

Small x Resummation - An Overview

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Abstract

I provide a summary of BFKL resummation applied to deep inelastic scattering. The origin of the small x problem is described, together with the various approaches that have evolved in recent years for dealing with the issue. Their technical details are briefly compared, and conclusions from global parton fits to scattering data examined.

1 Introduction

As is well known, hadronic cross-sections factorise into the following form:

$$\sigma = f_a(x_a, \mu_F^2) \otimes \hat{\sigma}_{ab} \otimes f_b(x_b, \mu_F^2), \quad a \in \{q, \bar{q}, g\},$$

where $\hat{\sigma}_{ab}$ are perturbatively calculable hard coefficient functions, and f_a non-perturbative parton distribution functions dependent on a longitudinal momentum fraction x and a factorisation scale μ_F . The latter are not calculable in perturbation theory, but evolve with μ_F according to the DGLAP equations:

$$\frac{\partial f_a(x, \mu_F^2)}{\partial \ln \mu_F^2} = \sum_b P_{ab}(x, \mu_F^2) \otimes f_b(x, \mu_F^2),$$

with P_{ab} perturbatively calculable splitting functions. A problem arises at small values of x in that both the splitting functions and hard coefficients contain terms of form $x^{-1} \alpha_S^n \log^m(1/x)$ with $n \geq m - 1$. Thus, when x is sufficiently small, each term in the perturbation series diverges even though α_S may be in the perturbative regime. One naïvely expects this to become a problem when:

$$\alpha_S \log\left(\frac{1}{x}\right) \simeq 1 \Rightarrow x \simeq 10^{-2},$$

and we will see later that this back of the envelope reasoning is essentially correct. At small x , one must reorder the perturbation expansion in terms of leading logarithmic (LL), next-to-leading logarithmic (NLL) terms and so on.

In this contribution, we focus on the solution to this problem in deep inelastic scattering, and will be able to answer the following questions. Firstly, how small is small x (i.e. can we confirm the above expectation)? Secondly, what approaches exist to deal with the problem, and how do they differ? Finally, what can scattering data tell us about their validity?

2 The BFKL equation

The small x divergence in deep inelastic scattering can be traced at LL order to ladders of gluon exchanges, with quark mixing at NLL order. The unintegrated gluon 4-point function $f(k^2, Q_0^2)$

which sums up these exchanges can be written as the solution to an integral equation - the BFKL equation [1]- which has the schematic form (up to NLL order)

$$Nf(k^2, Q_0^2) = Nf_I(Q_0^2) + \alpha_S(k^2) \int dk'^2 \left[\mathcal{K}_0(k^2, k'^2, Q_0^2) + \alpha_S(\mu^2) \mathcal{K}_1(k^2, k'^2, Q_0^2) \right] f(k'^2),$$

where we have performed a Mellin transform in x , with N the moment variable. Here k^2 and Q_0^2 are transverse momentum scales at the hard and soft ends of the gluon ladder respectively, and \mathcal{K}_i the BFKL kernel, which is currently known exactly up to NLL order [2] and approximately at NNLL order [3]. Solving this equation resums the problem logarithms at the level of the gluon density. This is then related to observable quantities (in this case proton structure functions) using the k_t -factorisation formula [4]

$$\mathcal{F}_i(k^2, N) = \int dk^2 h_i(k, N) f(k^2, Q_0^2),$$

where $h_i(k, N)$ is an impact factor coupling the incoming virtual photon to the gluon ladder. Thus, for physical predictions, one needs a knowledge of both the BFKL kernel to the desired order, and also the impact factors. The latter are known exactly at LL order in DIS [4, 5], and also at approximate NLL order in both the massless [6] and massive cases [7]. It has been known for some time that the LL BFKL formalism is insufficient to describe scattering data, thus any phenomenologically viable resummation approach must satisfy at least the following requirements:

1. Solve the BFKL equation at NLL order (preferably with running α_S).
2. Provide the complete set of splitting functions P_{ab} , and also coefficient functions for the structure functions F_2 and F_L .
3. Match the resummed results to the usual DGLAP results for correct moderate / high x behaviour.
4. Explain why NLO DGLAP appears to work well down to low x .

(strictly speaking one must also add the additional requirement of correctly implementing heavy quarks, as has been shown to be important in fixed order fits to current data). So far only three approaches achieve these aims to a reasonable extent (see also [8] for a recent alternative approach).

3 Resummation approaches

The formalisms which (broadly) satisfy the above requirements are known as the ABF [9], CCSS [10–12] and TW [7, 13, 14] approaches. Here, we summarise the main differences.

Firstly, not all of the approaches use the same factorisation scheme for the parton distributions. ABF present results in the the standard $\overline{\text{MS}}$ -bar scheme to all orders, whereas CCSS and TW present results in schemes which are the conventional $\overline{\text{MS}}$ -bar and DIS schemes respectively up to NLO in the fixed order expansion, but differ slightly from their fixed order counterparts in the resummed terms (ultimately arising from the fact that the regularisation of collinear singularities when solving the BFKL equation takes place in a so-called Q_0 -scheme, rather than dimensional regularisation). See the detailed studies in [9, 11, 12] for further details.

