High energy scattering in QCD vs. tiny black holes

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Abstract

We review the holographic conjecture which links the transition from a dilute to a dense system of partons in DIS with the formation of tiny black holes in the gravitational collapse of a perfect fluid. At small 't Hooft coupling and large center-of-mass energies the onset of unitarity in the Yang-Mills side is interpreted as the formation of a horizon due to nonlinear gravitational dynamics in the higher dimensional bulk. Recent progress in the study of critical behaviour present in the formation of closed trapped surfaces in the collision of gravitational shock waves is also presented.

1 Critical gravitational collapse of a massless scalar field and a perfect fluid

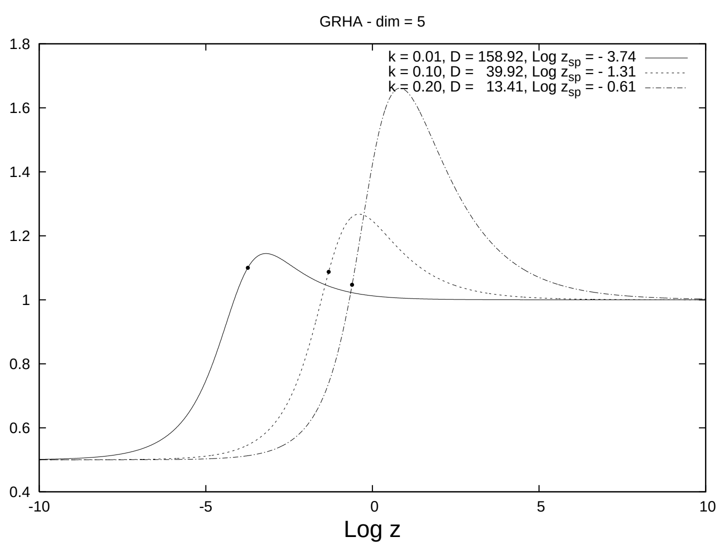

Fig. 1: Solutions for different CSS backgrounds.

In Ref. [1] it was remarked that the critical exponent characterizing the formation of a small black hole in the gravitational collapse of a massless scalar field is quite similar to the critical exponent present in the saturation line of DIS. This line marks the onset of saturation effects in the evolution of parton distribution functions at very small values of Bjorken x. It should be indicated that the calculation of the critical exponent in the QCD side suffers from some intrinsic uncertainties even if one stays in the leading logarithmic approximation. If the calculation of the saturation line is performed using an effective absorptive barrier implemented in the integration over transverse momenta [2] one obtains a critical exponent ~ 2.44. Using other approaches where unitarity takes the form of a nonlinear

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term in the evolution equation [3, 4] this number changes to ~ 2.28 [5]. Nevertheless it is encouraging that these are pure numbers independent of the value taken for the coupling and that they are quite similar to each other. Higher order corrections in the gauge theory side, such as next-to-leading order terms, are suppressed if we assume that the 't Hooft coupling is very small. The calculation of the so-called Choptuik exponent in the gravity side is robust and a recent study [6] has shown that its value for the scalar field in 5 dimensions is ~ 2.42.

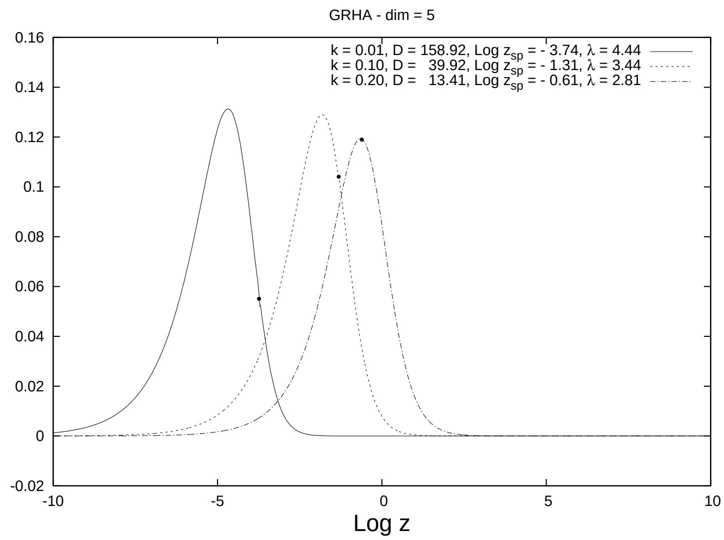

Fig. 2: Liapunov perturbations for different CSS backgrounds.

