The Lyth bound with a spectral tilt

Marco Scalisi

Van Swinderen Institute for Particle Physics and Gravity - RUG

university of groningen

work in collaboration with

Juan Garcia-Bellido (IFT-UAM-CSIC),
Diederik Roest (RUG),
Ivonne Zavala (RUG)

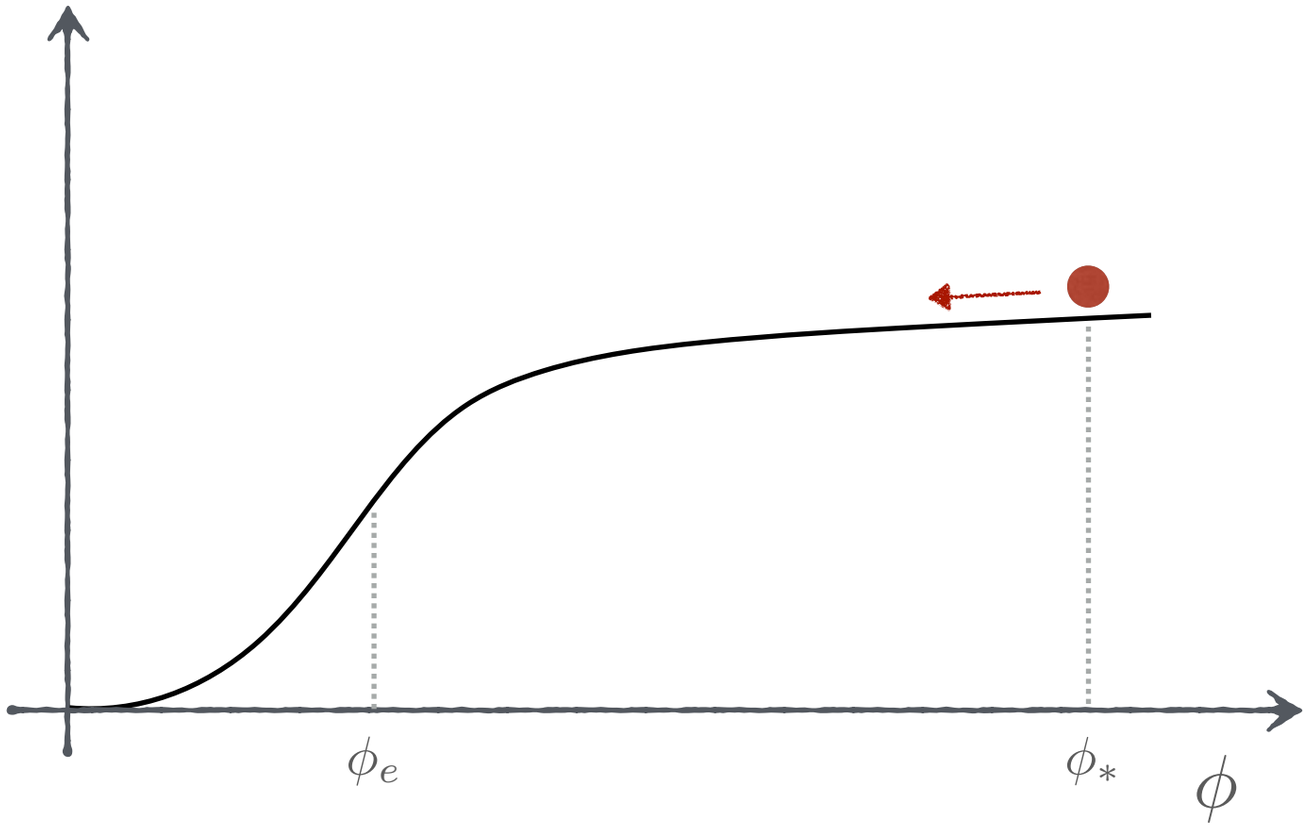
based on

"*The Lyth bound of inflation with a tilt*"
[arXiv: 1408.6839]
&
"*Can CMB data constrain the inflationary field range?*"
[arXiv:1405.7399], JCAP09(2014)006

