Introducing saturation effects into event generators

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Outline

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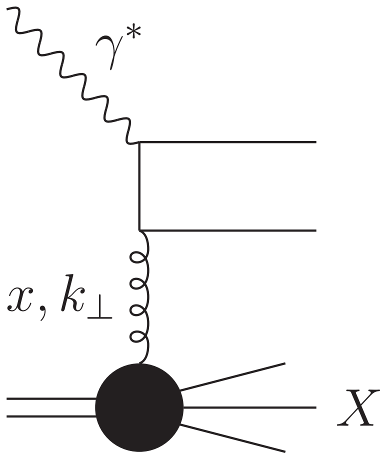
- QCD evolution at high energies:
  - Linear evolution: BFKL, CCFM
  - Nonlinear evolution: BK, JIMWLK
- BK and pulled fronts: Lessons from statistical physics
- Linear evolution with absorbtive boundary: a numerical study
- Applying the ideas to CCFM and to event generators

$k_\perp$ factorization and linear evolution

- In QCD at small x we have $k_\perp$ factorization:

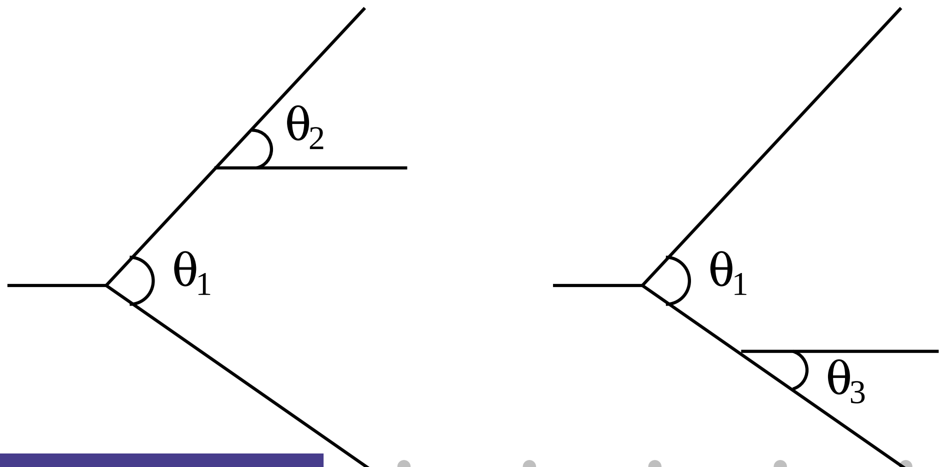
$$\sigma^{\gamma^* p} = \int d^2k_\perp dx f(x, k_\perp) \cdot \sigma(\gamma^* + g(x, k_\perp) \to q + \bar{q})$$



- BFKL: $\partial_Y f(x, k_\perp) = K_{BFKL} \otimes f(x, k_\perp)$, $Y =$ ln $1/x$

Coherence and final states

- Although BFKL is suitable for studying the evolution of $f(x, k_\perp)$ at (not too) low x, it is not so for studying the properties of the exclusive final states.
- In e^+e^- annihilation, it has been known for a long time that coherence is important for the FSR. QM interference leads to $\theta_2, \theta_3 < \theta_1$



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- At low x, CCFM gives same intercept as LO BFKL.
- Thus, $f \sim 1/x^\lambda$ in both cases, with $\lambda \approx 0.5$ at LO.
- However, CCFM also contains some DGLAP physics, and thus it also mimics NLO corrections to BFKL and vice versa.

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$$T(x,r) = r^2 \int d^2k_\perp e^{ik_\perp \cdot r} f(x,k_\perp)$$

