# Monte Carlo and large angle gluon radiation

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**Abstract**
I discuss the problem of incorporating recoil effects into the probabilistic QCD evolution scheme based on the picture of colour dipoles as done in recent Monte Carlo programs.

## 1 Introduction

Generation of events using Monte Carlo methods is an indispensable tool for planning, running and analysing the results of modern high energy experiments. A possibility to generate multiparticle production as a Markov chain of successive independent parton splittings is based on the general property of *factorization of collinear singularities*. For inclusive parton distributions this leads to the DGLAP evolution equations [1] whose generalisation to multi-particle distributions can be achieved in the spirit of jet-calculus [2], with additional account of soft gluon coherence [3–6]. This is done essentially by ordering the angles of successive gluon emissions, which ordering takes full care of the destructive interference contributions in the soft region and preserves the probabilistic parton multiplication picture. In particular, this procedure was applied to construct the HERWIG event generator [5].

An alternative way of dealing with soft gluon interference effects is provided by the "dipole scheme" [4, 7] in which an independently radiating *parton* is replaced by a colourless *dipole* formed by two partons neighbouring in the colour space. Gluon radiation off a dipole is automatically suppressed at angles exceeding the dipole opening angle thus reproducing the angular ordering. The dipole formulation offers a possibility to improve the treatment by taking into consideration logarithmically enhanced effects due to multiple emission of soft gluons at *large angles* with respect to jets. Non-collinear soft gluons dominate inter-jet particle flows in various hard processes. They also complicate the analysis of the so-called *non-global* QCD observables [8], i.e. in observables in which recorded radiation is confined in geometrically definite phase space regions. It is then interesting to involve these corrections into a Monte Carlo code based on dipole emission, including soft radiation away from jets.

In this talk, based on the paper [9] with Yuri Dokshitzer, I discuss a dipole scheme (in the large-$N_c$ approximation) well suited for deriving improved analytic predictions for observables that incorporate large-angle soft gluon radiation effects. However we observe that, once one aims at *beyond the no-recoil (soft) approximation*, treating colour dipoles as independently evolving entities is likely to conflict the collinear factorization. Taking energy ordering as natural from the multi-parton distributions in [4] one does not obtain the correct DGLAP equation. However considering transverse momentum ordering one obtains the DGLAP evolution to leading order with important non-leading corrections. But this ordering does not really reproduce the multiparton distributions [4].

## 2 Multiple soft gluons and Monte Carlo

Consider, for simplicity, the generation of quark-antiquark pair $p_a p_b$ plus an ensemble of $n$ secondary *soft* gluons $\gamma^* \to p_a p_b\, q_1\, q_2\, \dots q_n$. In the planar approximation [4, 10] the distribution is given by a sum of permutation of

$$\frac{(p_a p_b)}{(p_a q_1)\cdots(q_n p_b)}\,. \tag{1}$$

Selecting soft emission $\omega_i \ll E_a \simeq E_b \simeq E = Q/2$ (with $E_a, E_b$ and $\omega_i$ the c.m. quark antiquark and gluon energies) one obtains for the generating functional

$$E\partial_E \mathcal{G}(p_a, p_b; E) = \int \frac{d\Omega}{4\pi}\bar{\alpha}_s \frac{\xi_{ab}}{\xi_{aq}\xi_{qb}}\Big[u(q)\,\mathcal{G}(p_a, q;\omega)\mathcal{G}(q, p_b;\omega) - \mathcal{G}(p_a, p_b, \omega)\Big]_{\omega=E} \tag{2}$$

with $\bar{\alpha}_s = \frac{N_c \alpha_s}{\pi}$ and $\xi_{ik} = 1 - \cos\Theta_{ik}$. The "source functions" $u$ attached to each parton help to extract an arbitrary final state observable. The fully inclusive measurement, that is when one allows for production of any number of particles with arbitrary momenta, corresponds to setting all $u = 1$. This gives $\mathcal{G}(Q, u = 1) = 1$ corresponding to normalization to the total cross section. Iteration of this equation can be interpreted as parton branching which can be realised as a Monte Carlo (Markov) process. We now discuss first the multiplicities (large angle contributions are correctly included) then the distributions in which recoil is needed.