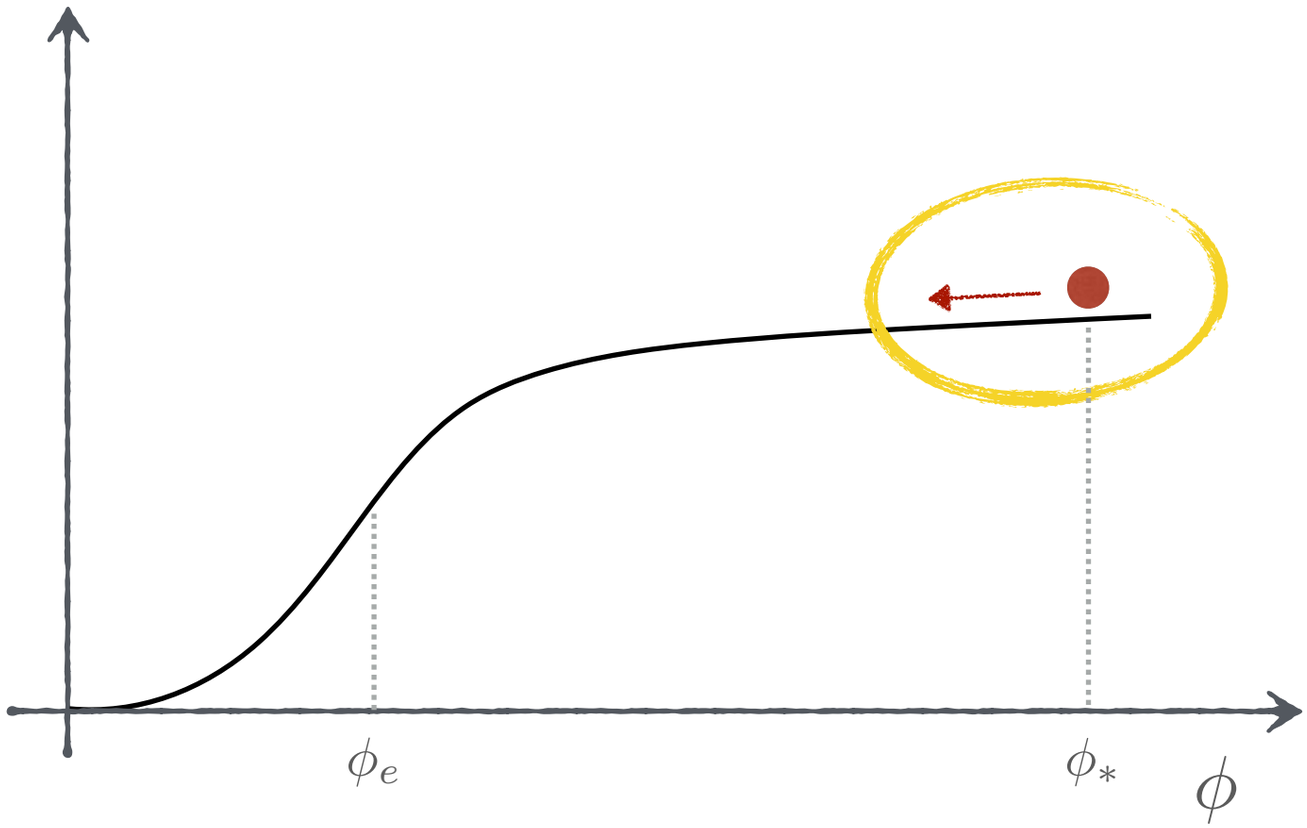
September 24th, 2014 - DESY Theory Workshop - Hamburg

