Saturation in lepton- and hadron induced reactions

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Abstract
Possible saturation of the matter density in two different classes of reactions, those induced by hadrons and leptons are studied. They may have common dynamical origin and be of the same nature.

1 Hadron-induced reactions: The Black Disc Limit at the LHC?

Unitarity in the impact parameter b representation reads:

$$\Im h(s,b) = |h(s,b)|^2 + G(s,b),$$

where $h(s,b)$ is the elastic scattering amplitude at the center of mass energy $\sqrt{s}$, $\Im h(s,b)$ is the profile function, representing the hadron opacity and $G(s,b)$, called the inelastic overlap function, is the sum over all inelastic channel contributions. Integrated over b, the above equation reduces to a simple relation between the total, elastic and inelastic cross sections $\sigma_{tot}(s) = \sigma_{el}(s) + \sigma_{in}(s)$.

Unitarity imposes the absolute limit

$$0 \le |h(s,b)|^2 \le \Im h(s,b) \le 1,$$

while the so-called black disc limit $\sigma_{el}(s) = \sigma_{in}(s) = \frac{1}{2}\sigma_{tot}(s)$, or

$$\Im h(s,b) = 1/2,$$

is a particular realization of the optical model, namely it corresponds to the maximal absorption within the eikonal unitarization, when the scattering amplitude is approximated as

$$h(s,b) = \frac{i}{2}(1 - \exp\left[\mathrm{i}\omega(\mathrm{s},\mathrm{b})\right]),$$

with a purely imaginary eikonal $\omega(s,b)$.

Eikonal unitarization corresponds to a particular solution of the unitarity equation, with $\Re h(s,b) = 0$,

$$h(s,b) = \frac{1}{2}\left[1 \pm \sqrt{1 - 4G_{in}(s,b)}\right],$$

the one with the minus sign.

An alternative solution, that with a plus sign in front of the square root, is known and realized within the so-called U-matrix approach, where the unitarized amplitude is a ratio rather than an exponential typical of the eikonal approach:

$$h(s,b) = \frac{U(s,b)}{1 - i\,U(s,b)},$$

where U is the input Born term, the analogue of the eikonal ω.

In the U-matrix approach, the scattering amplitude $h(s,b)$ may exceed the black disc limit as the energy increases. The transition from a (central) black disc to a (peripheral) black ring, surrounding a gray disc, for the inelastic overlap function in the impact parameter space corresponds to the transition from shadowing to antishadowing.

The impact parameter amplitude $h(s,b)$ can be calculated either directly from the data (where, however, the real part of the amplitude was neglected) or by using a particular model that fits the data sufficiently well.

In the dipole Pomeron (DP) model [1], logarithmically rising cross sections are produced with a Pomeron intercept equal to unity, thus respecting the Froissart-Martin bound.

Apart from the conservative Froissart-Martin bound, any model should satisfy also s-channel unitarity. We show that both the D-L and DP models are well below this limit and will remain so for long, in particular will so at the LHC.

The elastic scattering amplitude corresponding to the exchange of a dipole Pomeron reads

$$\begin{aligned} A(s,t) = & \frac{d}{d\alpha}\left[\mathrm{e}^{-i\pi\alpha/2}G(\alpha)(s/s_0)^\alpha\right] \\ = & \ \mathrm{e}^{-i\pi\alpha/2}(s/s_0)^\alpha[G'(\alpha)+(L-i\pi/2)G(\alpha)] \ , \end{aligned}$$

where $L \equiv \ell n\dfrac{s}{s_0}$ and $\alpha \equiv \alpha(t)$ is the Pomeron trajectory.

The elastic amplitude in the impact parameter representation in our normalization is

$$h(s,b) \ = \ \frac{1}{2s}\int_0^\infty dq\ qJ_0(bq)A(s,-q^2)\ , \qquad q=\sqrt{-t}\ .$$