## 3 Mean multiplicity

In order to obtain an equation for the multiplicity of secondary partons, one applies to (2) the variational derivative over the probing function $u(q)$ and integrates over $q$, while setting $u \equiv 1$ for all remaining probing functions (one-particle inclusive measurement). We derive

$$E\partial_E \mathcal{N}(\xi_{ab}) = \int \frac{d\Omega}{4\pi}\,\bar{\alpha}_s\,\frac{\xi_{ab}}{\xi_{aq}\xi_{qb}}\Big[\mathcal{N}(\xi_{qa}) + \mathcal{N}(\xi_{qb}) - \mathcal{N}(\xi_{ab})\Big]. \tag{3}$$

This formulation leads to the following results:

- resummation of collinear logs and large angle corrections are correctly included here;
- large angle $Q\bar{Q}$ emission in which soft singularities are important are correctly resummed (see [11]);
- non-global jet corrections [8] are correctly treated by (2).

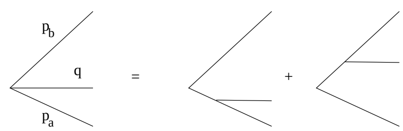
## 4 Attempt to include recoil in dipole multiplication

The dipole-based evolution equation (2) remains insufficient for building a realistic Monte Carlo event generator as we shall discuss in the following. The simplest observable to consider is the inclusive distribution of the final-state quark. This is obtained by studying the generating functional $\mathcal{G}_{ab}$ with $u = 1$ as function of the energy fraction $x$ of the final-state quark $p_a$ after the emission of any number of soft gluons $y_i$ (here $P_a$ is the incoming quark at the photon vertex)

$$p_a \simeq x\,P_a\,, \qquad x = 1 - \sum_i y_i\,, \qquad \omega_i = y_i\,E\,. \tag{4}$$

We shall restrict ourselves to configurations in which all radiated gluons have small emission angles with respect to the quark direction. This — quasi-collinear — approximation is sufficient for the analysis of the *anomalous dimension* which accumulates collinear singularities of the fragmentation function in all orders and describes the scaling violation. Here one has to consider recoil (see (4)). We start from the elementary process $P_a + P_b \implies p_a + p_b + q$. In order to properly formulate a recoil strategy, one must split the soft dipole radiation function into two pieces which incorporate the collinear singularity when $\vec{q}$ collinear to $\vec{p}_a$ or $\vec{p}_b$, respectively

$$\frac{\xi_{ab}}{\xi_{aq}\xi_{qp_b}} = \widehat{W}_{ab}^{(a)}(q) + \widehat{W}_{ab}^{(b)}(q)\,, \tag{5}$$

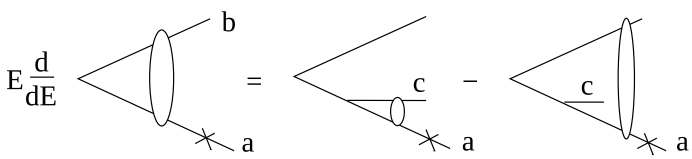



(6)

In the above final-state quark distribution one needs to consider only the splitting $\widehat{W}_{ab}^{(a)}(q)$ which is singular for $\xi_{aq} \to 0$. This distribution essentially becomes irrelevant when the emission angle $\xi_{aq}$ exceeds the opening angle of the parent dipole, $\xi_{aq} > \xi_{ab}$, that is away from the angular ordered kinematics. Upon averaging over the azimuthal angle $\phi_{qa}$ of the gluon momentum $\vec{q}$ around the singular direction $\vec{p}_a$ one has

$$\int \frac{d\phi_{aq}}{2\pi}\widehat{W}_{ab}^{(a)}(q) = \frac{1}{\xi_{qa}} \cdot \vartheta(\xi_{ab} - \xi_{aq}). \tag{7}$$

