Small-x Resummation - an Overview

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Introductory Questions

- What is small x physics and why is it a problem?
- How small is small x?
- Why should we be worried / interested now?
- What approaches exist to deal with the problem?
- What can data tell us about their validity?

Collinear Factorisation in QCD

- Hadronic cross-sections factorise:

$$\sigma = f_a(x_a, \mu_F^2) \otimes \hat{\sigma}_{ab} \otimes f_b(x_b, \mu_F^2), \quad a \in \{q, \bar{q}, g\}.$$

- Partons are process independent - allow predictive power once measured.
- Partonic cross-section $\hat{\sigma}_{ab}$ is perturbatively calculable, as is the scale dependence of the partons:

$$\frac{\partial f_a(x, \mu_F^2)}{\partial \ln \mu_F^2} = \sum_b P_{ab}(x, \mu_F^2) \otimes f_b(x, \mu_F^2)$$

  (the DGLAP equations).
- P_{ab} known to NNLO in α_S (Moch, Vogt & Vermaseren), but NLO already very successful in describing a wide range of data.

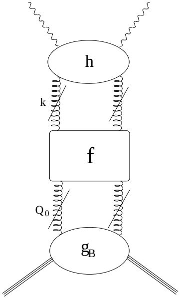
Small x Problem

- Splitting functions P_{ab} and partonic cross-sections $\hat{\sigma}_{ab}$ contain logarithms of partonic momentum fractions $x^{-1}\alpha_S^n \log^m(1/x)$, $n \geq m-1$.
- If x is small, perturbation theory breaks down in principle even if α_S is in the perturbative regime.
- Expect this to be a problem when:

$$\alpha_S \log\left(\frac{1}{x}\right) \simeq 1 \Rightarrow x \simeq 10^{-2}$$

- We will see later that this "back of the envelope" reasoning is essentially correct.
- Reorder perturbation expansion in terms of LL, NLL etc.

Solution - The BFKL Equation



- Divergence in splitting and coefficient functions due to t-channel gluon exchange at LL order, with some quark mixing at NLL order.

- Can sum LL, NLL terms to all orders by solving the BFKL equation.
- Integral equation for the (unintegrated) gluon density.

BFKL Equation

- The BFKL equation has the schematic form (up to NLL):

$$Nf(k^2, Q_0^2) = Nf_I(Q_0^2) + \bar{\alpha}_S(k^2) \int dk'^2 \left[\mathcal{K}_0(k^2, {k'}^2, Q_0^2) \right.$$
$$\left. + \bar{\alpha}_S(\mu^2)\mathcal{K}_1(k^2, {k'}^2, Q_0^2)\right] f(k'^2),$$

- Gluon density related to structure functions using the k_T factorisation formula (Collins & Ellis; Catani, Ciafaloni & Hautmann):

$$\mathcal{F}_i(k^2, N) = \int dk^2 h_i(k, N) f(k^2, Q_0^2).$$

- For physical predictions, one needs the BFKL kernel and the impact factors h_i.

Some (selective!) history

- 1977-1979: BFKL equation presented at LL.
- 1990s: Small x data from HERA: $x \gtrsim$ few $\times 10^{-5}$.
- Early to mid 1990s: k_t factorisation.
- 1998: Full NLL corrections to BFKL equation presented. Flurry of activity.
- 2000s: Three approaches to resummed splitting / coefficient functions at NLL emerge, giving similar results.
- 2006: First global parton fit including NLL BFKL resummations.
- Now / near future: Global fits using results from more than one approach.
- Around the corner: LHC operates at $x \gtrsim$ few $\times 10^{-6}$.

Small x Resummation Approaches

- It has been known for some time that at least NLL BFKL is needed.
- Thus any viable approach to resummed coefficient and splitting functions must:
  1. Solve the BFKL equation at NLL (preferably with running α_S).
  2. Provide the complete set of splitting functions P_{ab}, and also coefficient functions for F_2 and F_L.
  3. Match the resummed results to the usual DGLAP results for correct moderate / high x behaviour.
  4. Explain why NLO DGLAP appears to work well down to low x.
- So far only three approaches do this.