Field range bounds in inflation

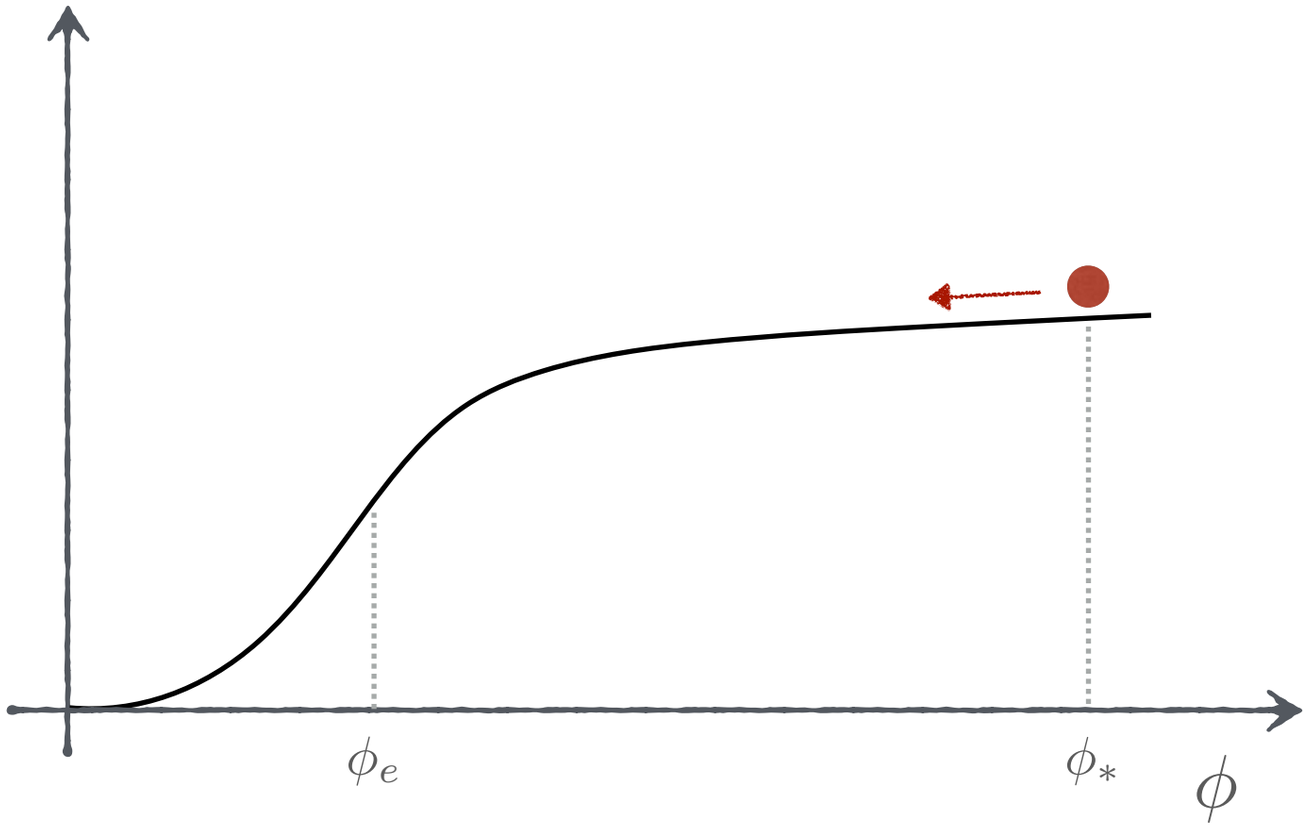
Slow-roll Inflation



