Transport Coefficients for Non Newtonian Fluids

Tomoi Koide and Takeshi Kodama

Univ. Rio de Janeiro

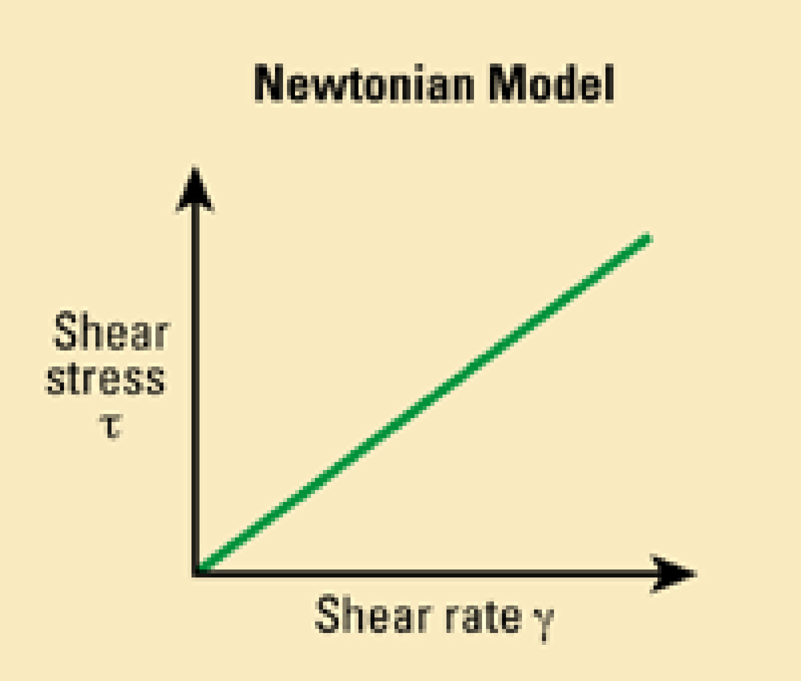
Outline

- What is Non Newtonian ?
- Viscosity and Relativistic Hydro
- Memory Effects vs. Israel-Stewart thoery
- Stability
- Extension of Green-Kubo-Nakano Formula
- Possible problems in Viscosity Coefficient for RHIC physics

Non Newtonian Fluid

Newtonian:

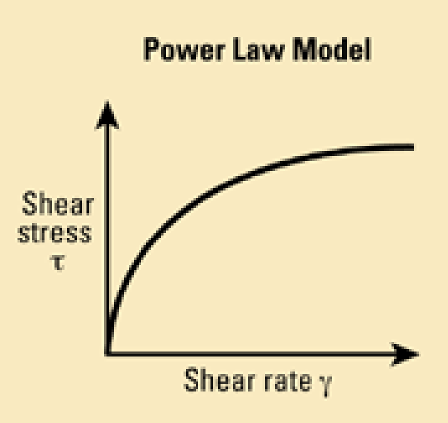
Thermodaynac force ∝ irreversible current

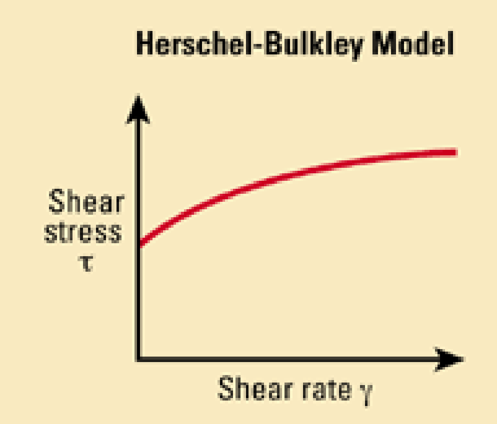


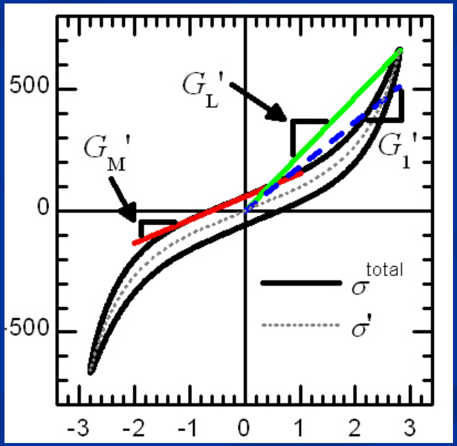
Non Newtonian Fluid

NonNewtonian:

