Relativistic viscous hydrodynamics

and AdS/CFT correspondance

Rudolf Baier

Faculty of Physics, University of Bielefeld

A fluid which has no shear stresses, viscosity or heat conduction is called a

PERFECT FLUID

i.e. it looks isotropic in its rest frame

Top physics story of 2005 is the RHIC discovery of the strongly interacting quark-gluon plasma (called sQGP), which behaves almost like a perfect fluid, with very low viscosity

[T. D. Lee 06]

STILL IT IS -

BUT IT HAS TO BE CAREFULLY TESTED

CONTENT

- viscous relativistic hydrodynamics - approximations
- viscosity η - relaxation time τ_π
- conformal hydrodynamics - Anti-de-Sitter/Conformal Field Theory (AdS/CFT) correspondence
- some results and comparison with RHIC data

mainly based on more recent work by:

R. Baier, P. Romatschke, D. T. Son, A. O. Starinets and M. A. Stephanov

“Relativistic viscous hydrodynamics, conformal invariance, and holography”

JHEP 0804 (2008) 100 (arXiv: 0712.2451 [hep-th])

and

M. Luzum and P. Romatschke

“Conformal relativistic viscous hydrodynamics: applications to RHIC”

arXiv: 0804.4015 [nucl-th]

hydrodynamics

energy momentum tensor:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} + \Pi^{\mu\nu}$$

ϵ energy density and p pressure

u^μ fluid velocity, $u_\mu u^\mu = -1$, and $\Pi^{\mu\nu}$ shear tensor

with

$$u_\mu \Pi^{\mu\nu} = 0, \quad \Pi^\mu_\mu = 0$$

local conservation law (covariant derivative ∇_μ):

$$\nabla_\mu T^{\mu\nu} = 0$$

(assume: no net charge in the system)

approximation

only retaining shear viscosity terms

- Navier - Stokes $=$ first-order theory in gradients with kinetic equation:

$$\Pi^{\mu\nu} = -2\eta < \nabla^\mu u^\nu > \equiv -\eta\sigma^{\mu\nu}$$

with

$$\sigma^{\mu\nu} \equiv (\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$$

and projection

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Navier-Stokes

projection $\Delta^\mu_\alpha \nabla_\beta T^{\alpha\beta} = 0 \quad \rightarrow$
relativistic Navier-Stokes equation in first-order theory

$$(\epsilon + p) u^\alpha \nabla_\alpha u^\mu = \nabla^\mu p - \Delta^\mu_\alpha \nabla_\beta \left[-2\eta < \nabla^\alpha u^\beta > \right]$$

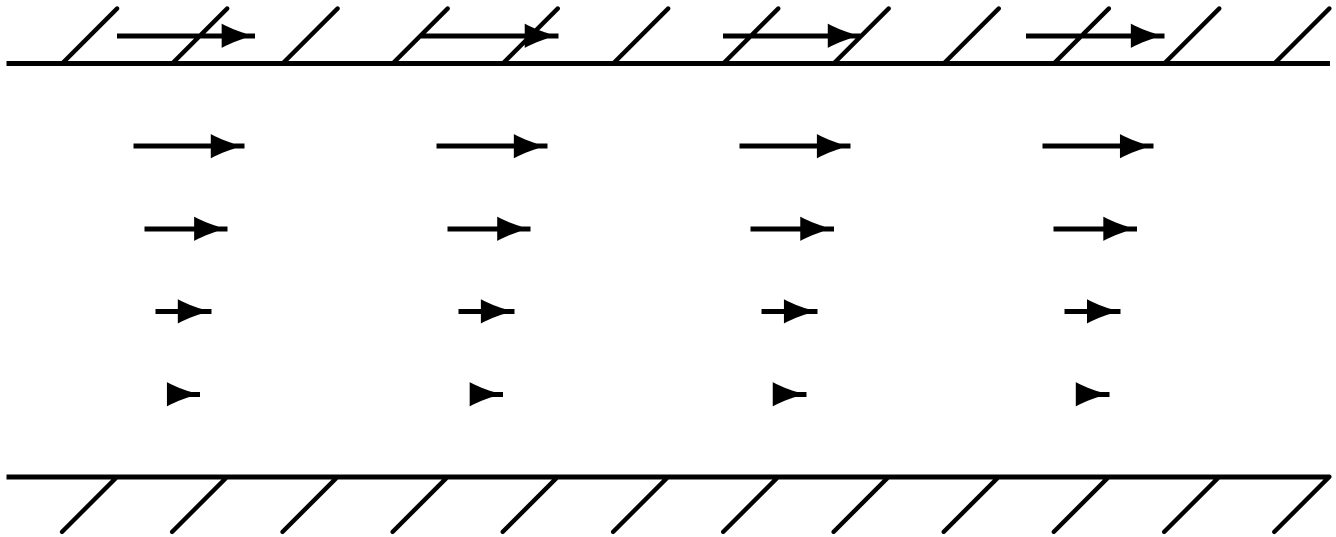
i.e. parabolic equation:

time derivative is of first order ($u^\alpha \nabla_\alpha \rightarrow \partial / \partial t$)
while
space derivative is of second order (∇^2)

“relativistic first-order dissipative theory is highly pathological, and therefore should be discarded in favor of the second-order one”

[Hiscock and Lindblom 1983-1985]

shear viscosity η



shear flow in $x-$ direction $\quad \rightarrow$

force per unit area/momentum transfer

$$\frac{F_x}{A} = -\Pi_{xz} = \eta \frac{\partial u_x}{\partial z}$$

transverse mode

small linear perturbation and first-order $\rightarrow$
diffusion equation in shear channel:

$$\delta u_\perp = \sqrt{\frac{(\epsilon+p)}{4\pi\eta t}} \exp\left[-\frac{(\epsilon+p)x^2}{4\eta t}\right]$$

i.e. propagates outside the light-cone (Gaussian)
starting from $\delta u_\perp(x, t=0) = \delta(x)$!!

minimal modification $\rightarrow$ second-order:
relaxation time $\tau_\pi > 0$ $\rightarrow$ hyperbolic equation

$$\left[\tau_\pi \partial_t^2 + \partial_t - \frac{\eta}{(\epsilon+p)}\partial_x^2\right] \delta u_\perp = 0$$

[talk by T. Kodama]

a new result

all second order terms classified by conformal symmetry,
Weyl transformation: $g^{\mu\nu} \to e^{2\omega(x)}\, g^{\mu\nu}$, $\quad T^{\mu\nu} \to e^{6\omega}\, T^{\mu\nu}, ...$