There is a more serious complication to map perturbative saturation with the critical collapse of a scalar field. In this case the solutions to Einstein's equations for any scaleless quantity have a discrete self-similarity. This means that they reproduce themselves after a simultaneous fixed discrete rescaling in the time and radial components. Such a discrete scaling, also known as "echoing", is not present in the Yang-Mills side. However, not everything is lost since it is well known that DIS data for the total cross section in the collision of a virtual photon with a proton manifests what is known as "geometric scaling". This behaviour appears for a large range of values of the virtuality of the photon, Q^2, when Bjorken $x \simeq Q^2/s$ is smaller than 0.01, with s being the center-of-mass energy in the process. In this region the HERA data is a function only of the ratio of Q^2 over x to some power [7]. In this way we can consider this scaling as continuous self-similar (CSS) because the cross section is invariant under any shift in Q^2, compensated by a similar one in x. This CSS can be interpreted within perturbative QCD as a consequence of saturation effects where the parton multiplicity is so large that a simple linear evolution cannot hold any longer and recombination effects must be taken into account. These effects are of non-perturbative origin, in the sense that they are related to the dynamics of the formation of a high density system, but not related to confinement since the typical transverse scale in the problem is set by Q^2, always above $\Lambda^2_{\rm QCD}$.

The natural question now is whether there exists any gravitational system with CSS collapse which is characterized by a similar critical exponent to that found in the case of the scalar field. This question motivated us to study [8] the critical gravitational collapse of a perfect fluid with barotropic equation of state $p = k\rho$ and spherical symmetry in arbitrary dimensions. In this type of collapse black hole singularities are formed with a radius given by

$$r_{\rm BH} \sim (p - p^*)^\gamma,$$

where p parameterizes generic values of the initial radial density of collapsing fluid. p^* denotes a critical region of densities for which, if p is above but close to p^*, a singularity appears. We are particularly interested in this system because the critical solutions at $p = p^*$ are CSS and can be directly calculated from Einstein equations sourced by the perfect fluid imposing that they depend only on the variable $z = -r/t$. For fine tuned values of p^* we numerically obtained these background solutions at different values of the speed of sound $\sqrt{k}$ in the fluid. An example of our numerical results is shown in Fig. 1 where we plot the ratio of the fluid local density at the point r over the global density up to that point. We show the behaviour of this function versus the variable z for different values of k in five dimensions. Note that D, which is related to p^*, has to be fine tuned in order to cross the so-called "sonic point", where the surfaces of constant z move at a speed equal to $\sqrt{k}$. The main constraint to select the correct value of D is to have analyticity at this sonic point, indicated by a dot in the figure.

The critical exponent γ in the formula for the radius of the black hole can be found by introducing a Lyapunov perturbation around the CSS critical line $y(z)$ of the form

$$y(t,z) = y(z)\left(1 + \epsilon\,(-t)^{-\frac{1}{\gamma}}\,y_1(z)\right).$$

To calculate the perturbations $y_1(z)$ it is again crucial to have analyticity at the sonic point. This condition fixes the value for the single unstable mode γ. The form of the perturbations can be seen in Fig. 2 where $\lambda = 1/\gamma$. The corresponding γ modes which we calculated for different dimensions are shown in Table 1. The results for dimension four coincide with those found in Ref. [9]. Thinking of a possible holographic interpretation we have also investigated how these critical exponents vary with the dimension. To match the numbers obtained in QCD we would be looking for a range of $\gamma \in (0.41, 0.44)$. Of course we now face the problem of selecting the correct k. A possible candidate would be that corresponding to a conformal fluid of traceless energy-momentum tensor for which $k = 1/(d-1)$. An heuristic motivation for this choice is that in the linear growth of parton distributions and in the transition vertex from two reggeized gluons to four reggeized gluons, which is a fundamental piece in the unitarization corrections, there is an associated $SL(2,C)$ invariance [10]. We are currently investigating the extension of Table 1 up to dimension ten since it is possible that the holographic dual might live in a $AdS_5 \times S^5$ geometry where all dimensions would be equally important since the critical black holes here discussed can be arbitrarily small. Preliminary studies show that γ is close to the QCD range of results in the conformal limit of ten dimensions.