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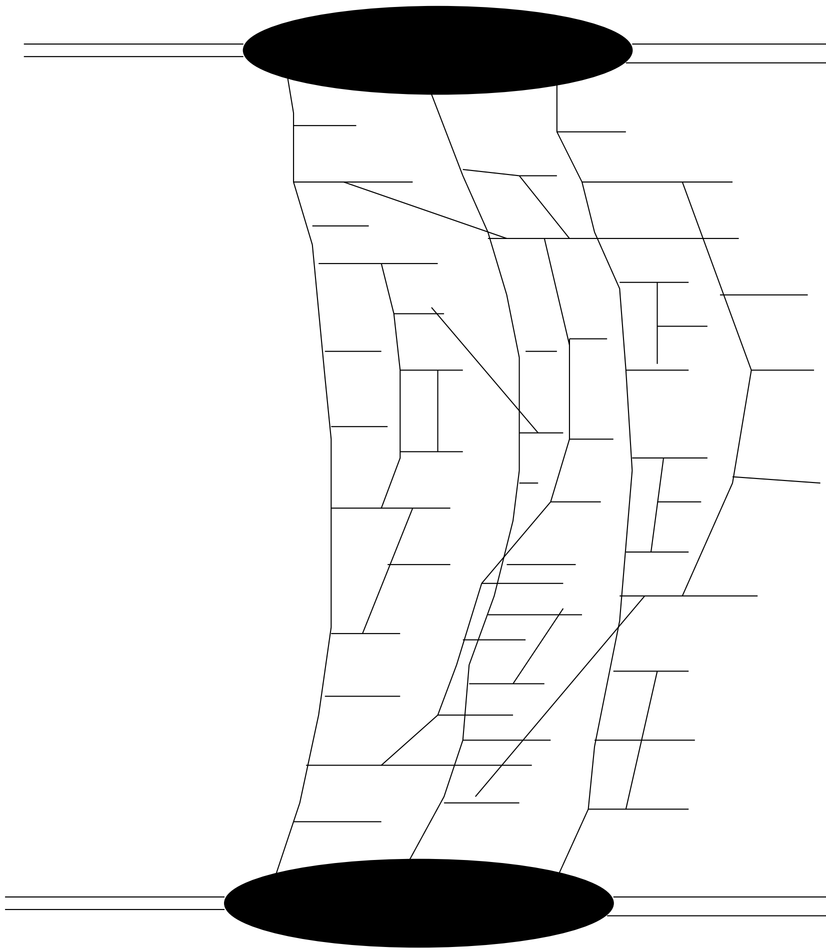
- However, still very complicated! formulated in terms of gauge fields.
- Also, it is not known how to extract the exclusive final states.
- BK is a simplified eq. which neglects correlations.

A picture of what happens



Relation to statistical physics

- Small-x evolution in QCD in same universality class as pulled fronts (R-D processes).
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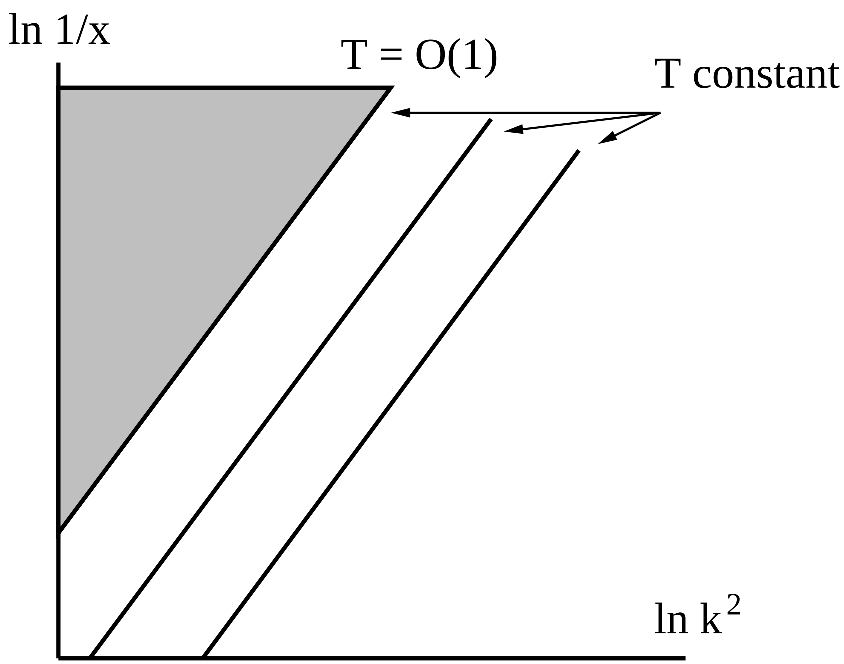
- Thus, correct r and x dependence of T for $T < 1$, and x dependence of saturation momentum $Q_s(x)$ can be determined from the linear dynamics alone...
- provided there is a saturation mechanism.

BFKL with absorbtive boundary

- Mueller $\&$ Triantafyllopoulos: Use BFKL$+$ absorbtive boundary to mimic nonlinear evolution.
- Here, evolution along lines of constant T. Gives different anom. dim. from usual BFKL saddle point (fix $k_\perp$, larger Y).
- To keep T constant, $k_\perp$ has to be increased (r decreased) when Y increases.
- When $T = \mathcal{O}(1)$, line of constant T identified with $Q_s(x)$.