- causal viscous hydrodynamics -

$$
\begin{aligned}
\Pi^{\mu\nu} \quad = \quad & -\eta\sigma^{\mu\nu} + \eta\tau_\pi \left[\langle D\sigma^{\mu\nu}\rangle + \frac{1}{3}\sigma^{\mu\nu}(\nabla_\alpha u^\alpha) \right] \\
& +\kappa \left[R^{<\mu\nu>} - 2u_\alpha R^{\alpha<\mu\nu>\beta} u_\beta \right] \\
& +\lambda_1 \sigma^{<\mu}{}_\lambda \sigma^{\nu>\lambda} + \lambda_2 \sigma^{<\mu}{}_\lambda \Omega^{\nu>\lambda} + \lambda_3 \Omega^{<\mu}{}_\lambda \Omega^{\nu>\lambda}
\end{aligned}
$$

$R^{\alpha\beta\gamma\delta}$... Riemann tensor, $\quad R^{\alpha\beta}$... Ricci tensor
$\Omega^{\alpha\beta}$... antisymmetric vorticity tensor

[Baier, Romatschke, Son, Starinets, Stephanov 07]

[Bhattacharyya, Hubeny, Minwalla, Rangamani 07, Loganayagam 08]

Müller - Israel - Stewart theory

keeping esssentially one term in the derivative expansion
up to second order

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta\, \tau_\pi\, \langle D\sigma^{\mu\nu}\rangle \quad , \quad D = u \cdot \nabla$$

remark: does not match with AdS/CFT $\mathcal{N} = 4$ SYM

$$sound: \quad \tau_\pi = \frac{2 - \ln 2}{2\pi T}$$

$$Bjorken\ flow: \quad \tau_\pi = \frac{1 - \ln 2}{2\pi T}$$

$\Rightarrow$ all second-order terms consistent with
conformal symmetry have to be included !

relaxation phenomena/strong coupling limit

hydrodynamic transport coefficients by gauge/gravity duality:
near extremal black $D3-$ brane metric with horizon $r = r_0$

$$ds^2 = \frac{r^2}{R^2}(-f(r)dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f(r)}dr^2, \quad f(r) = 1 - \frac{r_0^4}{r^4}$$

compare with quite involved AdS/CFT-gravity calculations at Hawking temperature T for momentum $\omega, k << T = \frac{r_0}{\pi R^2}$, e.g. from sound channel dispersion up to $O(k^3)$, etc :

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad \tau_\pi = \frac{2 - \ln 2}{2\pi T}, \quad \kappa = \frac{\eta}{\pi T} = 2\lambda_1 \ ,$$

$$\lambda_2 = -\frac{\ln 2}{2\pi T}\eta \ , \quad \lambda_3 = 0$$

acoustic absorption

consider small linear perturbations $\delta\epsilon, \delta\vec{u}, \delta\Pi_{ij}$ with space-time dependence

$$\exp\left(-i\omega t + ikx\right)$$

long-wave length modes ($k \to 0$)

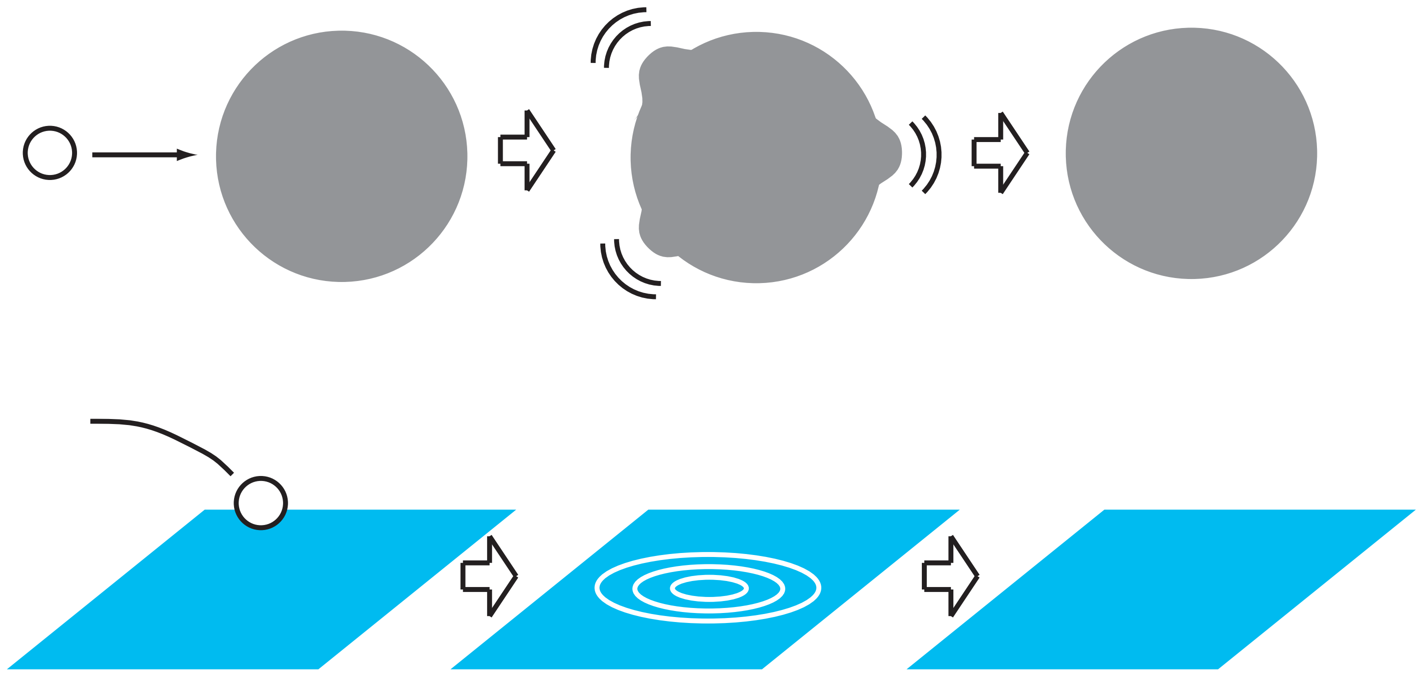
- two “longitudinal” (sound channel) modes

$$\omega \simeq \pm c_s k - \frac{i\Gamma_s}{2}k^2 \pm \frac{\Gamma_s}{2c_s}(c_s^2\tau_\pi - \frac{\Gamma_s}{4})k^3 + O(k^4)$$