Small x Resummation Approaches

- These are known as ABF (Altarelli, Ball & Forte), CCSS (Ciafaloni, Colferai, Salam & Stasto) and TW (Thorne & White).
- Each of them solves the BFKL equation in a different manner, thus include different sets of subleading terms.
- Different factorisation schemes are considered.
- Heavy quark effects are considered in some approaches and not others.
- Remarkably, results are very similar.

Factorisation Schemes

- DGLAP splitting and coefficient functions are calculated in dimensional regularisation, usually in $\overline{\text{MS}}$ scheme.
- BFKL equation is usually solved in a "Q_0" scheme i.e. off-shell gluon used to regulate collinear singularities.
- It is difficult in general to relate these two schemes, and /or disentangle impact factors in terms of coefficient and splitting functions in a general scheme.
- See detailed studies by CCSS, ABF.

Collinear Resummation

- The BFKL kernel expansion (LL, NLL...) is potentially unstable due to logarithms of collinear origin. E.g. in Mellin space, one has terms:

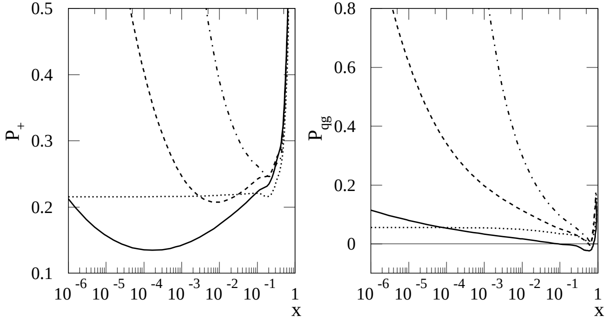
$$\sim \frac{1}{\gamma^3} + \frac{1}{(1-\gamma)^3}$$

  in the NLL kernel.
- These can be resummed to all orders (Salam).
- Different schemes exist.
- ABF and CCSS adopt this resummation, whereas TW do not.
- Not strictly necessary in DIS (see later), but certainly important in processes with two hard scales.

Summary of Approaches

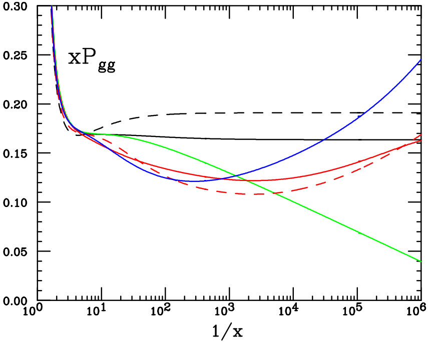
	ABF	CCSS	TW
Full set of P_{ab} and C_a (for light flavours)	$\surd$	$\surd$	$\surd$
Resummation of BFKL kernel (Salam)	$\surd$	$\surd$	$\times$
Factorisation scheme	DIS, $\overline{\text{MS}}$	"$\overline{\text{MS}}(Q_0)$"	DIS(Q_0)
Heavy quark effects	$\times$	$\times$	$\surd$
Global fit carried out	$\times$	$\times$	$\surd$

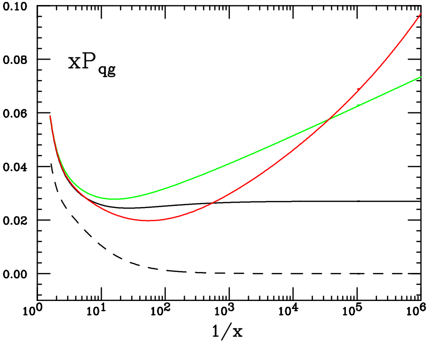
Results for Splitting Functions - TW



- Results shown at $n_f = 4$, $t = 6$.
- Running coupling suppresses low x divergence.
- NLL kernel and impact factor effects lead to even more suppression.
- Main feature is a dip below the NLO DGLAP result - thus can see why NLO DGLAP also works.