	ABF	CCSS	TW
Full set of P_{ab} and C_a (for light flavours)	✓	✓	✓
Resummation of BFKL kernel [15]	✓	✓	×
Factorisation scheme	DIS, MS-bar	MS-bar(NLO+ Q_0)	DIS(NLO+ Q_0)
Heavy quark effects included	×	×	✓
Global fit carried out	×	×	✓

Table 1: Summary of the ABF, CCSS and TW approaches. See the text for further details.

Secondly, the BFKL kernel is known to contain potentially unstable terms of collinear origin. These can be resummed to all orders in the kernel [15], as is done in the ABF and CCSS approaches. The TW group do not implement this further resummation, although could do so in principle. The latter group also include higher order terms from the impact factors [6, 7], which are not included by the other approaches (consistent with their definition of NLL order).

These and other differences are summarised in table 1. Results for splitting and coefficient functions cannot be directly compared between the approaches due to factorisation scheme ambiguities. However, the main qualitative feature of the resummed splitting functions in all the approaches is a pronounced dip below the NLO DGLAP result at moderate x values, followed by an eventual rise at very small x (see figures 1-2, 5 and 3-4 in [9, 10, 14] respectively).

4 Fits to scattering data

So far, only the TW approach above has been implemented in a global fit to scattering data, although work is in progress involving the other approaches. Nevertheless, the qualitative similarity between the results (i.e. the dip in evolution when resummations are included in the splitting functions) means that similar results to those presented here should presumably be seen in all the approaches.

The dip in the resummed evolution qualitatively changes the shape of the gluon distribution, as seen in figure 1. The gluons agree at high x , as is required by consistent matching of the resummed and fixed order descriptions. At low x (and the input scale for evolution $Q^2 = 1\text{GeV}^2$), the resummed gluon is positive definite and slightly growing as $x \rightarrow 0$. This has several phenomenologically desirable features. Firstly, a raised gluon at low x decreases the tension between the Tevatron jet data [16] and the small x HERA data [17]. It also stabilises the longitudinal structure function F_L at low x and Q^2 , and consequently reproduces the correct shape for the HERA reduced cross-section data at high inelasticities y .

One notes that the gluon distributions (resulting from a fit to data) move away from each other below $x \sim 10^{-2}$. Thus, this can be taken as an empirically determined upper bound below

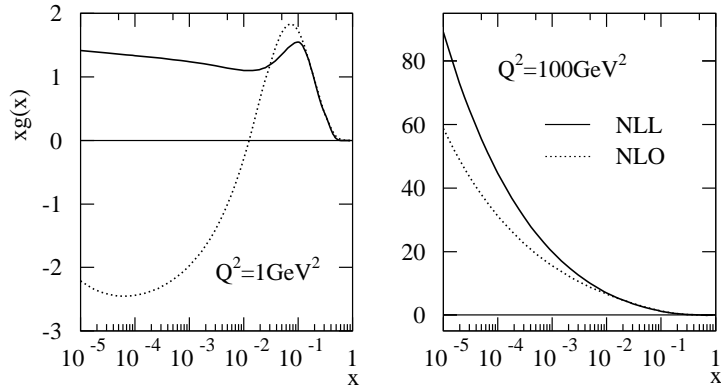


Fig. 1: Gluons obtained in a NLO fit to DIS and related data, with and without NLL small x resummations.

which small x resummation becomes important. Note that this value is in rough agreement with the back of the envelope calculation described in the introduction. It is also clear why NLO DGLAP has worked well down to very low x values ($\sim 10^{-5}$) - the DGLAP splitting function sits roughly in the middle of the resummed dip found in each of the above approaches.

Although the above describes compelling evidence for the need for small x resummation, there are not yet inclusive observables in DIS which defy description using the traditional DGLAP theory. However, resummed and fixed order predictions can be quite different. The case of F_L (shown alongside the recent H1 data [18]) is shown in figure 2 with a current NNLO prediction [19]. One sees from the plot that the two descriptions cannot yet be distinguished, although points at slightly lower Q^2 (thus also lower x) could indeed discriminate between them.

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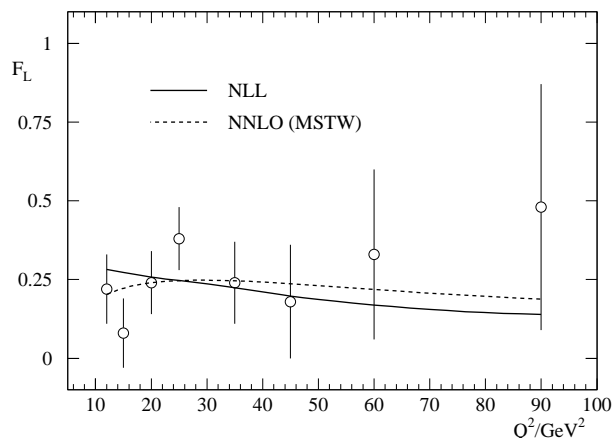


Fig. 2: Resummed prediction for F_L , shown alongside the recent H1 data.

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