sound velocity $c_s = \frac{1}{\sqrt{3}}$

attenuation length $\Gamma_s = \frac{4\eta}{3(\epsilon+p)} = \frac{4\eta}{3Ts}$

relaxation time τ_π



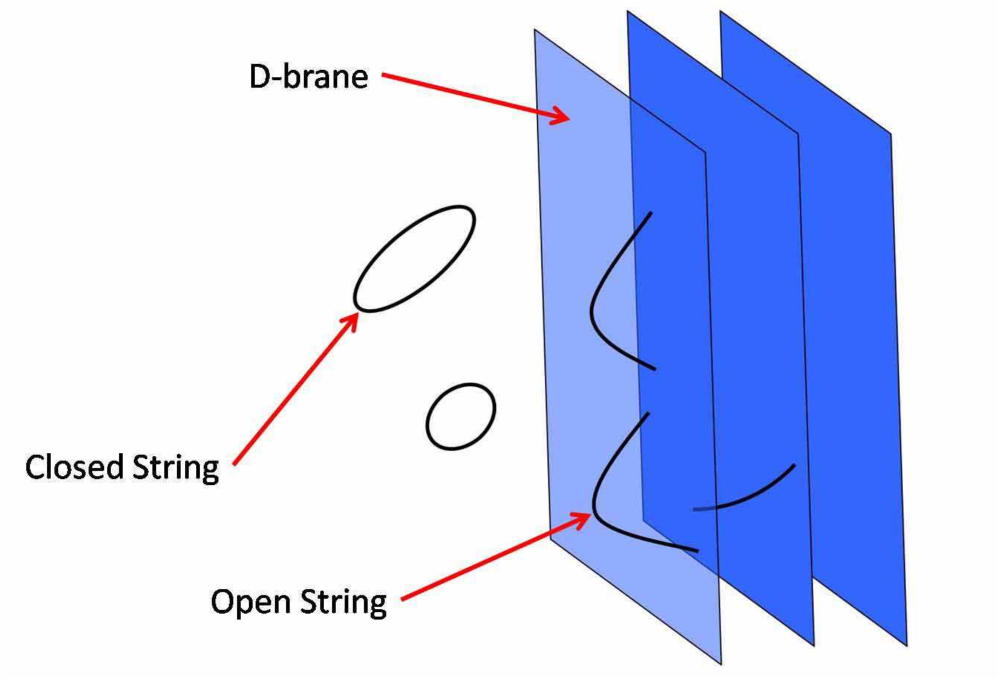
[from M. Natsuume]

quasinormal modes:

gravitational perturbation to a black hole

and to a hydrodynamic system

$D-$branes



[from Myers and Vazquez 08]

dynamical walls on which strings can end:

theory of open strings living on $D3-$branes ($\mathcal{N} = 4$ SYM, $d = 4$) $\Longleftrightarrow$ gravity theory of fields living in the space curved by the branes ($AdS_5,\ d = 5$)

AdS/CFT [Maldacena 98]

strongly coupled quantized conformal gauge theory
in $d = 4$ dimensions ($\mathcal{N} = 4$ SYM with $8N_c$ (1 gauge and 6 scalar) bosons and ($4N_c$) Weyl fermions) [[NOT QCD !]]

$$\Longleftrightarrow$$

weakly coupled classical supergravity (type IIB)
in $d = 10$ dimensions (on $AdS_5 \times S^5$)
via holographic property: radial coordinate $r_0 \leq r < \infty$ with gauge theory on the boundary at ∞
in the limit:
’t Hooft coupling $\lambda = g^2_{YM} N_c$ is large, $N_c \to \infty, \;\; g^2_{YM} << 1$
i.e. string coupling $g_s = \frac{g^2_{YM}}{4\pi} << 1$ – NO LOOPS

and

small curvature $\frac{l_s^4}{R^4} = \frac{1}{\lambda} << 1$ – RADIUS R of CURVATURE is LARGE compared to the STRING SCALE l_s

Bjorken flow

boost-invariant (irrotational) $1+1$ flow [Bjorken 83]
second-order equations (proper time τ, Φ... viscous flow):

$$\partial_\tau \epsilon = -\frac{4\epsilon}{3\tau} + \frac{\Phi}{\tau}$$

$$\tau_\pi \partial_\tau \Phi = \frac{4\eta}{3\tau} - \Phi - \frac{4\tau_\pi}{3\tau}\Phi - \frac{\lambda_1}{2\eta^2}\Phi^2$$

non-linear term NOT in MIS theory! [BRSSS 07]

compare with AdS/CFT calculation:

$$\frac{\lambda_1 T}{\eta} = \frac{1}{2\pi}\left[1 + \frac{215\,\zeta(3)}{8}\lambda^{-3/2} + \ldots\right]$$

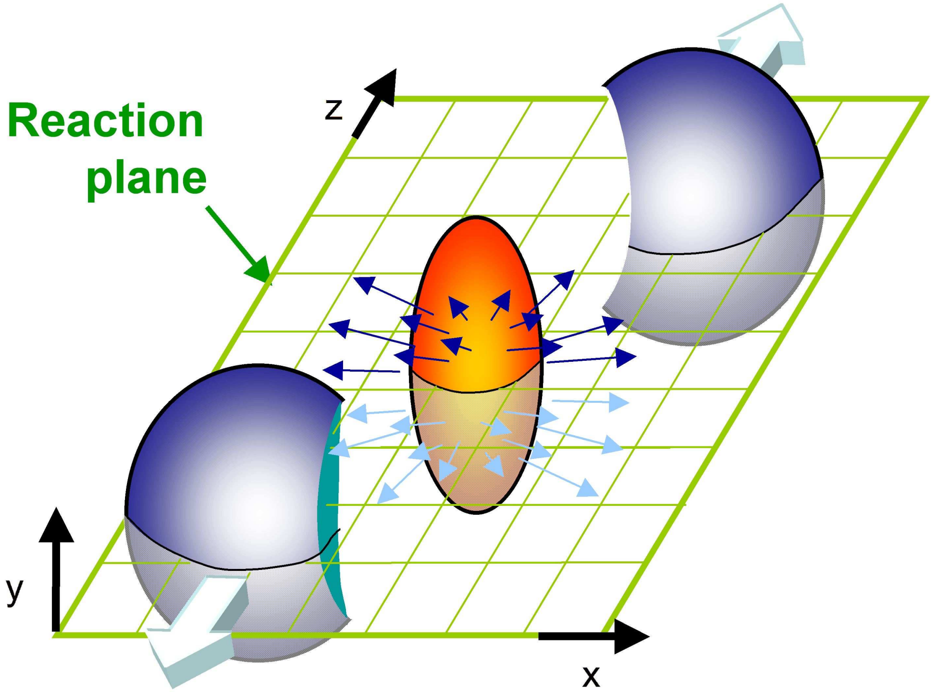
[Janik, Peschanski, Heller 06; Buchel, Paulos 08]

hydro $+$ RHIC

heavy-ion collisions require beyond hydrodynamics:

- hydrodynamics $=$ differential equations
  initial conditions !
- initial $=$ equilibration time
- distribution of energy density (Glauber? CGC ?)
- QCD equation of state
- hadronisation prescription (Cooper-Frye?)