The impact parameter representation for linear trajectories is calculable explicitly for the DP model. We have

$$h(s,b) \ = i\ g_0\ [e^{r_1^2\delta}\ e^{-b^2/4R_1^2}\ -\ \epsilon\ e^{r_2^2\delta}\ e^{-b^2/4R_2^2}]\ ,$$

where

$$R_i^2 = \alpha' r_i^2 \ \ (i=1,2);\ g_0 = \frac{a}{4b_p\alpha' s_0}\ .$$

Asymptotically (when $L \gg b_p$, i.e. $\sqrt{s} \gg 2.$ TeV, with the parameters quoted in Table 1) we get

$$h(s,b)_{s\to\infty} \to i\ g(s)\ (1-\epsilon)\ e^{-\frac{b^2}{4R^2}}\ ,$$

where

$$R^2 = \alpha' L \quad ;\ g(s) = g_0\left(\frac{s}{s_0}\right)^\delta\ .$$

It is important to note that the unitarity bound 1 for $Imh(s,b)$ will not be reached at the LHC energy, while the black disc limit 1/2 will be slightly exceeded, the central opacity of the nucleon being $\Im\mathrm{m}h(s,0)=0.54$.

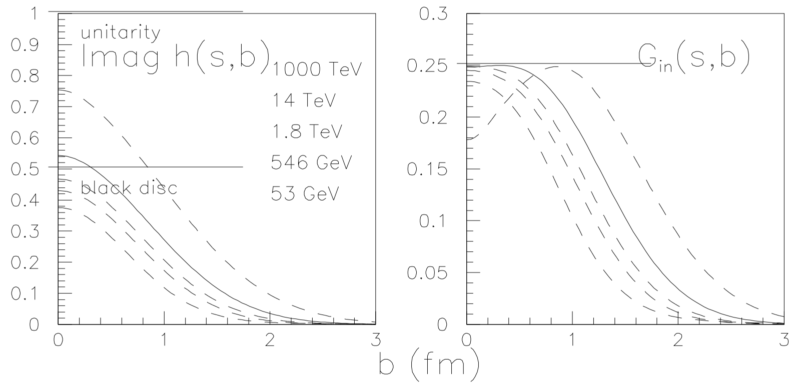

Fig. 1: A family of curves showing the imaginary part of the amplitude in the impact parameter-representation as well as the calculated inelastic overlap function $G(s,b)$ at various energies.

$\sqrt{s}$	53 GeV	546 GeV	1800 GeV
exp	0.36	0.420 ± 0.004	0.492 ± 0.008
th	0.36	0.424	0.461

Table 1: Central opacity of the nucleon $Imh(s,0)$ calculated at the ISR, SPS and Tevatron energies compared with experiment.

The black disc limit is reached at $\sqrt{s} \sim 2$ TeV, where the overlap function reaches its maximum $\frac{1}{4}$. While $Imh(s,b)$ remains central all the way, $G_{in}(s,b)$ is getting more peripheral as the energy increases starting from the Tevatron. For example at $\sqrt{s} = 14$ TeV, the central region of the antishadowing mode, obtained from the U matrix unitarization, below $b \sim 0.4$ fm is discernible from the peripheral region of shadowing scattering beyond $b \sim 0.4$ fm, where $G_{in}(s,b) = \frac{1}{4}$. The proton will tend to become more transparent at the center (gray, in the sense of becoming a gray object surrounded by a black ring).

The s channel unitarity limit will not be endangered until extremely high energies (10^5 for the Donnachie-Landshoff model and 10^6 GeV for the DP), safe for any credible experiment.

2 Lepton-induced reactions: DIS

An ansatz interpolating between the soft (VMD, Pomeron) Regge behavior and the hard (GLAP evolution) regime, given by the explicit solution of the DGLAP equation in the leading-log approximation, for the small-x singlet part of the proton structure function,

$$F_2 \approx \sqrt{\gamma_1 \ell n(1/x)\ \ell n \ell n Q^2}\ ,$$

with $\gamma_1 = \frac{16N_c}{(11-2f/3)}$ (for 4 flavours $(f = 4)$ and three colours $(N_c = 3)$, $\gamma_1 = 5.76$) was suggested in Ref. [2]

$$F_2^{(S,0)}(x,Q^2) = A\left(\frac{Q^2}{Q^2+a}\right)^{1+\widetilde{\Delta}(Q^2)} e^{\Delta(x,Q^2)},$$

with the "effective power"

$$\widetilde{\Delta}(Q^2) = \epsilon + \gamma_1 \ell n\left(1 + \gamma_2 \ell n\left[1 + \frac{Q^2}{Q_0^2}\right]\right),$$

and

$$\Delta(x,Q^2) = \left(\widetilde{\Delta}(Q^2)\ell n \frac{x_0}{x}\right)^{f(Q^2)},$$

where

$$f(Q^2) = \frac{1}{2}\left(1 + e^{-Q^2/Q_1^2}\right).$$

At small and moderate values of Q^2, the exponent $\widetilde{\Delta}(Q^2)$ can be interpreted as a Q^2-dependent effective Pomeron intercept.

