

Theoretical predictions for the LHC

S. Moch

Deutsches Elektronensynchrotron DESY, Platanenallee 6, D-15738 Zeuthen, Germany

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Abstract

We review the status of QCD predictions for the Large Hadron Collider. We include recent theoretical developments for cross sections calculations to higher orders and discuss various Standard Model reactions such as W^\pm/Z -boson, Higgs boson or top quark production.

1 Introduction

The Large Hadron Collider LHC is built to explore the energy frontier as it operates at a center-of-mass energy of $\sqrt{s} = 14$ TeV. It will realize a major leap forward in collision energy compared to all other colliders thus far and it allows searches for the Higgs boson and tests of proposed extensions of the Standard Model, such as supersymmetry or models with large extra dimensions. The experimental signatures of many of the new physics scenarios are characterized by a high multiplicity of particles in the final state. They consist of multiple jets, leptons and missing transverse energy, see Fig. 1.

The cross sections for Standard Model scattering processes such as the production of b -quarks, W^\pm and Z -bosons, multiple jets and top quarks at LHC are large. These Standard Model reactions lead to similar final states as those encountered for instance in the decay of the Higgs-boson. Thus, the challenge for theory at LHC is to provide precise predictions for the known physics, i.e. the Standard Model background in order to extract and identify signs of any kind of new physics. In particular, the theoretical predictions have to match or exceed the accuracy of the LHC data. To quote numbers, let us consider some rough estimates. The total cross section for W -boson production amounts to $\sigma_W \sim 150$ nb. With a branching ratio $\text{BR}(W \rightarrow e + \mu) \sim 20\%$ this leaves approximately 300M leptonic events at a luminosity of 10 fb^{-1} or, in other words, a production rate of 30 Hz in the initial low luminosity run. Likewise, for Z -boson production we

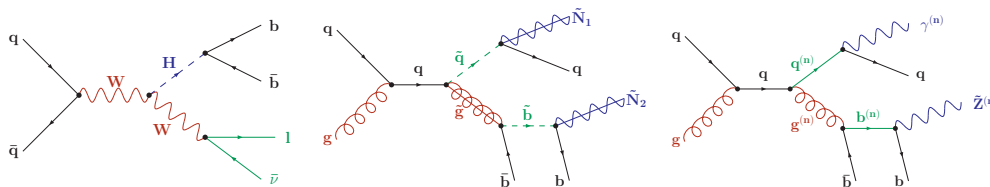


Fig. 1: Sample of Feynman diagrams for different physics scenarios at LHC with multiple jets, leptons and missing transverse energy. Left: Higgs-strahlung $q\bar{q} \rightarrow W(Z)H$ in the dominant decay mode $H \rightarrow b\bar{b}$. Middle: neutralino pair-production $\tilde{N}_{1,2}^0$ in the Minimal Supersymmetric Standard Model (with R -parity). Right: pair-production of excited Kaluza-Klein-modes $\gamma^{(n)}, Z^{(n)}$ in a model with large extra dimensions.

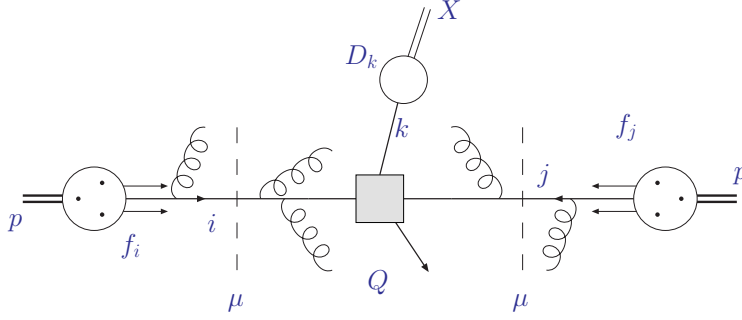


Fig. 2: Factorization for the hard-scattering cross sections in Eq. (1) in the QCD improved parton model.

have $\sigma_Z \sim 50$ nb and with $\text{BR}(Z \rightarrow ee + \mu\mu) \sim 6.6\%$ a total of 33M leptonic events at 10 fb^{-1} . For the low luminosity run this implies a rate of 3.5 Hz. In comparison, the typical rates for new physics signals are often of order $\sigma_{\text{new physics}} \sim \mathcal{O}(1 - 10)$ pb.

For precision predictions, much of the physics is actually dominated by the gauge theory of the strong interactions, Quantum Chromodynamics (QCD) and its perturbative formulation is an essential and established part of our theory toolkit. See e.g. Ref. [1] for a recent review on hard QCD at LHC. The basic prerequisite is QCD factorization which rests on the ability to separate dynamics from different scales. This is sketched schematically in Fig. 2.