A phase space diagram



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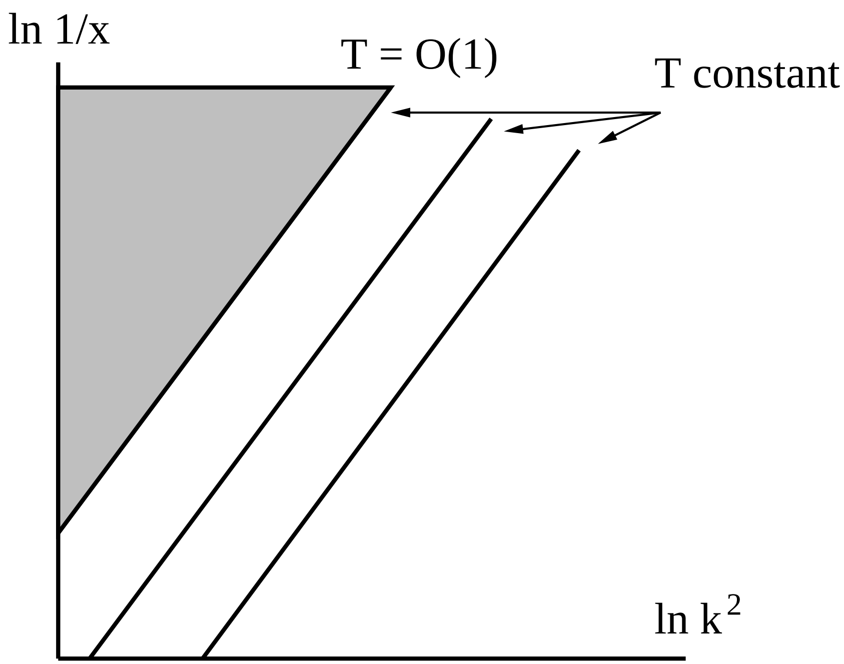
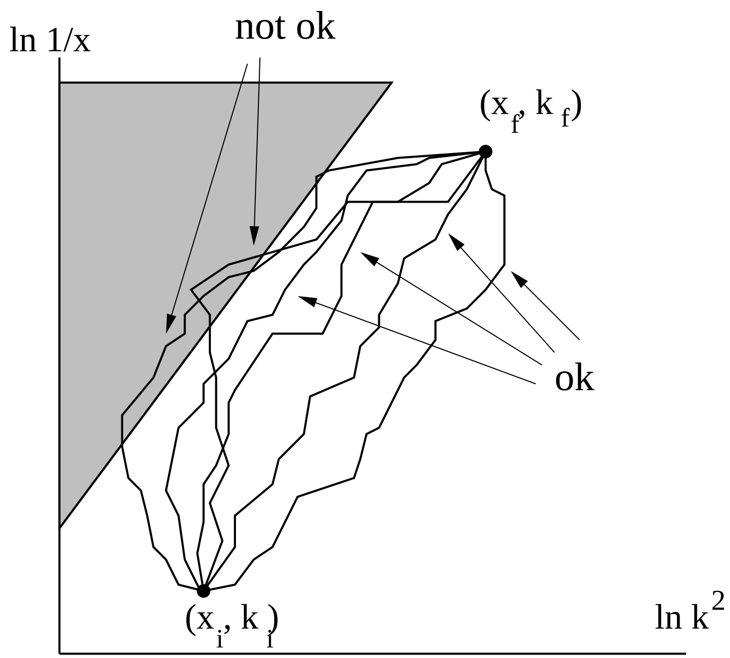


- However, saddle point solution along such a line recieves contribution also from saturation region.

Which paths are allowed?