Thermodaynac force $\not\propto$ irreversible current

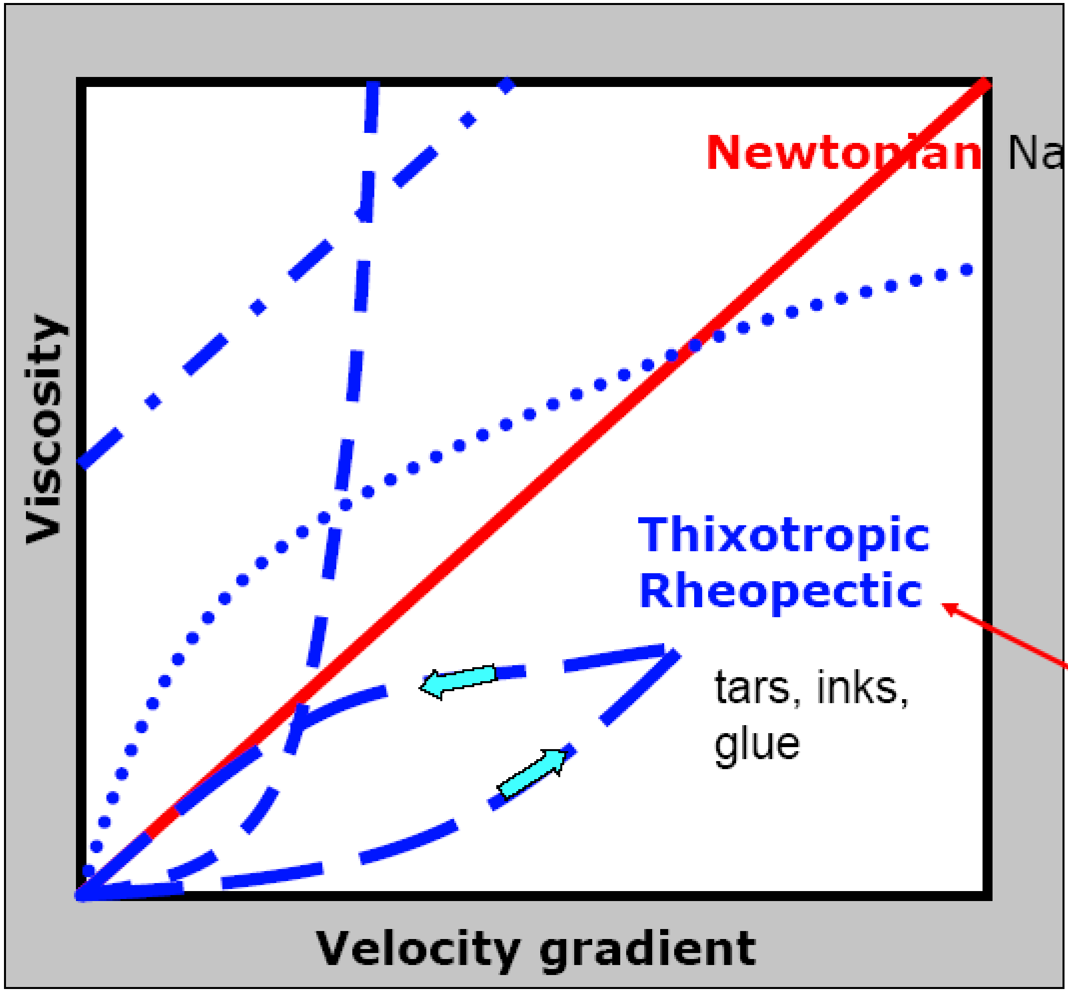






Anomalous viscosity

Pseudoplastic latex, paper pulp, clay solns.