QCD factorization allows a proton-proton scattering cross section of some hadronic final state X to be written as,

$$\sigma_{pp \rightarrow X} = \sum_{ijk} \int dx_1 dx_2 dz f_i(x_1, \mu^2) f_j(x_2, \mu^2) \times \hat{\sigma}_{ij \rightarrow k}(x_1, x_2, z, Q^2, \alpha_s(\mu^2), \mu^2) D_{k \rightarrow X}(z, \mu^2), \quad (1)$$

where Q is the hard scale and details of the integration range in the convolution depend on the kinematics of the hard scattering process under consideration.

The parton luminosity $f_i \otimes f_j$ ($i, j = q, \bar{q}, g$) is given as a convolution of the parton distribution functions (PDFs) in the proton. The latter depend on the parton momentum fractions x_1, x_2 and on the factorization scale μ . The PDFs are universal and have to be determined by fits to reference processes at low scales μ . The scale μ of the PDFs then has to be evolved from those of the reference processes to the one appropriate for applications at LHC. As the LHC probes the energy frontier this implies a scale evolution over two to three orders, see e.g. Ref. [2] for the current status of the parton luminosity. This evolution (μ -dependence) of the PDFs is governed by the perturbatively calculable splitting functions P_{ij} , now known through next-to-next-to-leading order (NNLO) in QCD [3, 4]. The (hard) parton cross section $\hat{\sigma}_{ij \rightarrow k}$ ($i, j, k = q, \bar{q}, g$) describes how the constituent partons from incoming protons interact at short distances of order $\mathcal{O}(1/Q)$. It is calculable in perturbative QCD as a series in the strong coupling constant α_s at leading order (LO) in QCD or, including quantum corrections, at next-to-leading order (NLO) or even NNLO. $\hat{\sigma}_{ij \rightarrow k}$ is the main quantity of interest in discussing the accuracy

process ($V \in \{\gamma, W^\pm, Z\}$)	background to	reference
$pp \rightarrow VV + 1 \text{ jet}$	$t\bar{t}H$, new physics	$WW + 1 \text{ jet}$ [5, 6]
$pp \rightarrow H + 2 \text{ jets}$	H production by vector boson fusion (VBF)	$H + 2 \text{ jets}$ [7]
$pp \rightarrow t\bar{t}b\bar{b}$	$t\bar{t}H$	$q\bar{q} \rightarrow t\bar{t}b\bar{b}$ [8]
$pp \rightarrow t\bar{t} + 2 \text{ jets}$	$t\bar{t}H$	
$pp \rightarrow VVb\bar{b}$	VBF $\rightarrow VV$, $t\bar{t}H$, new physics	
$pp \rightarrow VV + 2 \text{ jets}$	VBF $\rightarrow VV$	
$pp \rightarrow V + 3 \text{ jets}$	various new physics signatures	
$pp \rightarrow VVV$	SUSY tripleton	ZZZ [9], WWZ [10]

Table 1: Scattering processes at LHC for which the radiative corrections to NLO in QCD are needed.

of theoretical predictions at LHC. For completeness, we mention that X may denote any final state, e.g. hadrons, mesons or jets. The transition from the perturbative hard partons k in Eq. (1) to the observed particles is again non-perturbative and $D_{k \rightarrow X}$ can therefore be a fragmentation function or also a jet algorithm. Here the interface with showering algorithms (based on a Monte Carlo approach) becomes particularly crucial.

Physical observables like the cross section $\sigma_{pp \rightarrow X}$ in Eq. (1) cannot depend on the factorization scale, which implies that any dependence on μ in $\sigma_{pp \rightarrow X}$ has to vanish at least to the order in α_s considered.

$$\frac{d}{d \ln \mu^2} \sigma_{pp \rightarrow X} = \mathcal{O}(\alpha_s^{l+1}). \quad (2)$$

This variation defines the commonly adopted approach to quantify uncertainties in theoretical predictions based on the scale variation.

Let us now turn to hard parton scattering cross sections $\hat{\sigma}_{ij \rightarrow k}$. As mentioned, there exist various levels of accuracy for predictions building on exact matrix elements. At LO, we have at our disposal many highly automated programs for tree level calculations in the Standard Model, in its minimal supersymmetric extension (MSSM) or in other BSM models, which allow easy interfacing of LO calculations with parton shower Monte Carlos. These LO estimates based on exact matrix elements seem mandatory in search scenarios for studies of distributions, e.g. in p_T or the (pseudo-)rapidity (η) and for assessing the effects of kinematical cuts.