Iterating this procedure for all splittings, for the final quark distribution one is left to consider

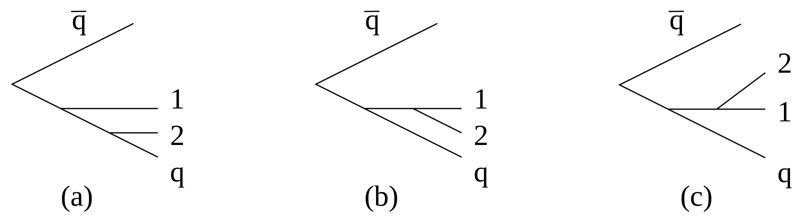



(8)

with only gluons emitted in the lower blob involving the parton $a$.

And here comes the crucial observation: the emission angle of the gluon $c$ is essentially limited from above by the aperture of the parent dipole $\xi_{ab}$, see (7). Therefore, soft gluons generated by the evolution equation (8) turn out to be ordered, simultaneously, in energies *and* in angles with respect to the radiating quark. Instead we know that the DGLAP equation is obtained from ordering only in collinear variables, disregarding the relative energies of emitted partons.

The variation is already at two loop order. Consider the splitting (5). For the term $\widehat{W}_{ab}^{(a)}$, following the Catani–Seymour prescription [12], one has $p_b^{(a)} = (1-\eta)P_b,\ p_a^{(a)} = zP_a + (1-z)\eta P_b - k_t$ and $q = (1-z)P_a + z\eta P_b + k_t$ with $k_t$ orthogonal to $P_a$ and $P_b$. Here $z$ is the light-cone fraction of the parent momentum $P_a$ carried by the final quark $p_a$. In the collinear limit $\eta \to 0$. In the soft limit both $\eta \to 0$ and $z \to 1$. Consider now the emission of two soft gluons off the parton $P_a$. The antenna functions that potentially contribute in the collinear limit (recall that we keep all gluon angles with respect to the quark $P_a$ to be small) are displayed here:




(9)

The first two graphs correspond to the splitting of the dipole $(a1)$:

$$W_{a1}^{(a)}(2) \to \vartheta(\xi_{a1} - \xi_{a2}) \qquad \text{and} \qquad W_{a1}^{(1)}(2) \to \vartheta(\xi_{a1} - \xi_{12}),$$

while the third one is the relevant part of the large-aperture dipole $(1b)$:

$$W_{1b}^{(1)}(2) \to \vartheta(\xi_{1b} - \xi_{12}),$$

Due to the *local* recoil prescription used, only the contribution $(a)$ affects the momentum of the quark $q$. In the two remaining ones, (b) and (c), the gluon 2 borrows its energy–momentum from the gluon 1 and does not produce any quark recoil. Therefore, these contributions cancel against corresponding virtual corrections in the inclusive quark measurement. In conclusion, within the adopted recoil strategy, only the graph (a) should be kept, and we obtain the following phase space for the two-gluon emission:

$$\xi_{a2} < \xi_{a1} < \xi_{ab} \qquad \text{and} \qquad \omega_2 < \omega_1 < E. \tag{10}$$

The first condition comes from the angular ordering in the graph (9(a)), and the second condition from the energy ordering of successive emissions. We know, however, that in order to obtain the DGLAP equation that properly resums collinear singular contributions, one needs to assemble angular (or, transverse momentum) ordered emissions (the first ordering), regardless to the order of gluon energies (the second one). At the same time, the dipole logic is leading us to the *double-ordered* gluon ensemble, according to (10). What is missing here is actually the coherence of QCD radiation. As well known, a soft gluon $\omega_2$, with $\omega_2 \ll \omega_1 \ll E_a$, could be emitted at large angles ($\xi_{a2} \gg \xi_{a1}$) directly by the original parton $q + p_1 \simeq q$. In the language of Feynman amplitudes, such radiation occurs as a coherent sum of the graphs (b) and (c).