Newtonian Navier-Stokes

Dilatant

quicksand,
candy compou

Bingham flow

sludge, paint,
blood, ketchup

QGP

IN RELA-
TIVISTIC
REGIME ?

Viscosity in Relativistic Regime

Non Newtonian (memory) effect is necessary to be consistent with the causal propagation of sound. Also non-linear term for stabilities.

T.Koide, G.S. Denicol, Ph. Mota and T.K, Phys.Rev. C75: 034909, 2007
G.S. Denicol, T. Kodama, T. Koide and Ph. Mota, PhysRevC.78.034901, 2008
G.S. Denicol, T. Kodama, T. Koide and Ph. Mota, J. Phys.G, 2008, in press

Ideal Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0,$$

$$\partial_\mu N^\mu = 0.$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - pP^{\mu\nu}, \qquad P^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$N^\mu = nu^\mu.$$

5 Independent variables, 5 equations.

$$\varepsilon,\ n,\ u^\mu$$

If Equation of State is given to calculate *p* and *T*, etc.

Viscous Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0,$$

$$\partial_\mu N^\mu = 0.$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - P^{\mu\nu}\left(p + \Pi\right) - P^{\mu\nu\alpha\beta}\pi_{\alpha\beta},$$

$$N^\mu = n u^\mu - P^{\mu\nu} \nu_\nu .$$

New Variables

$$\varepsilon,\ n,\ u^\mu,\ \Pi,\ \pi^{\mu\nu},\nu^\mu$$

Viscous Hydrodynamics

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Projection Operators

New Variables

$$\varepsilon,\ n,\ u^\mu,\ \Pi,\ \pi^{\mu\nu},\nu^\mu$$

Entropy Production

From $$\partial_\mu T^{\mu\nu} = 0,\ \partial_\mu N^\mu = 0.$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - P^{\mu\nu}\left(p + \Pi\right) - P^{\mu\nu\alpha\beta}\pi_{\alpha\beta},$$

$$N^\mu = n u^\mu - P^{\mu\nu} \nu_\nu .$$

Local Entropy production rate :

$$\partial_\mu \left(s u^\mu - \alpha \nu^\mu\right) =$$

$$\frac{1}{T}\left(-\Pi P^{\mu\nu}\partial_\mu u_\nu + \pi^{\mu\nu} P_{\mu\nu}{}^{\alpha\beta}\partial_\mu u_\nu\right) + \nu^\mu \partial_\mu \alpha$$

Landau Prescription

$$\frac{1}{T}\left(-\Pi P^{\mu\nu}\partial_{\mu}u_{\nu} + \pi^{\mu\nu}P_{\mu\nu}{}^{\alpha\beta}\partial_{\mu}u_{\nu}\right) + \nu^{\mu}\partial_{\mu}\alpha \geq 0$$

$$\Pi \propto \partial_{\mu}u^{\mu},$$

$$\pi^{\mu\nu} \propto \partial^{\mu}u^{\nu},$$

→ Covariant description of Navier-Stokes Theory

$$\nu^{\mu} \propto \partial^{\mu}\alpha.$$

Run-away solution !!

W. A. Hiscock and L. Lindblom, Ann. Phys. (N.Y.) 151, 466 (1983)
G.S. Denicol, T. Kodama, T. Koide and Ph. Mota, J. Phys.G, 2008, in press

Diffusion Equation

$$\frac{\partial n}{\partial t} = \zeta \nabla^2 n$$

Infinite velocity of propagation

How to solve? Cattaneo (1953)