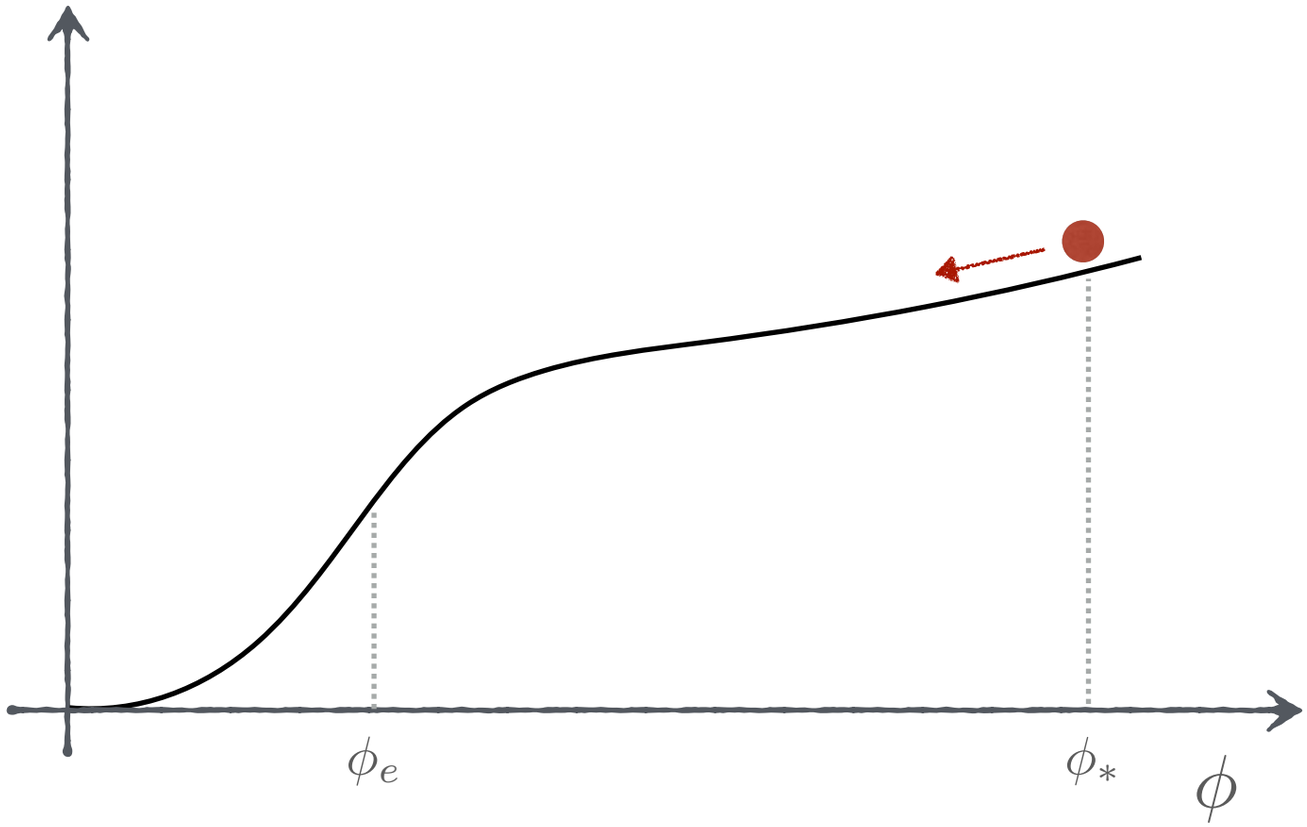
Slow-roll Inflation



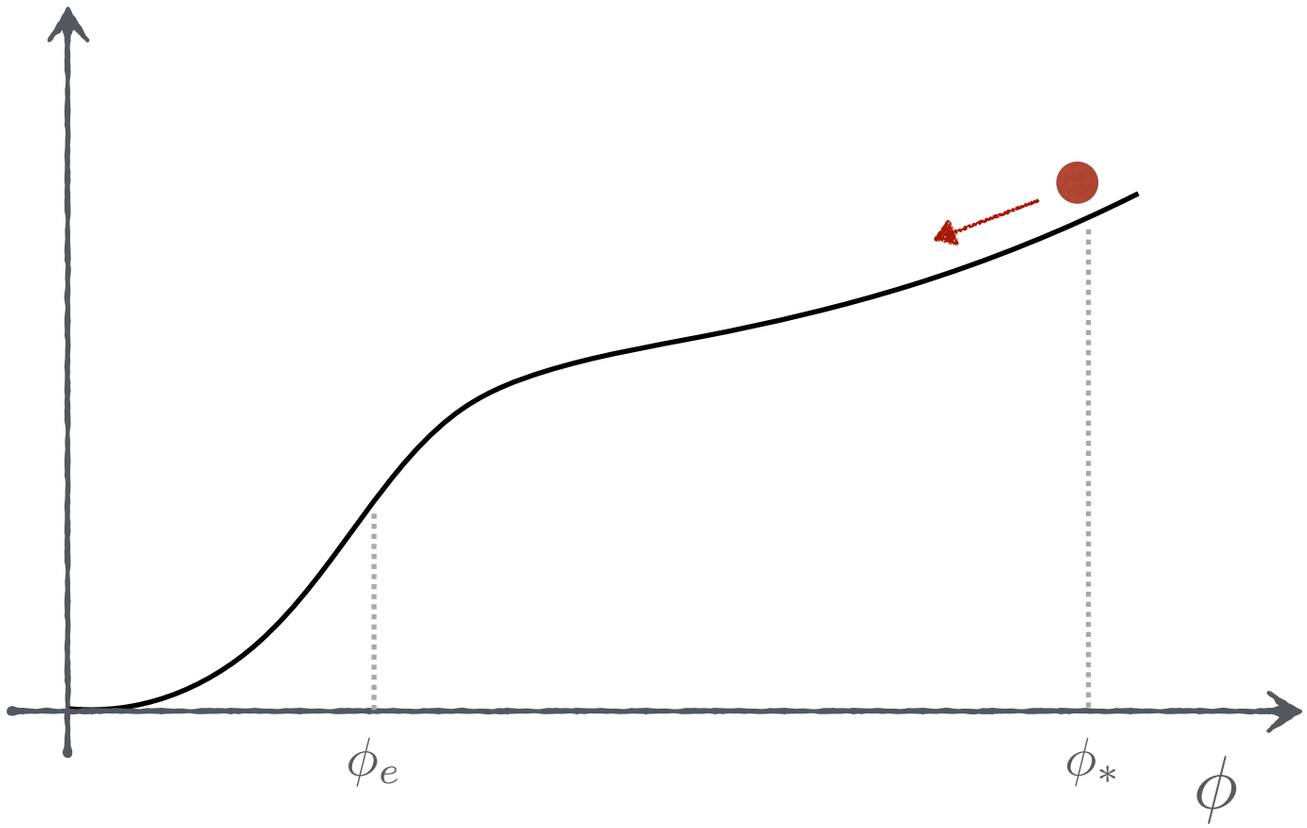