Although these investigations show encouraging results we are still far from having a holographic picture of the problem at hand. There are many unanswered questions and probably the most pressing one is to find the geometry corresponding to the perturbative hard pomeron. In Ref. [11] this problem was addressed from the large 't Hooft coupling perspective arguing that the main features of the BFKL kernel cannot change too much in the transition from weak to strong coupling since it is protected by conformal invariance. Our research targets a more complicated problem, not only because we handle perturbative results in the Yang-Mills side but also because we are at the transition region from a single pomeron picture to a regime dominated by multiple pomeron exchanges.

k	$\gamma_{d=4}$	$\gamma_{d=5}$	$\gamma_{d=6}$	$\gamma_{d=7}$
0.01	0.114	0.225	0.290	0.330
0.02	0.123	0.233	0.296	0.336
0.03	0.131	0.241	0.303	0.342
0.04	0.140	0.248	0.309	0.348
0.05	0.148	0.256	0.316	0.353
0.06	0.156	0.263	0.322	0.359
0.07	0.164	0.270	0.328	0.364
0.08	0.172	0.277	0.334	0.369
0.09	0.180	0.284	0.340	0.375
0.10	0.187	0.291	0.346	0.380
0.11	0.195	0.298	0.352	0.385
0.12	0.203	0.304	0.358	0.390
0.13	0.210	0.311	0.364	0.396
0.14	0.218	0.318	0.369	0.401
0.15	0.225	0.324	0.375	0.406
0.16	0.232	0.330	0.381	0.411
0.17	0.240	0.337	0.386	0.416
0.18	0.247	0.343	0.392	0.421
0.19	0.254	0.347	0.397	0.426
0.20	0.261	0.356	0.403	0.431
0.21	0.259	0.362	0.408	0.435
0.22	0.276	0.368	0.414	0.440
0.23	0.283	0.375	0.419	0.445
0.24	0.290	0.381	0.425	0.450
0.25	0.297	0.387	0.430	0.454

Table 1: Values of the Choptuik exponent with precision ± 0.001 as a function of k for $d = 4$, 5, 6 and 7.

2 Closed trapped surfaces in shock wave collisions

Some light might be shed on these issues if we focus our attention on a related problem which shares some features with the physics of saturation. In Ref. [12] a gravity dual of a boosted Woods-Saxon nuclear energy density for heavy ions was proposed. For example, if we consider gold with a typical size L and energy E the corresponding energy momentum tensor on the gauge theory side, $T_{\mu\nu}$, is associated with a bulk gravitational source of the form $\rho(x_i, z) \sim E\delta(x_i)\delta(z - L)$, where x_i are the transverse coordinates and z the holographic direction in a H_3 space. The $R^{3,1}$ boundary lies at $z = 0$. The solution to the Einstein equations in this space

$$\left(\Box_{H_3} - \frac{3}{L^2}\right)\Phi(x_i, z) = \delta(u)\rho(x_i, z)$$

can be used to construct a five dimensional AdS shock wave bulk geometry with metric

$$ds^2 = \frac{L^2}{z^2}\left(-dudv + \sum_i dx_i^2 + dz^2 + \frac{z}{L}\Phi(x_i, z)\delta(u)du^2\right).$$

It is very interesting that in [12] when head-on collisions of heavy ions, which correspond to the gravitational collision of two shock waves, are considered, a closed trapped surface is formed. The area of the trapped surface is of the order of the entropy generated in the collision, which is itself related to the number of generated charged tracks. The total energy in the system can be written as $E \sim \int_{H_3} \rho(x_i, z)$. If a $O(3)$ symmetry in the H_3 plane is assumed then the source can be written as a function of the chordal coordinate $q(x_i, z) = \left(\sum_i x_i^2 + (z - L)^2\right)/(4zL)$. In this coordinate the trapped surface is characterized by a density function $\rho(q)$ describing the strong gravity collision region such that $E \sim \int_0^{q_c} \rho(q)$ with the horizon defined by the surface $q = q_c$.