Transform the parabolic equation to hyperbolic one: Telegraphist's Equation

$$\frac{\partial n}{\partial t}+\tau_R\frac{\partial^2 n}{\partial t^2}=\zeta\nabla^2 n$$

$$v_{\max}=\sqrt{\frac{\zeta}{\tau_R}}$$

Physical Interpretation

- Origin of diffusion equation

Continuity equation: $$\frac{\partial n}{\partial t} + \nabla \bullet \vec{j} = 0,$$

Thermodynamical force $$\vec{F} \propto \nabla n,$$

Fick's law $$\vec{j} \propto \vec{F},$$

$$\frac{\partial n}{\partial t} \propto \nabla^2 n$$

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Thermodynamical force $$\vec{F} \propto \nabla n,$$ OK

Fick's law $$\vec{j} \propto \vec{F},$$??

$$\frac{\partial n}{\partial t} \propto \nabla^2 n$$

Should be..

$$\frac{d\vec{j}(t)}{dt} \propto \zeta \vec{F}(t)$$

Including the viscous force,

$$\frac{d\vec{j}}{dt} = -\frac{1}{\tau_R}\vec{j} + \frac{1}{\tau_R}\zeta\vec{F},$$

For $\tau_R \rightarrow 0$,

$$\vec{j} \rightarrow \zeta\vec{F}$$

Then ..

$$\frac{d\vec{j}}{dt} = -\frac{1}{\tau_R}\vec{j} + \frac{1}{\tau_R}\zeta\vec{F}, \qquad \frac{\partial n}{\partial t} + \nabla \bullet \vec{j} = 0,$$

$$\frac{\partial n}{\partial t} + \tau_R \frac{\partial^2 n}{\partial t^2} = \zeta \nabla^2 n$$

This is equivalento to the retardation effect

$$\vec{j}(t) = \int_{-\infty}^{t} G(t,t')\zeta \vec{F}(t')dt'$$

With memory kernel function

$$G(t,t') = \frac{1}{\tau_R} e^{-\frac{t-t'}{\tau_R}}$$

Retardation is also essential for the unitarity in the quantum mechanical derivation of diffusion equation (violation of sum-rule) !!!

T.Koide Phys. Rev. E72, 026135 (2005)

However, exist ambiguities

How to identify $\vec{j}$ and $\vec{F}$

For bulk viscosity

$$j \to \Pi, \quad F \to \zeta \partial_\mu u^\mu$$

$$j \to \frac{\Pi}{\sigma}, \quad F \to \frac{\zeta}{\sigma} \partial_\mu u^\mu, \qquad \sigma:$$ Density

$$j \to \sqrt{\frac{\tau_R}{\zeta T \sigma}} \Pi, \quad F \to \sqrt{\frac{\tau_R}{\zeta T \sigma}} \zeta \partial_\mu u^\mu, \qquad T:$$ temperature

However, exist ambiguities

How to identify $\vec{j}$ and $\vec{F}$

For bulk viscosity

$$j \to \Pi, \quad F \to \zeta \partial_\mu u^\mu$$ Truncated IS, Minimum Causal

$$j \to \frac{\Pi}{\sigma}, \quad F \to \frac{\zeta}{\sigma} \partial_\mu u^\mu,$$ Exensiveness of memory

$$j \to \sqrt{\frac{\tau_R}{\zeta T \sigma}} \Pi, \quad F \to \sqrt{\frac{\tau_R}{\zeta T \sigma}} \zeta \partial_\mu u^\mu,$$ Full IS theory

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Extensivity of Energy

Analogy: Magnetization energy $dE = H\ dM$

Magnetization in equilibrium $dM = \mu H$

Extensivity of Energy

Analogy: Magnetization energy

$$\underbrace{dE}_{\text{Extensive}} = \underbrace{H}_{\text{Intensive}} \; \overbrace{dM}^{\text{Extensive}}$$