Slow-roll Inflation



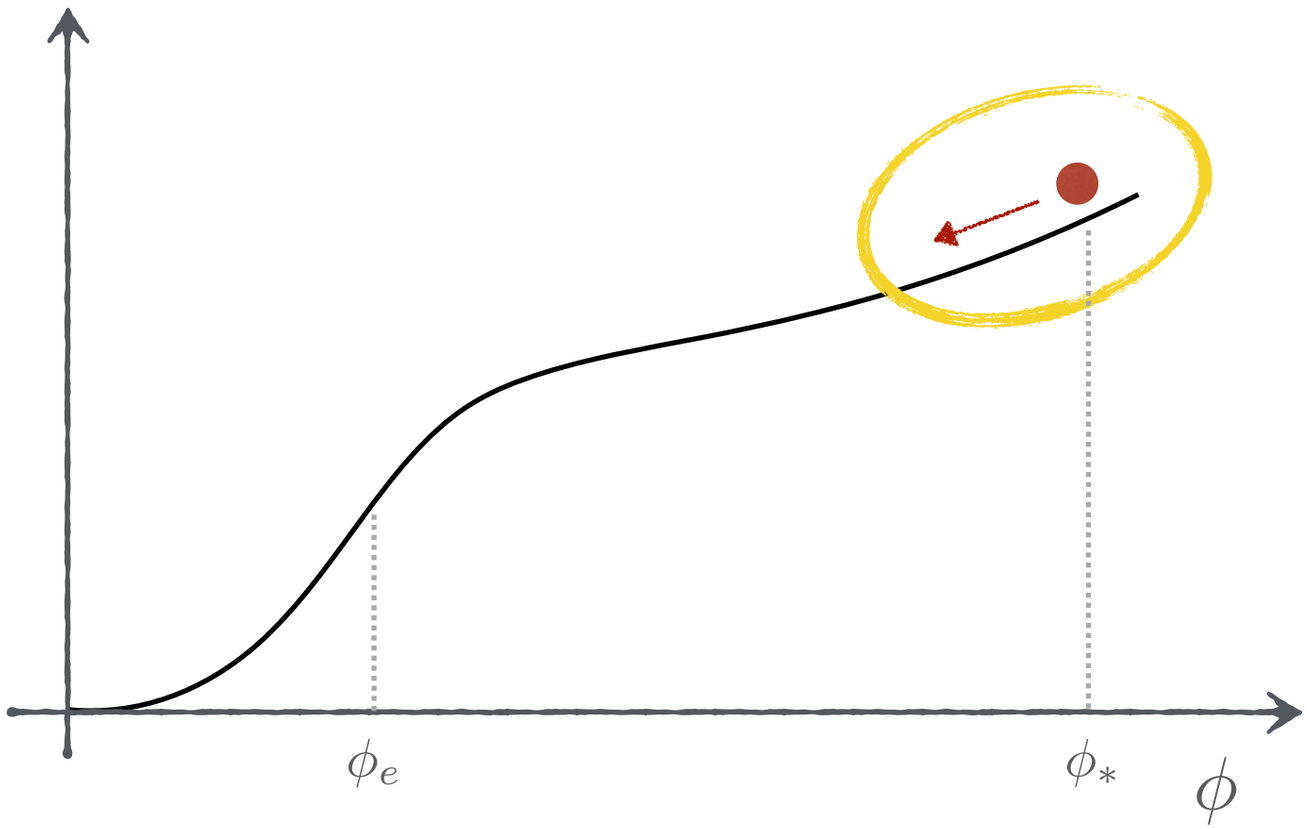
Slow-roll Inflation