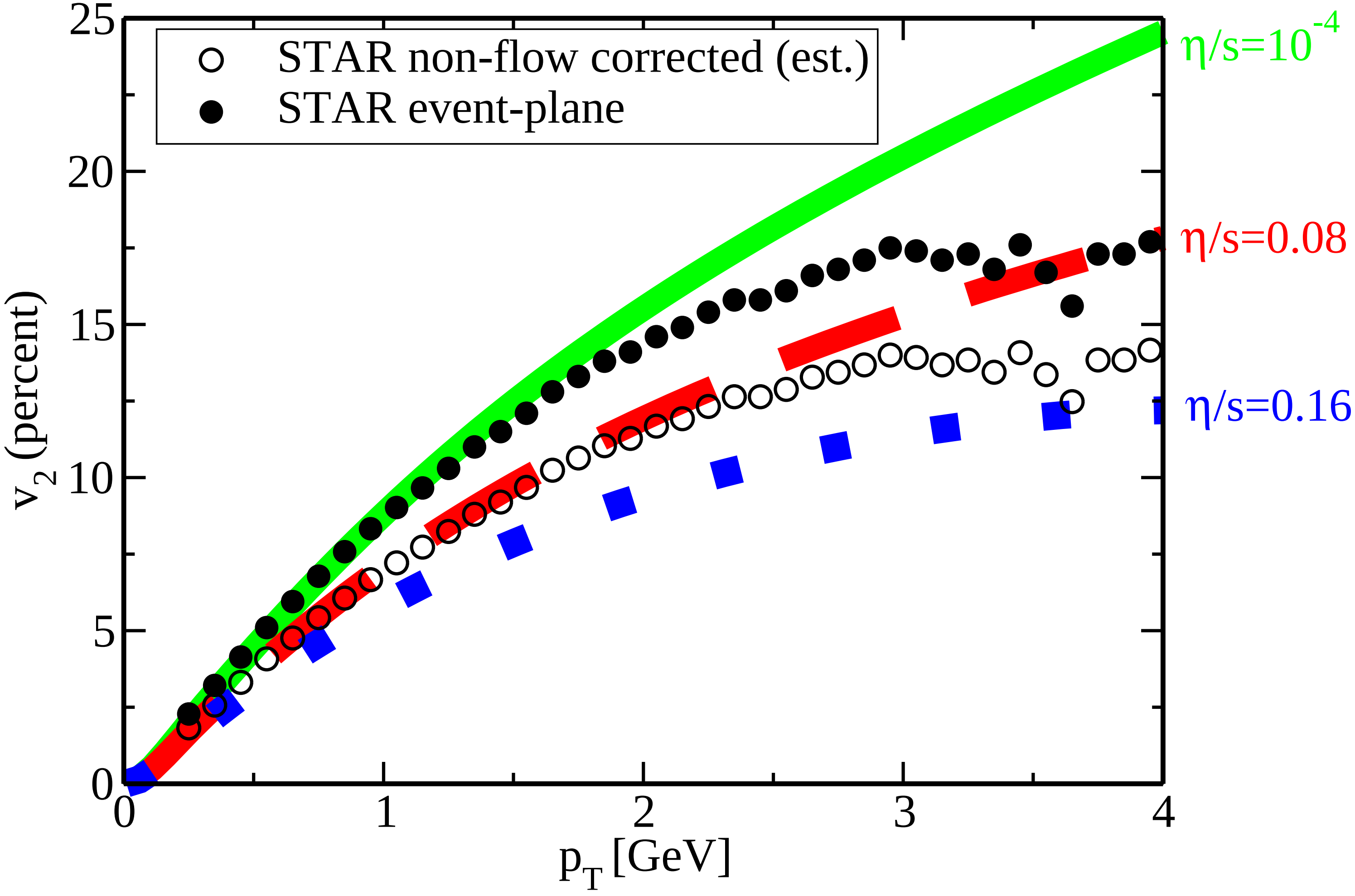
as important example



spatial asymmetry

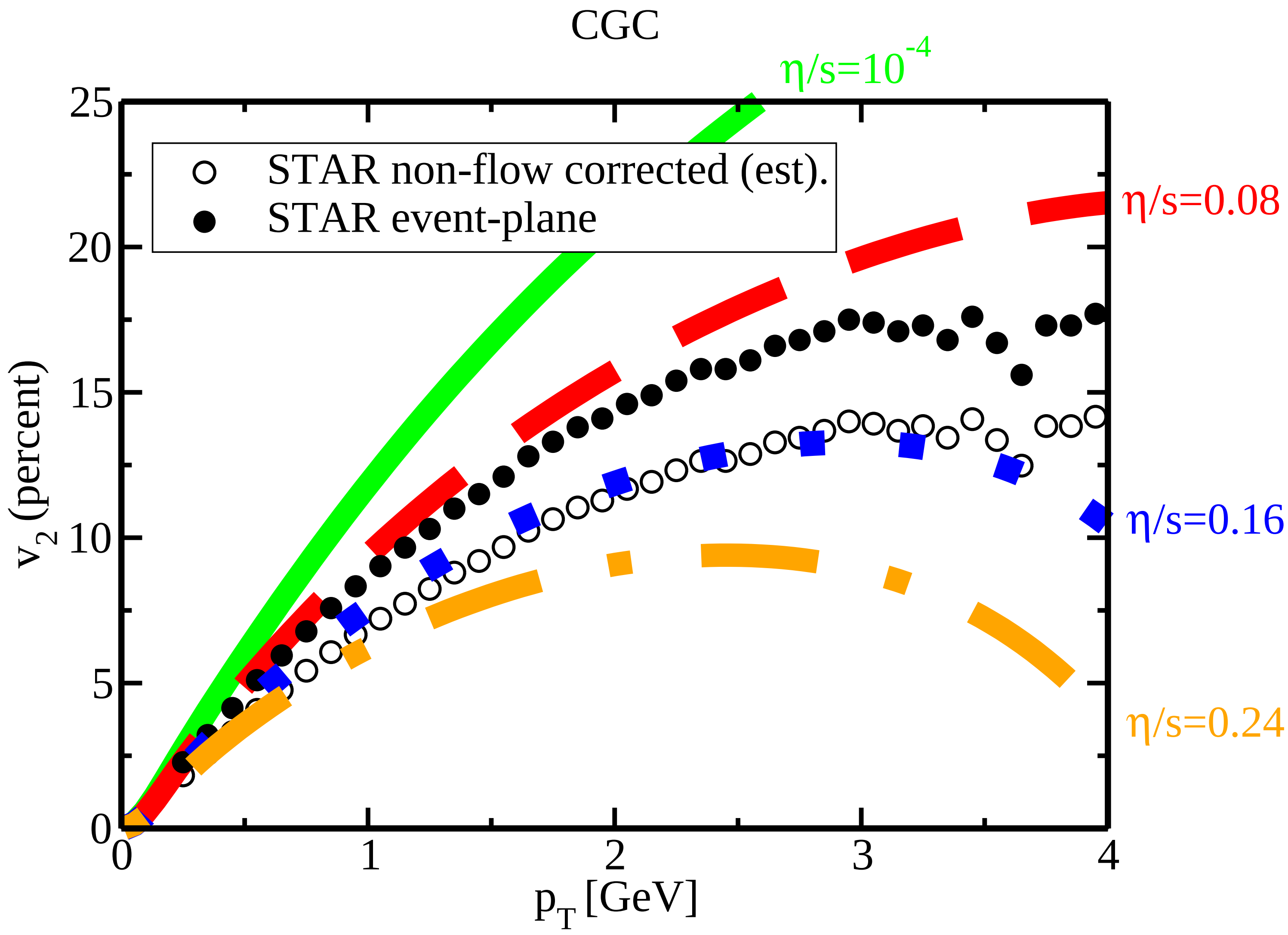
- $$\frac{dN}{dy dp_\perp d\phi} = \langle \frac{dN}{dy dp_\perp d\phi} \rangle_\phi \left(1 + 2 v_2(p_\perp) \cos(2\phi) + \ldots \right)$$
- elliptic flow: $v_2(p_\perp)$