Magnetization in equilibrium

$$dM = \mu H$$

Extensivity of Energy

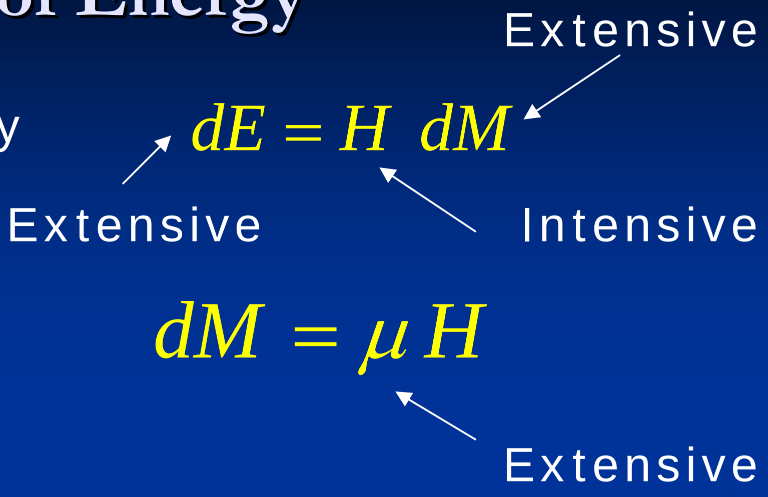
Analogy: Magnetization energy

$$\underset{\text{Extensive}}{dE} = \underset{\text{Intensive}}{H}\ \overset{\text{Extensive}}{dM}$$

Magnetization in equilibrium

$$dM = \underset{\text{Extensive}}{\mu}\, H$$

Extensivity of Energy



Analogy: Magnetization energy

Extensive

$$dE = H\ dM$$

Extensive

Intensive

Magnetization in equilibrium

$$dM = \mu H$$

Extensive

Presence of Histeresis

$$dM = \int^{t} dt' G(t,t') \mu H(t)$$

Extensivity of Energy

Analogy: Viscous energy

$$\underset{\text{Extensive}}{\underset{\nearrow}{E}} = \underset{\text{Intensive}}{\underset{\nwarrow}{\partial_\mu u^\mu}} \; \overset{\text{Extensive}}{\overset{\swarrow}{\frac{\Pi}{\sigma}}}$$

First order theory

$$\Pi = \underset{\text{Intensive}}{\underset{\nwarrow}{\zeta}} \, \partial_\mu u^\mu$$

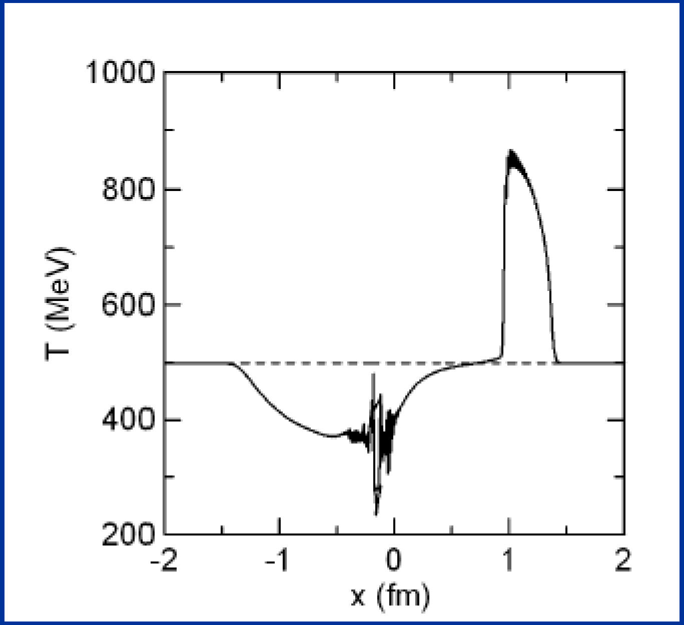
Memory in Extensive Quantity

$$\frac{\Pi}{\sigma} = \int^t dt' \, G\left(t - t'\right) \, \frac{\zeta}{\sigma} \partial_\mu u^\mu \left(t'\right),$$