By construction, the model has the following asymptotic limits:

b) Low Q^2, fixed x:

$$F_2^{(S,0)}(x,Q^2\to 0) \to A\, e^{\Delta(x,Q^2\to 0)} \left(\frac{Q^2}{a}\right)^{1+\widetilde{\Delta}(Q^2\to 0)}$$

with

$$\widetilde{\Delta}(Q^2\to 0) \to \epsilon + \gamma_1\gamma_2\left(\frac{Q^2}{Q_0^2}\right) \ \to\ \epsilon,$$

$$f(Q^2\to 0) \to 1,$$

whence

$$F_2^{(S,0)}(x,Q^2\to 0) \to A\ \left(\frac{x_0}{x}\right)^{\epsilon} \left(\frac{Q^2}{a}\right)^{1+\epsilon} \propto (Q^2)^{1+\epsilon} \to 0\ ,$$

as required by gauge invariance.

c) Low x, fixed Q^2:

$$F_2^{(S,0)}(x\to 0,Q^2) \ =\ A\left(\frac{Q^2}{Q^2+a}\right)^{1+\widetilde{\Delta}(Q^2)} e^{\Delta(x\to 0,Q^2)}.$$

If $f(Q^2) \sim 1$ i.e. when $Q^2 \ll Q_1^2$, we get the standard (Pomeron-dominated) Regge behavior (with a Q^2 dependence in the effective Pomeron intercept)

$$F_2^{(S,0)}(x\to 0,Q^2) \to A\ \left(\frac{Q^2}{Q^2+a}\right)^{1+\widetilde{\Delta}(Q^2)} \left(\frac{x_0}{x}\right)^{\widetilde{\Delta}(Q^2)} \propto x^{-\widetilde{\Delta}(Q^2)}.$$

Within this approximation, the total cross-section for (γ, p) scattering as a function of the center of mass energy W is

$$\sigma_{\gamma,p}^{tot,(0)}(W) = 4\pi^2\alpha \left[\frac{F_2^{(S,0)}(x,Q^2)}{Q^2}\right]_{Q^2\to 0} = 4\pi^2\alpha\, A\, a^{-1-\epsilon}\, x_0^\epsilon\, W^{2\epsilon}.$$

Accounting for large x :

$$F_2^{(S)}(x,Q^2) = F_2^{(S,0)}(x,Q^2)\,(1-x)^{n(Q^2)},$$

with

$$n(Q^2) = \frac{3}{2}\left(1+\frac{Q^2}{Q^2+c}\right),$$

where $c = 3.5489$ GeV^2.

The non-singlet (NS) part of the structure function is also included:

$$F_2^{(NS)}(x,Q^2) = B\,(1-x)^{n(Q^2)}\,x^{1-\alpha_r}\,\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_r}\,.$$

The free parameters that appear with this addendum are c, B, b and α_r. The final and complete expression for the proton structure function thus becomes

$$F_2(x,Q^2) = F_2^{(S)}(x,Q^2) + F_2^{(NS)}(x,Q^2)\,.$$

Of great interest are the slopes:

$$\frac{\partial F_2}{\partial(\ell n Q^2)} \text{ as a functionof } x \text{ and } Q^2$$

and

$$\frac{\partial \ell n F_2}{\partial(\ell n(1/x))}$$

as a function of Q^2 for x fixed, showing explicitly the onset of the saturation in x and Q^2, namely the inflection point near $Q^2 = 100$ GeV^2, followed by its flattening around $Q^2 = 4\times 10^3$ GeV^2 for $x \le 10^{-3}$ (see Figs. 5-7 in Ref. [2]).

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References

[1] See: R. Fiore, L. Jenkovszky, R. Orava, E. Predazzi, A. Prokudin, and O. Selyugin, *Forward physics at the LHC; Elastic scattering.* arXiv: hep-ph/0810.2902 and earlier references therein.

[2] P. Desgrolard, L. Jenkovszky, F. Paccanoni, EPJ **C7**, 263 (1999).