Glauber



elliptic flow

[Luzum and Romatschke 08]



elliptic flow

[Luzum and Romatschke 08]

main results [Luzum and Romatschke 08]

- viscous hydrodynamics simulation give a good description of RHIC data for

$$\frac{\eta}{s} = 0.1 \pm 0.1(theory) \pm 0.08(experiment)$$

- modest estimate:

$$\frac{\eta}{s} < 0.5$$

- almost no dependence on the values of the second-order parameters $\tau_\pi, \lambda_1, ..$

- early thermalisation time is questioned, but

$$\tau_0 < 2\ fm$$

shear viscosity η

near equilibrium: $\eta \;\simeq\; \epsilon\bar{v}\lambda_f, \quad entropy\ density\ \ s \simeq \epsilon/m \quad \rightarrow$

$$\frac{\eta}{s} \simeq m\bar{v}\lambda_f \simeq \hbar\,\frac{mean\ free\ path}{deBroglie\ wavelength}$$

- dilute system (QFT $->$ kinetic theory $->$ hydro):
  scale $\lambda_f \;\rightarrow \frac{\eta}{s} >> \hbar$, e.g. pQCD $(N_f = 0)$

$$\frac{\eta}{s} \simeq 3.8\frac{1}{g^4\ln(2.8/g)} \simeq O(1)\ \ for\ g = 2.5$$

BUT with $\ln(2.8/g) \simeq O(1) : \frac{\eta}{s} \simeq 0.1 \quad \rightarrow$ sensitive to constant under the log !

[Arnold, Moore and Yaffe 03]

- strongly coupled system (QFT $->$ hydro):
  scale $1/T \;\rightarrow \frac{\eta}{s} = \frac{\hbar}{4\pi} \simeq 0.08$

[Policastro, Son and Starinets 01]

transport coefficients

kinetic Müller-Israel-Stewart theory (weak coupling):

$$\frac{\eta/s}{\tau_\pi T} = 1/6 = 0.167,\ \kappa = \lambda_1 = \lambda_3 = 0,\ \lambda_2 = -2\tau_\pi \eta$$

finite 't Hooft coupling $\lambda \equiv g_{YM}^2 N_c, \quad \lambda >> 1$ corrections to coefficients by gauge/gravity duality, e.g.:

[Buchel and Paulos 08]

$$\frac{\eta/s}{\tau_\pi T} = 0.383\ (1 - 3.52\ \lambda^{-3/2} + ...),$$

$$\kappa = \frac{\eta}{\pi T}\ (1 - \frac{145\zeta(3)}{8}\lambda^{-3/2} + ...)$$

excitement about gauge/gravity correspondence:

mainly to gain intuition into STRONG COUPLING

ASKING FOR MORE:

Is there an experiment whose outcome could cast strong doubts on the relevance of AdS/CFT to understand QCD ?

[P. Jacobs 08]

JET PHYSICS ? [talk by E. Iancu]

MOST CHALLENGING TASK of the theory is to find the microscopic mechanism for the rather RAPID EQUILIBRATION of matter in RHIC collisions

EXTRAS

COMPARISON

	QCD	$\mathcal{N}$=4 SYM
T=0	N_c=3=N_f , confinement, discrete spectrum, scattering, ...	N_c large, N_f/N_c small, deconfined, conformal, supersymmetric, ...
	very different !!	
$T>T_c$	strongly-coupled plasma of gluons & fundamental matter deconfined, screening, finite corr. lengths, . . .	strongly-coupled plasma of gluons & adjoint and fundamental matter deconfined, screening, finite corr. lengths, . . .
	very similar !!	
$T>>T_c$	runs to weak coupling	remains strongly-coupled
	very different !!	

QCD and $\mathcal{N} = 4\ SYM$ as a function of temperature

[from Myers and Vazquez 08]

master formula for AdS/CFT

schematically in terms of coinciding partition functions:

$$\int e^{iS_{4d}^{gauge}+\Phi_0 O} = \int e^{iS_{5d}[\Phi]} \simeq e^{iS^{classical}[\Phi_0]}$$