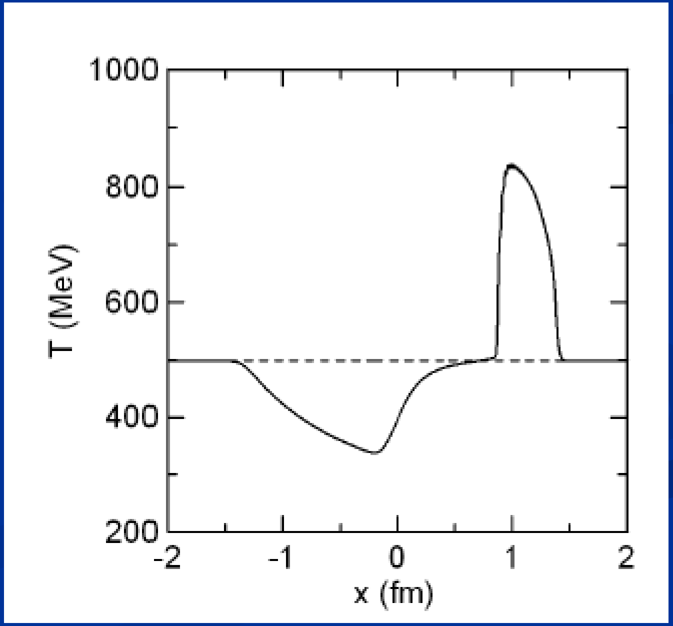
They are similar, but **NOT** equivalent.

Extensiveness in memory is proved to be always non-singular.

Example: 1D shock propagation



Minimum Theory



Extensiveness in Memory

Note that the so-called “full Israel-Stewart” theory in terms of memory function,

$$\sqrt{\frac{\tau_R}{\zeta T\sigma}}\Pi = \int_{-\infty}^{t} dt'\,G(t-t')\sqrt{\frac{\tau_R}{\zeta T\sigma}}\zeta\partial_\mu u^\mu, \qquad :$$

Whose thermodynamical interpretation is not clear. Also, the positivity of mass-tensor is not explicitly proven.

Microscopic approach

Is Green-Kubo-Nakano formula for viscosity consistent with non Newtonian behavior?

Linear response theory:

$$\left\langle J(t) \right\rangle = \int_{-\infty}^{t} dt\, \Xi(t-t')\, F(t')$$

$$\Xi(t) = \int_{0}^{\beta} d\lambda \left\langle A(-i\lambda)\, J(t) \right\rangle$$

where A is the operator corresponding to the required macroscopic observable.

Till here, exact within a linear response scheme

Usual Estimate of Viscous coefficient

$$J\left(t\right)=\eta F(t),$$

$$\eta \approx \int_{-\infty}^{\infty} dt\, \Xi\left(t-t'\right),$$

$$\Xi(t-t') \approx \eta\left(t\right)\delta\left(t-t'\right)$$

where memory effects are neglected (GKN formula).

Memory effects

$$J(t) = \int_{-\infty}^{t} dt' \Xi(t-t') F(t'),$$

$$\frac{dJ(t)}{dt} = \Xi(0) F(t) + \int_{-\infty}^{t} dt' \partial_t \Xi(t-t') F(t'),$$

Markov approximation

$$\frac{dJ(t)}{dt} = \Xi(0) F(t) + \int_{-\infty}^{+\infty} dt' \partial_t \Xi(t-t') F(t'),$$

Thus, we require 2 correlation functions,

$$\Xi_1(t) = \left\langle J(t)\, J(0) \right\rangle$$

$$\Xi_2(t) = \left\langle \rho(t)\, J(0) \right\rangle$$

and re-shuffles the definitions of transport coefficients !

Conclusion

- Viscous fluid behaves as non-Newtonian in relativistic regime.
- The second order theory, especially the form of the non-linear terms, is not well established yet.
- Boltzmann approach may not work for these terms.
- New derivation of GKN formula for non-Newtonian fluid is given.
- We have to be careful to identify viscosity and relaxation time for non-Newtonian fluids.