We compute directly the soft contribution (large $N$) to this second order distribution

$$\begin{aligned} D_N^{(2)}(Q) &\sim \bar{\alpha}_s^2 \int_{1/N}^{1} \frac{dy_1}{y_1} \int_{1/N}^{y_1} \frac{dy_2}{y_2} \int_{Q_0}^{Q} \frac{dq_{1t}}{q_{1t}} \int_{Q_0}^{Q} \frac{dq_{2t}}{q_{2t}} \, \Theta\left(\frac{q_{t1}}{y_1} - \frac{q_{t2}}{y_2}\right) \\ &= \frac{\bar{\alpha}_s^2}{4} \ln^2 N \ln^2 \frac{Q}{Q_0} - \frac{\bar{\alpha}_s^2}{6} \ln^3 N \ln \frac{Q}{Q_0}\,, \end{aligned} \tag{11}$$

Here the theta-function comes from averaging the distribution as in (7). As a result the first term does not provide the correct correction to the first order distribution $D_N^{(1)}(Q) \simeq -\bar{\alpha}_s \ln N \ln \frac{Q}{Q_0}$.

The situation changes if one considers transverse momentum ordering instead of energy ordering. This would give at two loop

$$D_N^{(2)}(Q) \sim \bar{\alpha}_s^2 \int_{1/N}^{1} \frac{dx_1}{x_1} \int_{1/N}^{1} \frac{dx_2}{x_2} \int_{Q_0}^{Q} \frac{dq_{1t}}{q_{1t}} \int_{Q_0}^{q_{1t}} \frac{dq_{2t}}{q_{2t}} \Theta\left(\frac{q_{t1}}{x_1} - \frac{q_{t2}}{x_2}\right) \tag{12}$$

$$= \frac{\bar{\alpha}_s^2}{2} \ln^2 N \ln^2 \frac{Q}{Q_0} - \frac{\bar{\alpha}_s^2}{6} \ln^3 N \ln \frac{Q}{Q_0}\,.$$

The first order term provides the correction to the leading order anomalous dimension. However the second term requires a compensation, possibly from Sudakov form factors, which are away from the leading order distribution (8). The relevant question here is that energy ordering does reproduce the multi-parton distribution (1) while transverse momentum ordering does not reproduce it, see [9].

## 5 Conclusions

Monte Carlo generation of QCD events is a quarter century old business, based on the structure of resummation of *collinear enhanced* Feynman diagram contributions. *Collinear-non-enhanced* ("large angle") soft gluon radiation provides significant corrections to global event characteristics (e.g., mean particle multiplicity) and determines the structure of various *non-global* observables [8]. Effects of multiple soft gluon radiation at large angles lie beyond the scope of the standard (collinear) approach and must be treated order by order in perturbation theory (while collinear enhanced contributions are resummed in all orders).

An elegant expression [4] for the multiple soft gluon production probability is valid for *arbitrary angles* and offers a possibility of improving the parton picture. The structure of multi-gluon distribution naturally suggests an interpretation in terms of a chain of *colour connected dipoles*. By using energy ordering of gluons this chain may be generated via a Markov process of successive dipole splittings, see [9]. The generating functional that we have constructed allows one to calculate specific effects due to multiple emission of soft gluons at large angles in the large-$N_c$ approximation. In order to construct a realistic Monte Carlo generator for multi-parton ensembles it is imperative, however, to formulate an adequate recoil prescription which would ensure energy–momentum conservation at every successive step of the parton (dipole) multiplication. In the present paper we addressed the question, whether the "dipole factorization" extends beyond the no-recoil approximation. By keeping energy ordering as a prescription to generate the multi-parton distributions (1), we have shown that a naive implementation of the dipole recoil strategy results in violating the collinear factorization, see (11) and [9].

There exists a number of Monte Carlo implementations of the dipole picture, see [13], which use transverse momentum ordering. This formulation does not produces the multi-parton distribution (1). It satisfies collinear factorization, see (12), but generates non leading pieces which needs to be cancelled. We are looking forward to learning from the experts.

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