S_{5d} is computed with non-trivial boundary condition

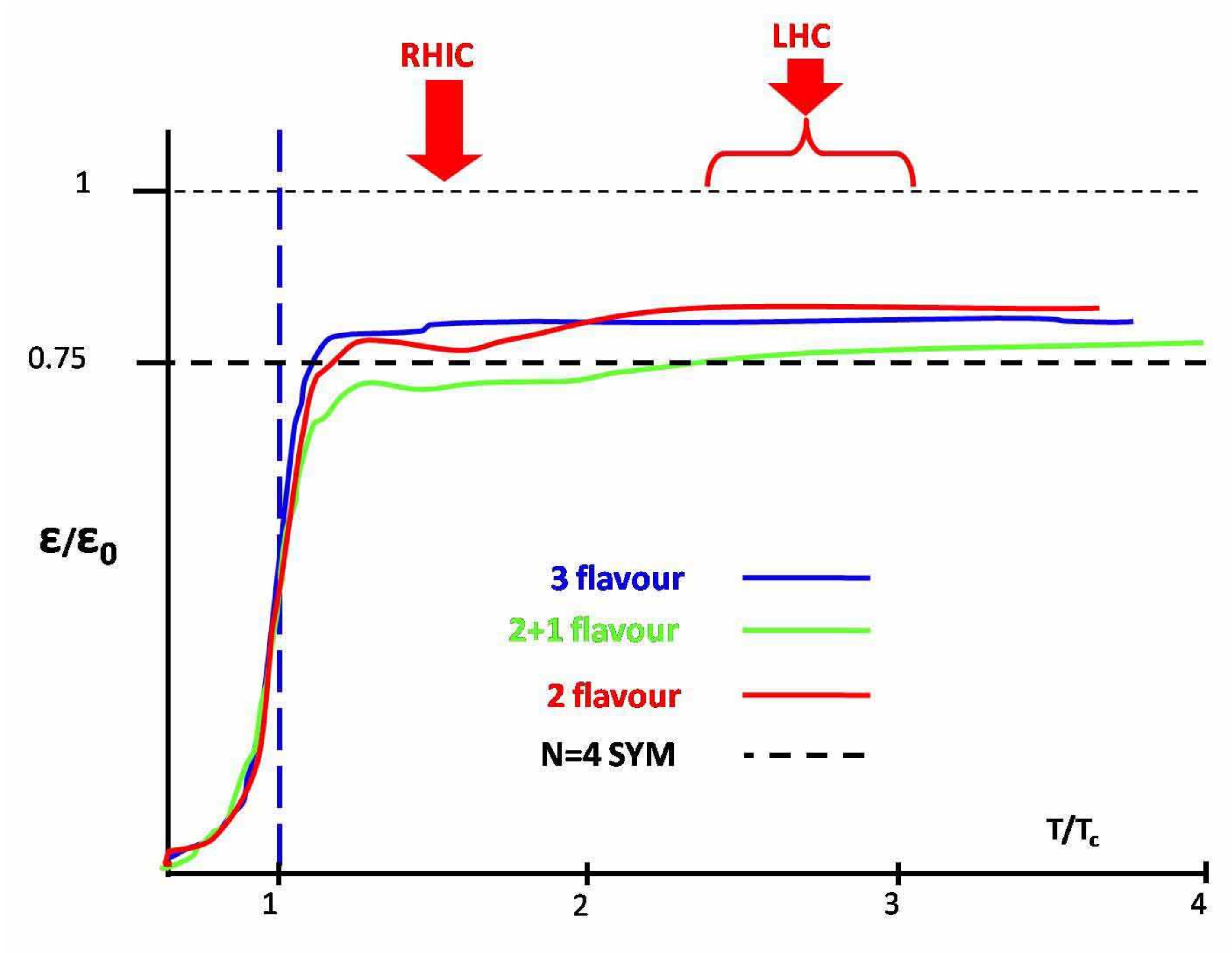
$$\Phi(t,\vec{x},r) \stackrel{r->\infty}{=} \Phi_0(t,\vec{x})$$

$$\Longrightarrow$$

quantum correlation $=$ classical two-point function

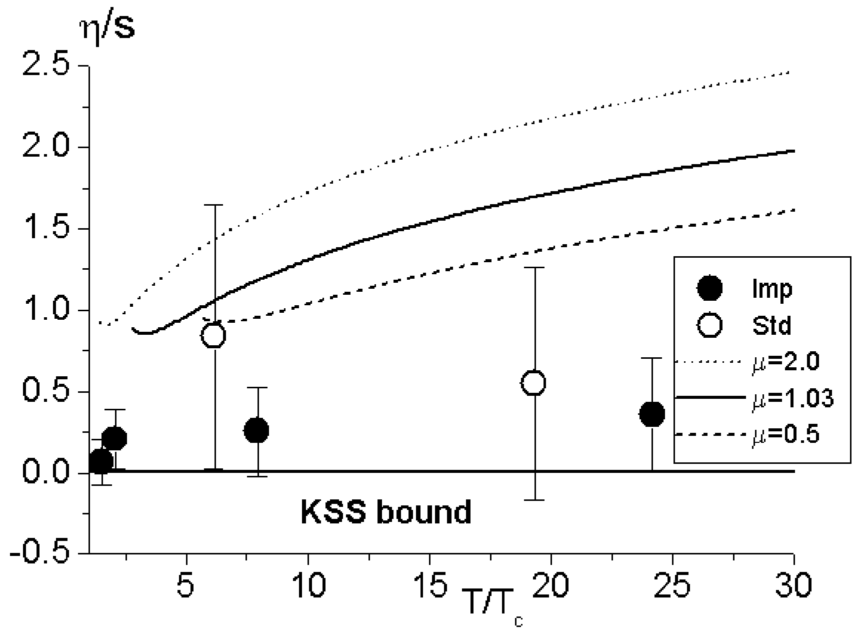
$$< TO(x)O(y) >= \frac{\delta^2 S^{classical}}{\delta\Phi_0(x)\delta\Phi_0(y)}|_{r=\infty}$$

gauge: $O = T_{\mu\nu}$ .. energy-momentum tensor
gravity: $\phi = g_{\mu\nu}$ .. graviton



energy density of QCD and SYM - via BH entropy

[from Myers and Vazquez 08]



η/s from quenched QCD lattice simulations [Nakamura and Sakai 05]

compared with the perturbative result

for strong coupling $\lambda = g_{YM}^2 N_c$ [Kovtun, Son and Starinets 05]
AdS/CFT universal (?) lower bound (times corrections)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \frac{5}{16}\frac{\lambda^{1/2}}{N_c^2} \pm \ldots\right]$$

[Buchel 08, Myers, Paulus and Sinha 08]

entropy current

Israel - Stewart 79: $s^\mu = (s - \frac{\tau_\pi}{4\eta T}\Pi_{\alpha\beta}\Pi^{\alpha\beta})u^\mu$

$\Rightarrow$ second law:

$$\nabla_\mu s^\mu = \frac{\Pi_{\alpha\beta}\Pi^{\alpha\beta}}{2\eta T} \geq 0$$

instead in causal viscous and conformal hydrodynamics
a more general current in terms of u^μ and its derivatives:

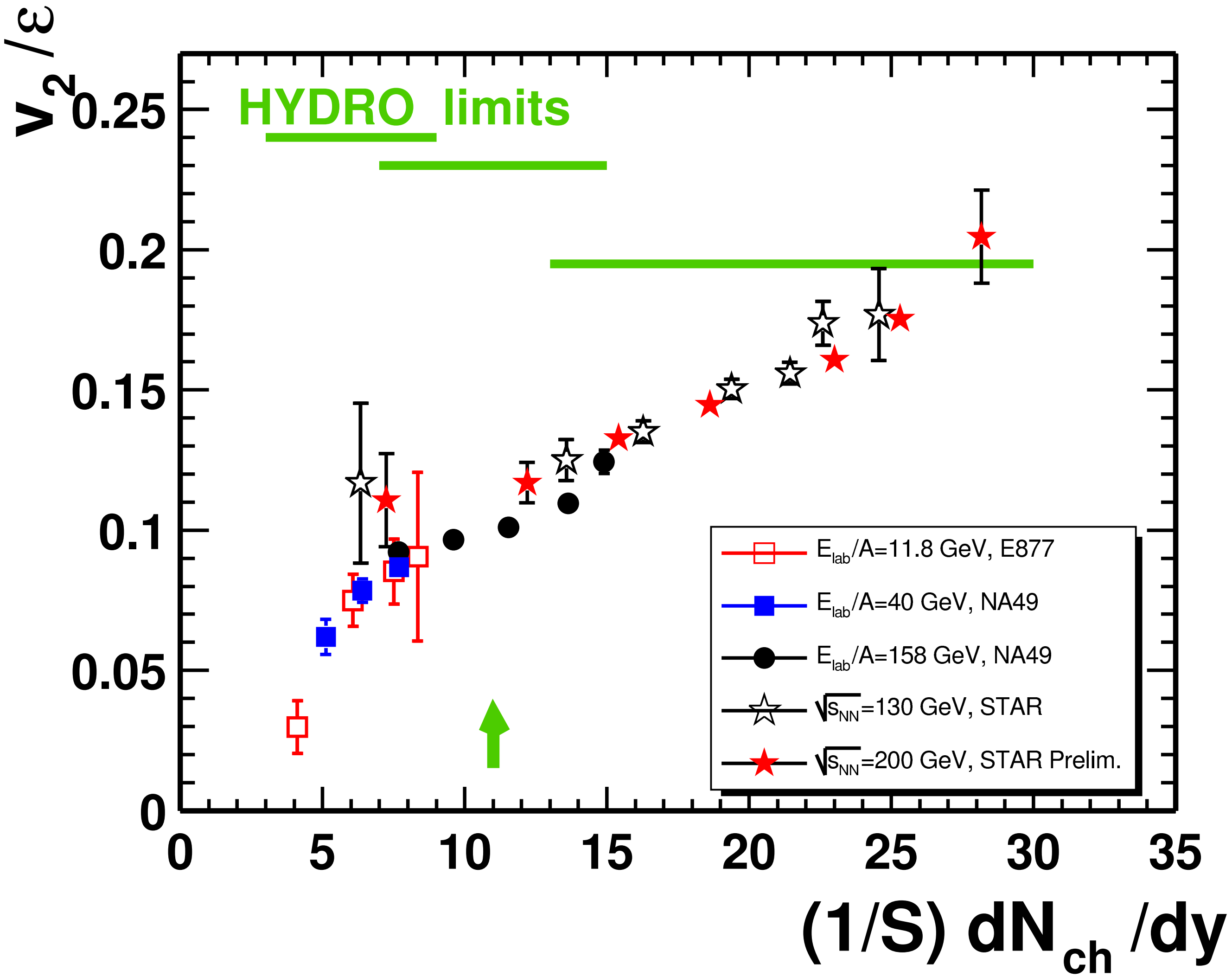
$$s^\mu = s u^\mu + (\sharp\, \sigma^2 + \sharp\, \Omega^2)u^\mu + O(u\nabla^2 u)$$

with

$$\nabla_\mu s^\mu = \frac{\eta}{2T}\sigma^{\mu\nu}\sigma_{\mu\nu} + \frac{1}{4T}(\kappa - 2\lambda_1)\ \sigma^\mu_\nu \sigma^\nu_\lambda \sigma^\lambda_\mu$$

in $\mathcal{N} = 4$ SYM: $\kappa = 2\lambda_1$

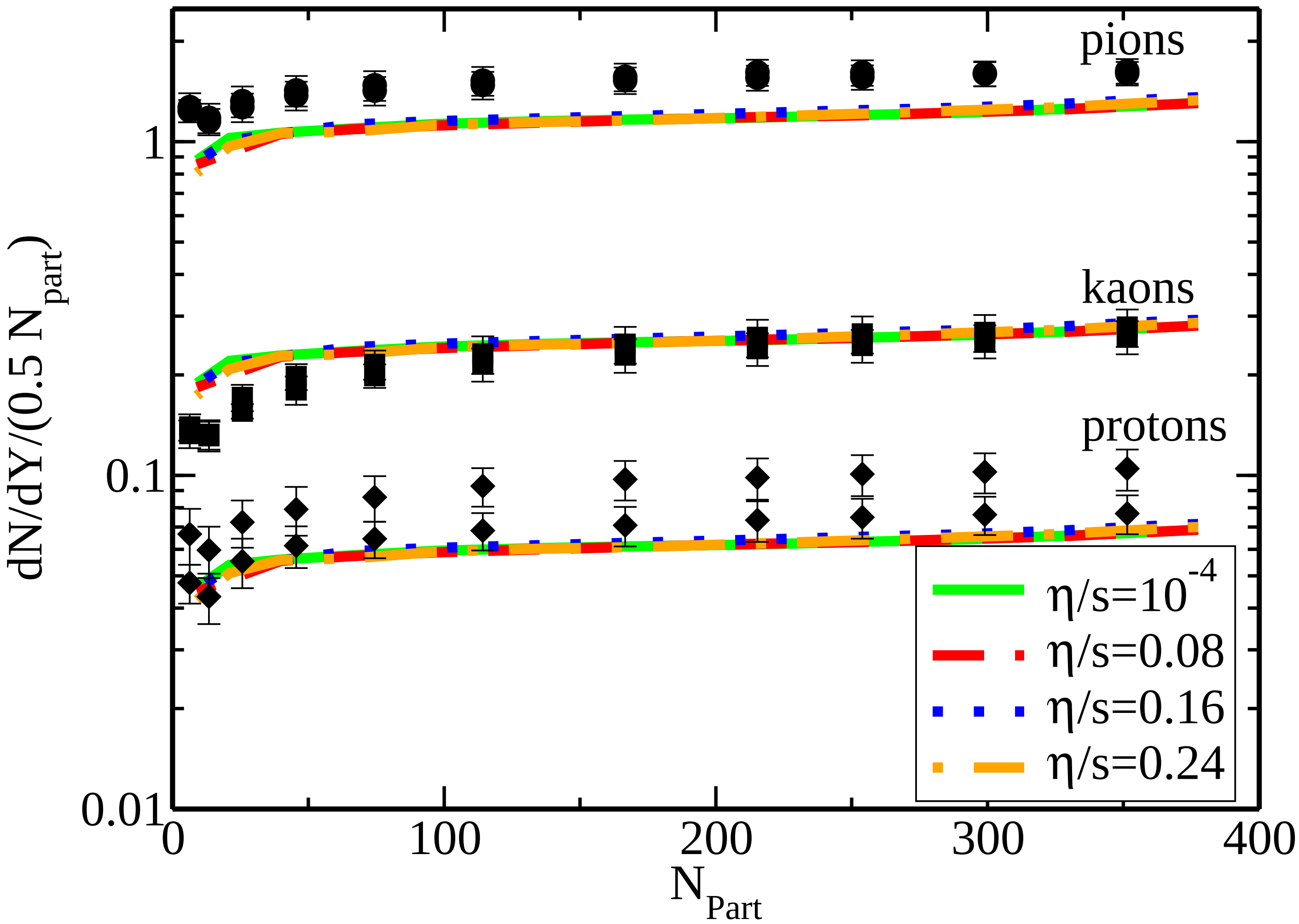
[Loganayagam 08]