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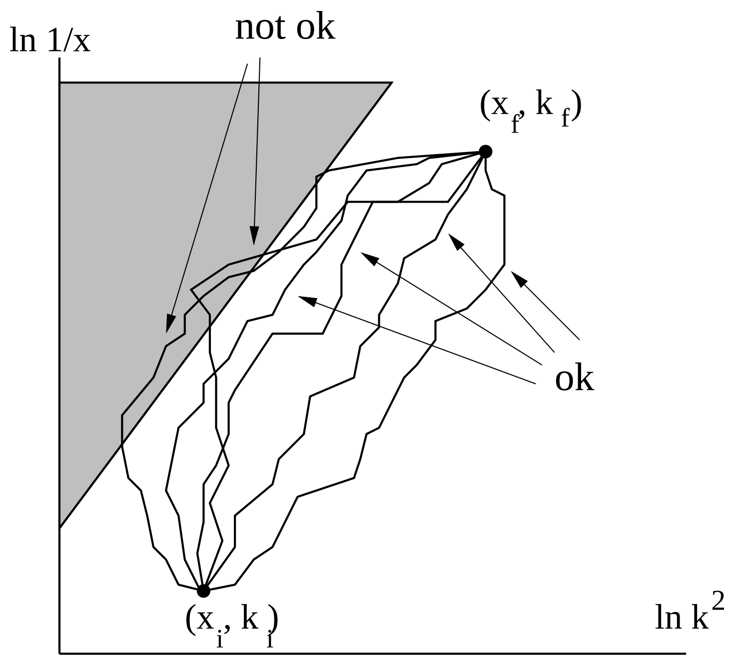

- $\ln Q_s^2 = C + c\bar{\alpha}Y - \frac{3}{2\gamma_s}\ln(\bar{\alpha}Y)$, with $c \approx 4.9$ and $\gamma_s \approx 0.63$.

FKPP equation

- Protoype of BK equation, the FKPP equation:

$$\partial_t T(t,x) = \partial_x^2 T(t,x) + T(t,x) - T^2(t,x)$$

$$t \leftrightarrow \bar{\alpha} Y, \quad x \leftrightarrow \ln(k_\perp^2)$$

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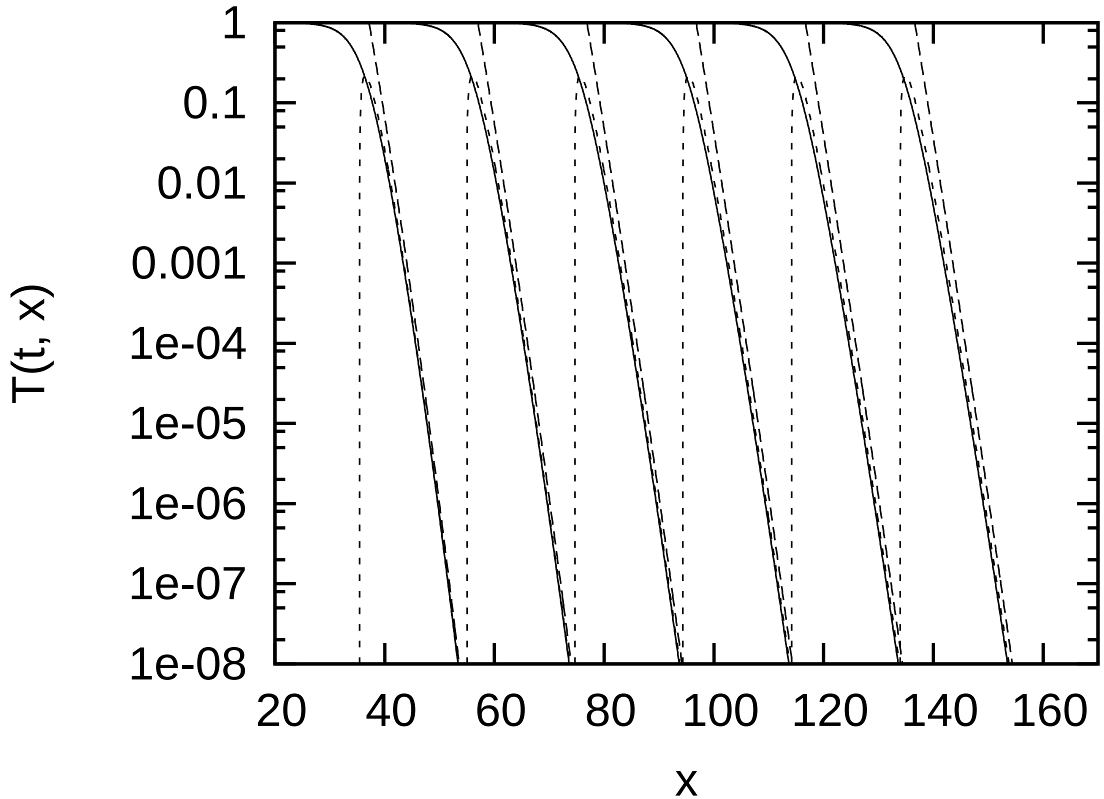
- Neglecting the T^2 term, one obtains a simplified version of the BFKL equation.
- We solve FKPP numerically. Absorbtive boundary applied such that $T(t,x) = 0$ for all $x \leq x_c - \Delta$ where $T(t,x_c) = c$.
- Δ and c free parameters.