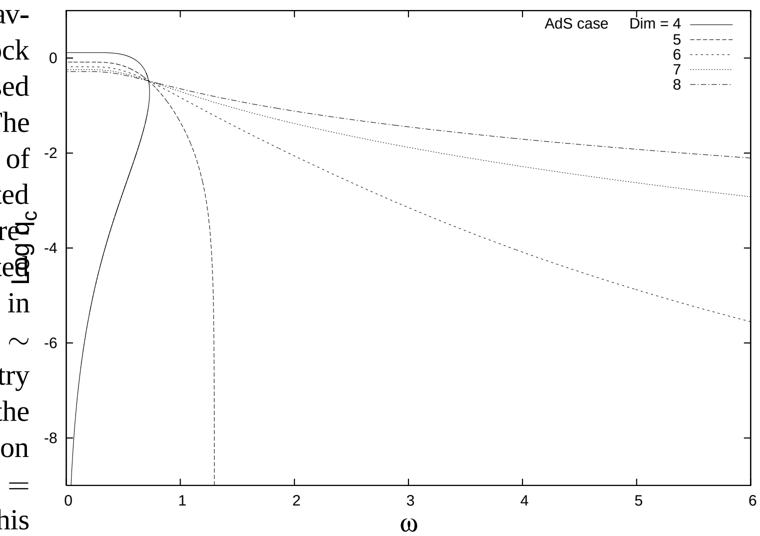

Fig. 3: Size of the trapped surface versus the diluting parameter ω.

It would be important to see if a similar set up could be used to describe DIS in the saturation region with the onset of nonlinear effects being related to the formation of a trapped surface. Indeed, we have found that in the formulation of [12] a critical phenomena resembling that found by Choptuik is present. In order to see this it is needed to smear the energy density in the chordal variable using, for example, a Gaussian distribution with width parametrized by a variable ω. With an AdS metric in different dimensions we have solved the equation to form a close trapped surface of size q_c as a function of ω. The results are plotted in Fig. 3.

We observe that in dimensions larger than five it is always possible to form an arbitrary small trapped surface by simply diluting the initial energy density. This is done by increasing ω while keeping the total energy constant. No critical behaviour is found in these dimensions. However, the situation is more interesting at $d = 4, 5$ since criticality kicks in. In both cases there exists a maximal $\omega = \omega_c$ beyond which it is not possible to form a trapped surface and in the region close to this point the relation

$$q_c \simeq q_c^* + (\omega_c - \omega)^\gamma$$

holds with q_c^* being different from zero in $d = 4$ and canceling for $d = 5$. The critical exponent γ is 1 in $d = 5$ and 0.5 in $d = 4$. When considering the same physics in a flat background we have found an equivalent behaviour, shown in Fig. 4. The only difference is that now the critical exponent γ is 0.5 in both dimensions 4 and 5. Further details on these results can be found in Ref. [13].

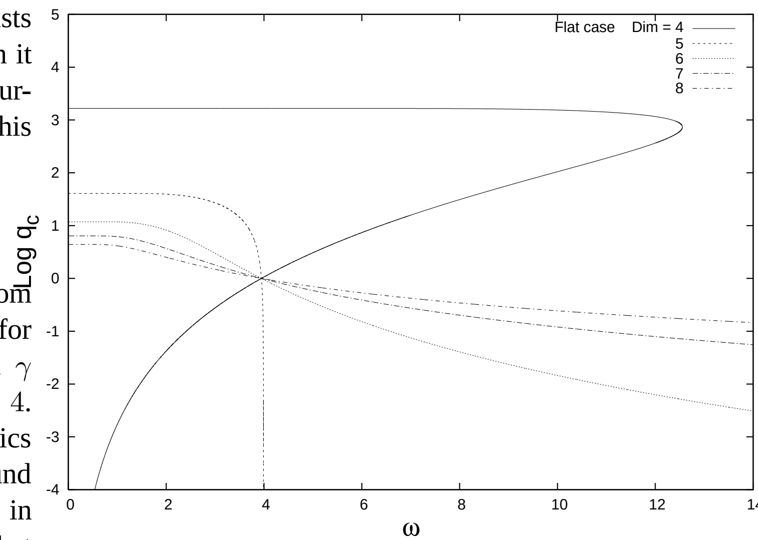

Fig. 4: Size of the trapped surface versus the diluting parameter ω.

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