However, any LO prediction has large theoretical uncertainties, typically estimated by the scale variation, Eq. (2). Consider, for instance, the cross section for $pp \rightarrow W + 4 \text{ jets}$, which is of $\mathcal{O}(\alpha_s^4)$ at LO. From a variation of the coupling of $\Delta(\alpha_s^{\text{LO}}) \simeq 10\%$ one can roughly estimate a cross section uncertainty of $\Delta(\sigma^{\text{LO}}) \simeq 40\%$. Thus, one needs to go beyond the Born approximation for scattering processes where quantum corrections at NLO may have an impact on the signal significance. Given the high multiplicity of final state particles at LHC (see Fig. 1) there exists a number of key processes at LHC which need to be known to NLO in QCD (see e.g. Ref. [11]). These are summarized in Tab. 1 and the computation of these radiative corrections is presently a very active field of research.

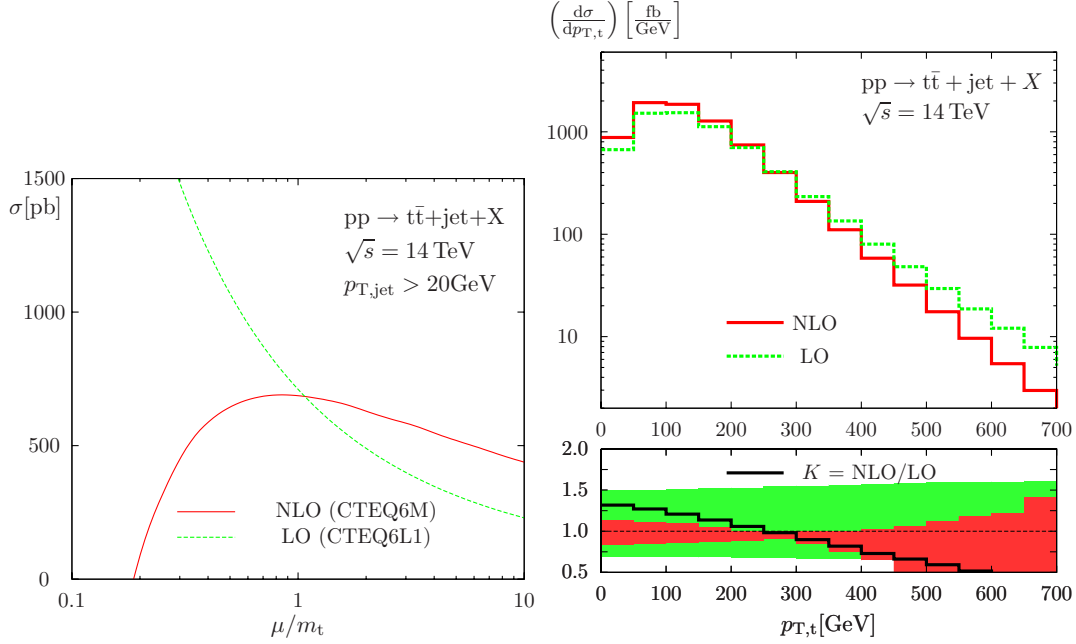


Fig. 3: The scale dependence of the LO and NLO cross sections for $t\bar{t} + 1$ jet production at LHC (left) and the transverse-momentum distribution of the top-quark $p_{T,t}$ along with the K -factor and the scale variation in the range $m_t/2 \leq \mu \leq 2m_t$.

Let us illustrate the effects of NLO radiative corrections in QCD with the results of the recent impressive state-of-the-art calculation for $t\bar{t} + \text{jet}$ production [12, 13]. Fig. 3 displays the much improved scale dependence and shows that the perturbative corrections are moderate for the nominal scale choice of the order of the top-quark mass, $\mu \simeq m_t$. It also shows the NLO differential distribution in the transverse momentum of the top-quark for this reaction along with the kinematics dependence of the K -factor and the uncertainty band due to the scale variation in the commonly adopted range $m_t/2 \leq \mu \leq 2m_t$. From Fig. 3 it is clearly visible, that the NLO corrections (i.e. the K -factor) are not a uniform function of the transverse momentum p_T .

