is suitable for the study of exclusive final states and is implemented in the CASCADE event generator. Although BFKL and CCFM are different formalisms there are nevertheless great similarities between the two. We have here no space to enter a detailed discussion on CCFM. As one of the most important similarities we shall however mention the following two points.

Numerical studies [10] have shown that CCFM, just like BFKL, shows a broadening of $k_\perp$. Infact this should come as no surprise. Denoting the momenta of the emitted real gluons by $q_\perp$ and that of the t-channel propagators by $k_\perp$, one is in CCFM free to go up and down in $k_\perp$ with the standard $d^2q_\perp/q_\perp^2$ bremstrahlung spectrum. In CCFM we also have angular ordering which prevents real gluons with very low momenta $q_\perp$ to be emitted, but this does not put much constraint on the virtual propagators $k_\perp$ which can again perform a random walk. The second point is that the CCFM gluon density grows as $\exp(\lambda Y)$ where to leading order $\lambda \approx 0.5$ just like in BFKL. Therefore the problem of unitarity is still there for CCFM, and in particular this shows that Q_s extracted from CCFM should be very similar to that extracted from BFKL. We are currently investigating a numerical solution of the CCFM equation.

In the MC program (for an early application see the talk by K. Kutak), the gluon distribution is first constructed by the standard forward evolution. The gluon ladder is then constructed via the backward evolution approach, starting from the hard scattering process. The unitarity constraint can be applied to the first step using the same strategy. This will give us a $k_\perp$ distribution which is cut below Q_s (which can be determined once the distribution is known). In the backward evolution one should then also make sure for consistency that no real gluon with $q_\perp < Q_s$ is emitted, as such gluons would have undergone saturation effects (basicly multiple scatterings).

In the application to event generators, the scales involved are not as large as the ones showed in figure 1, neither for $k_\perp$ nor for Y. In fact for $k_\perp$ the phenomenologically relevant part occupies a very small window in the figure. Here it can potentially be difficult to see any deviation from the linear physics, especially after full energy-momentum conservation is introduced. The precise choice of c and Δ can also be important in such a small window. We will come back to these issues in a lengthier paper.

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