Numerical results



- Here $t = 20 + 10 \cdot i$. Linear plot also shown.

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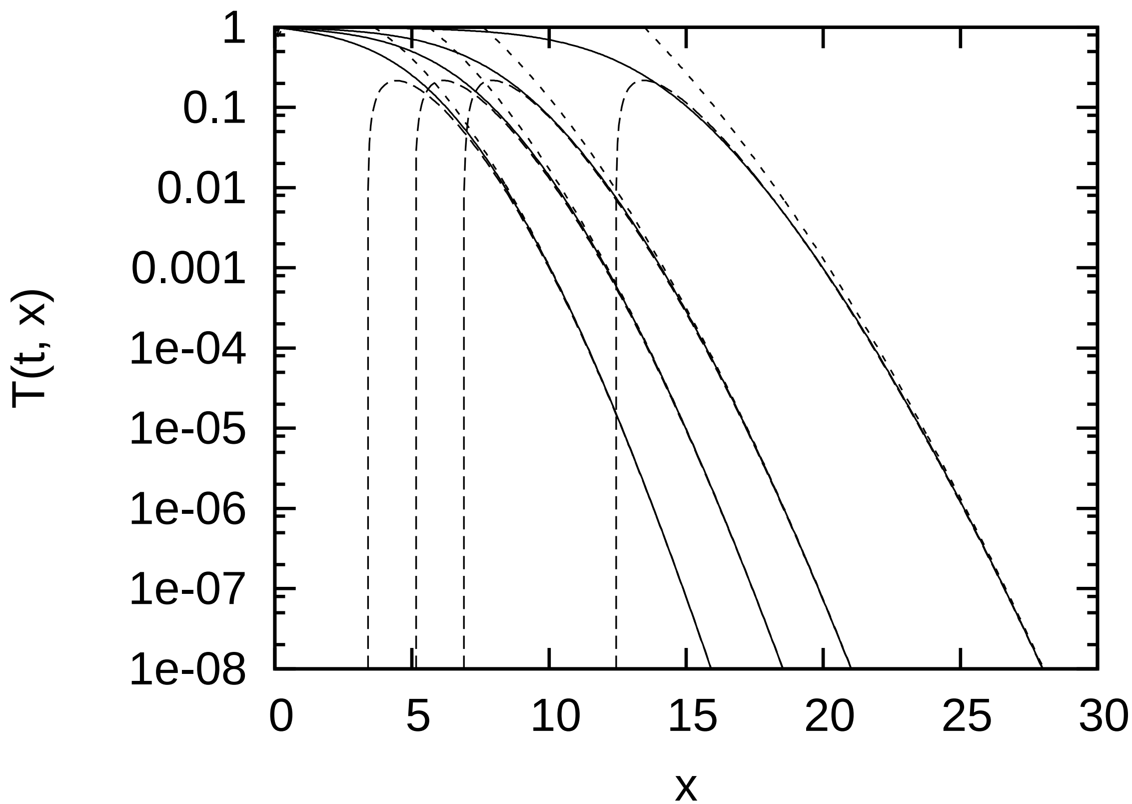

- Here $t = 3, 4, 5$ and 8.

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- However, we are not looking at this region. Interested in region where $T << 1$ and $k_\perp$-fact. ok.
- We can also not say anything about correlations etc.

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- However, to LO, BFKL and CCFM give same intercept, so Q_s extracted from CCFM should be similar to the one extracted from BFKL.
- Moreover, the space-like gluon emission in CCFM and BFKL identical to LO. Difference in total phase space subleading.

Back to CCFM

- Also virtual corrections similar. Between any pair of emissions n and $n+1$ we have

$$S_{ne}^2(n,n+1)\cdot S_{eik}^2(n,n+1)=\Delta_{ne}^{BFKL}(n,n+1).$$

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- Our ideas to be applied in the CCFM MC event generator CASCADE (Jung).
- First we should repeat FKPP analysis for real BFKL and BK. Work under progress.

Outlook

- At LHC, important to try estimate the size of the nonlinear effects.
- First time a formalism for nonlinear effects inspired from the actual QCD ev. eqs. would be included in an event generator.
- Existing MC generators include some saturation effects (e.g PYTHIA) but these are not really based on any QCD analysis.
- Our work will certainly not cover all aspects of the nonlinear physics.