Finally, there is of course demand for fully differential QCD predictions to NNLO for hadron collider processes. Currently, this scope has been achieved e.g. for the di-lepton pair production in Drell-Yan [14] or Higgs production in gluon fusion [15]. However, it remains a challenge for many other reactions which can potentially be measured very precisely at LHC, such as Higgs production in vector boson fusion, top-pair production and to $V + 1$ jet, where $V \in \{\gamma, W^\pm, Z\}$. However, also electroweak corrections become important once such an accuracy of a few percent is needed for an observable. As an example of the prospects at NNLO let us focus on the total cross section for top-quark pair-production, where currently complete NLO QCD predictions exist [16–18]. Based on soft gluon resummation though, it is possible to derive approximate NNLO results for the total cross section which combine the complete logarithmic dependence on the heavy quark velocity $\beta = \sqrt{1 - 4m^2/s}$ near threshold $s \simeq 4m^2$ with the complete two-loop Coulomb corrections as well as the exact dependence on the factorization

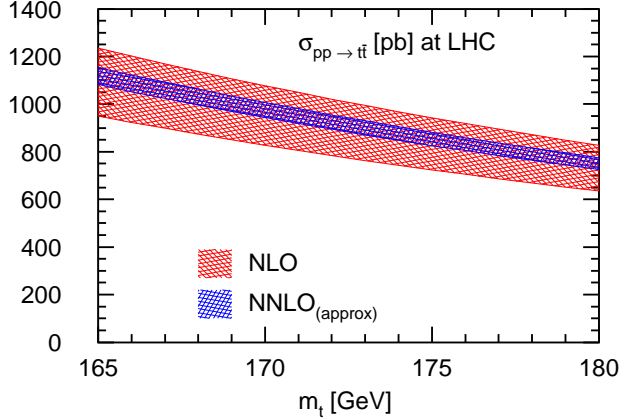


Fig. 4: The NLO and $\text{NNLO}_{\text{approx}}$ QCD prediction for the $t\bar{t}$ total cross section for LHC with $\sqrt{s} = 14$ TeV. The bands denote the total uncertainty from PDF and scale variations for the MRST06nnlo set according to Eq. (3).

scale at NNLO [19]. For phenomenology, this provides a very good approximation to the unknown exact NNLO result, because the parton luminosity in the convolution (1) emphasizes the threshold region of phase space, i.e. it gives much weight to parton energies of order $\hat{s} \simeq 4m^2$.

At this level of accuracy, it is interesting to account for both the scale variation according to Eq. (2) and the PDF uncertainty. We define the range as

$$\sigma(\mu = 2m_t) - \Delta\sigma_{PDF}(\mu = 2m_t) \leq \sigma(\mu) \leq \sigma(\mu = m_t/2) + \Delta\sigma_{PDF}(\mu = m_t/2), \quad (3)$$

where $\Delta\sigma_{PDF}$ is computed from the variation of the cross section with respect to the parameters of the global fit of PDFs. The NLO QCD corrections provide the first instance where a meaningful error can be determined in this way. In Fig. 4 we plot the uncertainty range (3) comparing NLO and NNLO accuracy. The latter enters in the approximation based on the soft corrections as detailed above. The residual scale dependence of σ_{NNLO} is 2%, which corresponds to a reduction by a factor of two compared to NLO. At LHC σ_{NNLO} leads only to a small shift of a few percent in the central value and the total $\text{NNLO}_{\text{approx}}$ band is about 6% for CTEQ6.6 PDFs [20] and about 4% for the MRST06nnlo PDF set [21], which exhibits a drastic reduction of the scale uncertainty and much improved perturbative stability as compared to the prediction based on NLO QCD.

2 Summary

We have briefly reviewed some aspects of the theoretical framework of hard QCD at LHC. Precision predictions rely on knowledge of the parton luminosity as well as the rates for the corresponding partonic subprocess. Improving the theoretical accuracy for the latter is currently an active field of research and a lot of ongoing activity is concentrated on processes with multi-particle production to higher orders for both, the (new physics) signal and the background at LHC with massive particles and jets.

We have illustrated the present status of perturbative QCD predictions with a few examples from top quark production and we have tried to convey the message that QCD theory is ready to meet the challenges of LHC.

Let us finish with a disclaimer. All aspects of Higgs production are covered extensively e.g. in Ref. [22]. Moreover, we have left out a detailed discussion of all aspects of W - and Z -boson production. We have also omitted details of specific hadronic final states, e.g. jet algorithms, b -quark (b -jet) production or aspects of b -quark fragmentation as well as parton showers in Monte Carlo simulations and any computational details of higher order radiative corrections. We have also skipped any discussion of resummation approaches meant to improve fixed order perturbation theory, be it threshold logarithms of Sudakov type or $\ln(p_T)$ -terms in transverse momentum. For all these remaining aspects as well as a broader coverage, the interested reader is referred e.g. to Refs. [1, 23, 24] and to the numerous references therein.

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