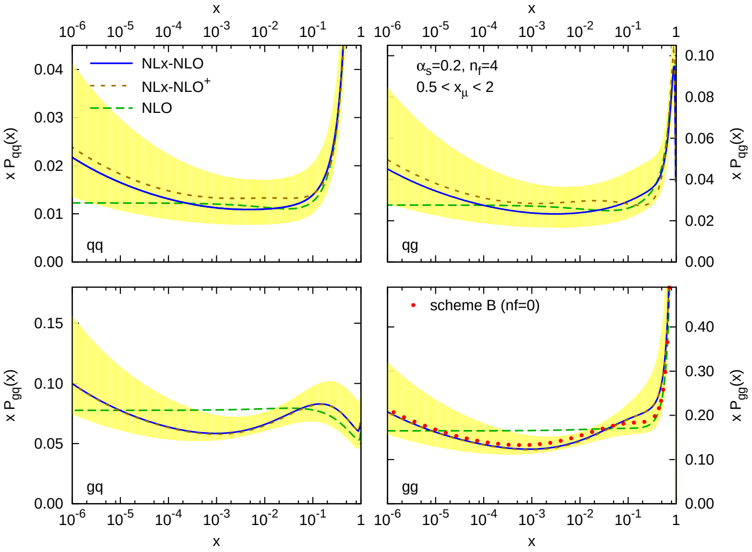
Results for Splitting Functions - ABF



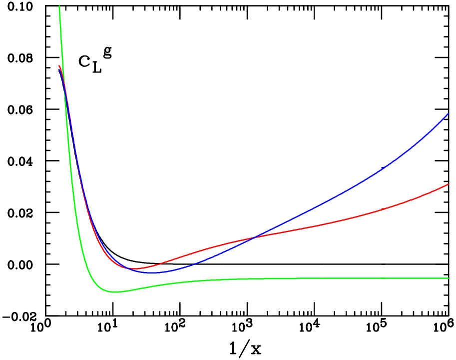


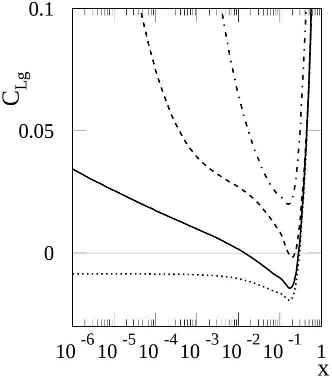
- Different factorisation scheme, but similar results obtained...

Results for Splitting Functions - CCSS



Results for C_{Lg} - CCSS & TW



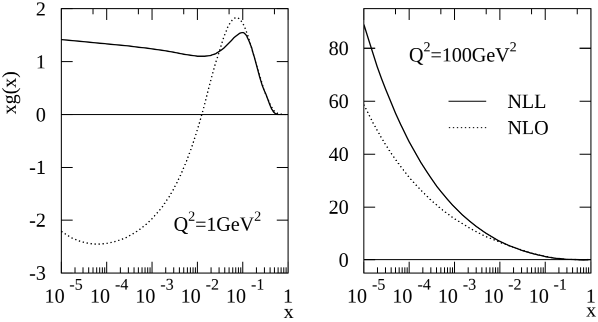


- Again see similar qualitative behaviour between approaches.
- Different factorisation schemes.

Effects of Resummation in a Global Fit

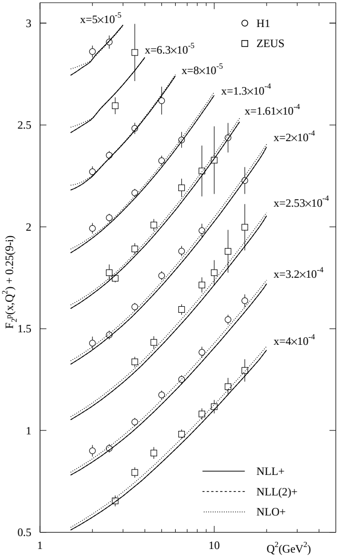
- Here we focus on TW approach. Other approaches should see similar results.
- Resummation qualitatively changes the gluon distribution...
- Recall the NLL splitting functions dip below their NLO counterparts.
- Allows increased gluon at both large and small x.
- Thus can improve fit to Tevatron jet data without compromising fit the HERA data.
- Gluon is qualitatively different - leads to more stable F_L.
- Correct turnover in reduced cross-section at high y.