v_2 : experiment vs. perfect hydro

[Heinz 04]

CGC



multiplicity

[Luzum and Romatschke 08]

equilibration time

REMINDER:

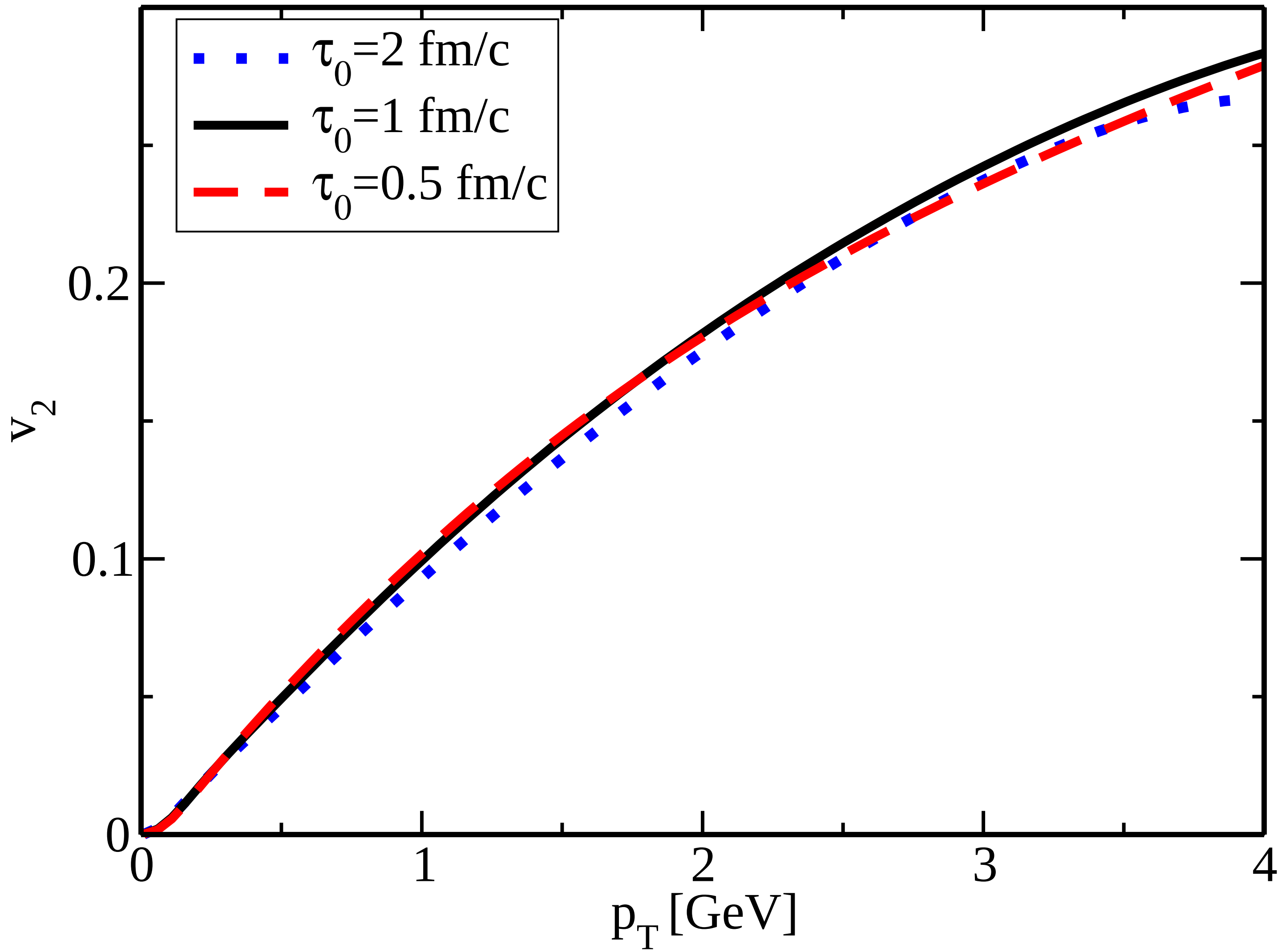
claim of short $\tau_0 \equiv \tau_{eq} \leq 0.5$ fm at RHIC

OPEN QUESTION:

$$CGC\ (\alpha_s << 1)\ \rightarrow\ sQGP\ (\alpha_s > O(1))$$

within a very short time < 0.5 fm ?

(b)



early thermalisation ?

[Luzum and Romatschke 08]

parametric pQCD estimate

for thermalisation in an expanding gluonic medium

near equilibrium at $T(\tau)$: Knudsen number *Kn*

$$\frac{1}{Kn} = \frac{longitudinal\ expansion\ time}{mean\ free\ path} = \frac{\tau}{\lambda_f} >> 1$$

from many gluon interactions (including saturation):

Arnold et al.: $\tau_{eq} Q_s \geq \alpha_s^{-7/3}$

'bottom-up' [Baier, Son, Mueller and Schiff 01]

$$\tau_{eq} Q_s \geq \alpha_s^{-13/5}$$

RHIC: $\tau_{eq} \geq 2 - 3$ fm