Gluon Distribution

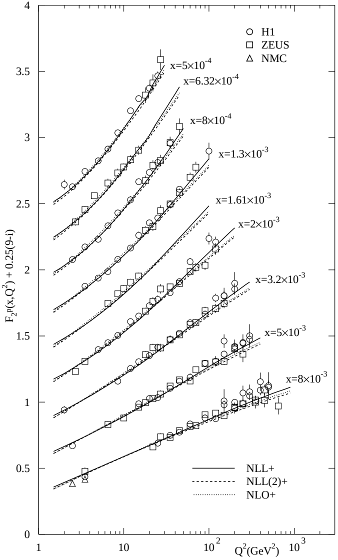


- Gluons differ for $x \lesssim 10^{-2}$.
- NLL resummed gluon positive and growing at small x!
- Not true at fixed order.
- Positive gluon avoids negative structure functions.

Results - F_2

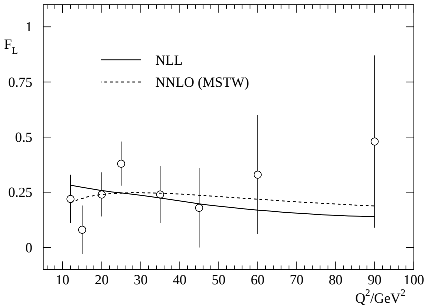


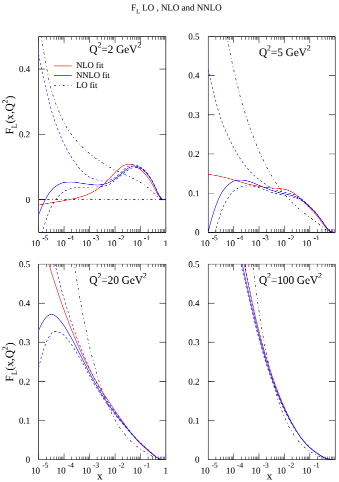
x=5×10-5
x=6.3×10-5
x=8×10-5
x=1.3×10-4
x=1.61×10-4
x=2×10-4
x=2.53×10-4
x=3.2×10-4
x=4×10-4
H1
ZEUS
NLL+
NLL(2)+
NLO+
F2p(x,Q2) + 0.25(9-i)
Q2(GeV2)



x=5×10-4
x=6.32×10-4
x=8×10-4
x=1.3×10-3
x=1.61×10-3
x=2×10-3
x=3.2×10-3
x=5×10-3
x=8×10-3
H1
ZEUS
NMC
NLL+
NLL(2)+
NLO+
F2p(x,Q2) + 0.25(9-i)
Q2(GeV2)

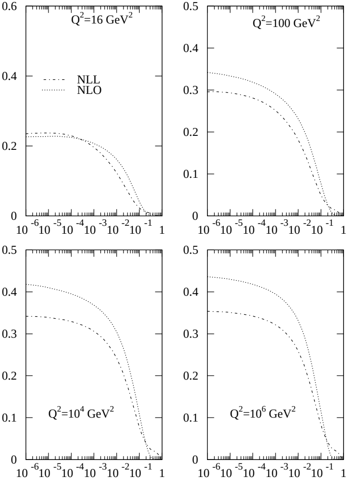
Results - F_L





- Data currently unable to distinguish between predictions.
- But fixed order looks unstable.

Charm contribution to F_2



- Clearly important at small x.
- Thus a consistent treatment of heavy flavours is necessary.

Conclusions

- Small x resummation is highly topical, due to new insights from HERA data and the x range of the LHC.
- Three main approaches have emerged in recent years. Very different methods, but similar final results.
- "Small x" means $x \lesssim 10^{-2}$, as seen by partons obtained from fit.
- NLL resummation is necessary, and resulting splitting functions dip below the NLO DGLAP ones.
- Simultaneously explains why NLO DGLAP works so well, but resummation still an improvement.
- Resummed gluon distribution shows a qualitatively different behaviour, which impacts σ_R, F_L etc.

Open Questions

- Can each approach be cast into the conventional $\overline{\text{MS}}$ factorisation scheme?
- Will the global fit results be replicated by other approaches?
- How should resummed partons be used? Resummed partonic cross-sections needed for processes other than DIS (e.g. Drell-Yan).
- Are resummed and NLO partons really incompatible, once uncertainties are taken into account?
- What are the implications for saturation, if linear BFKL evolution suppresses the gluon?