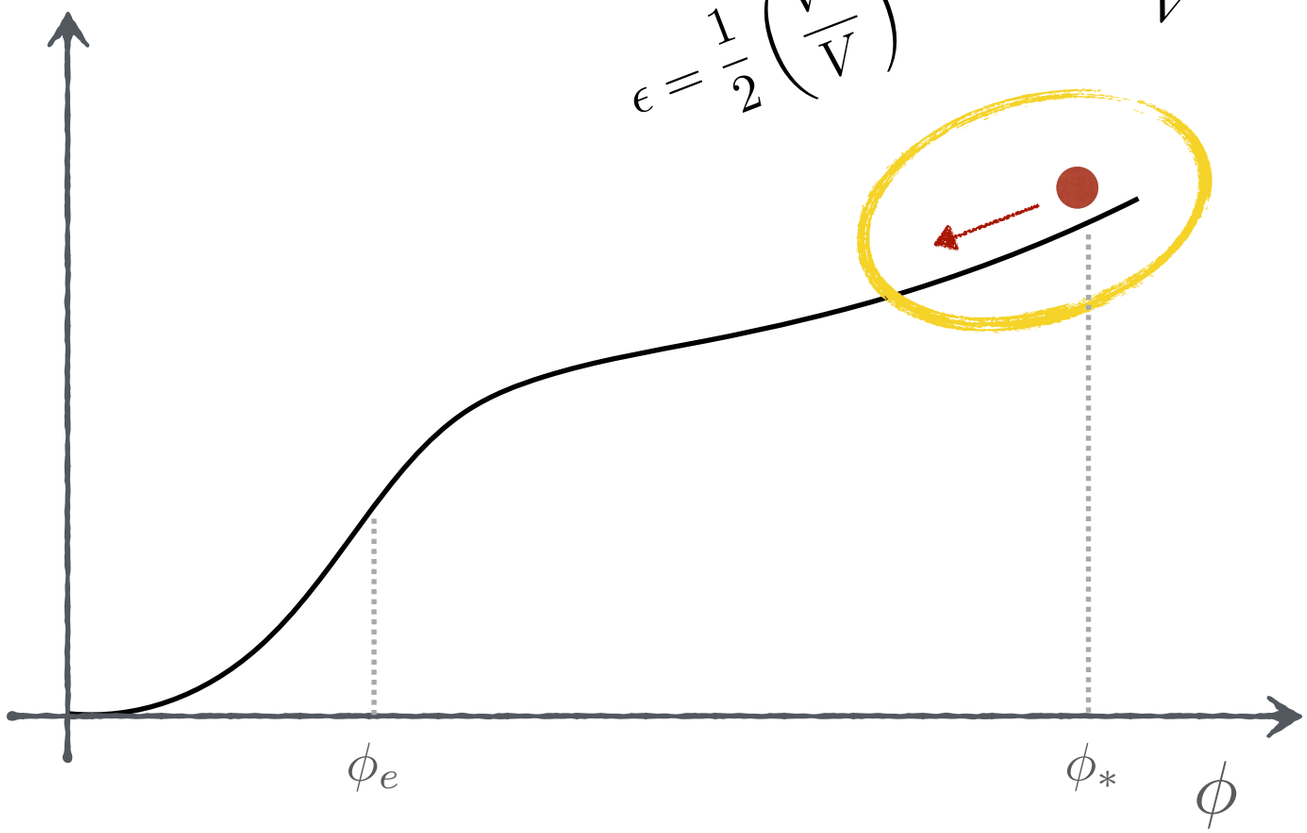
Slow-roll Inflation



Slow-roll Inflation



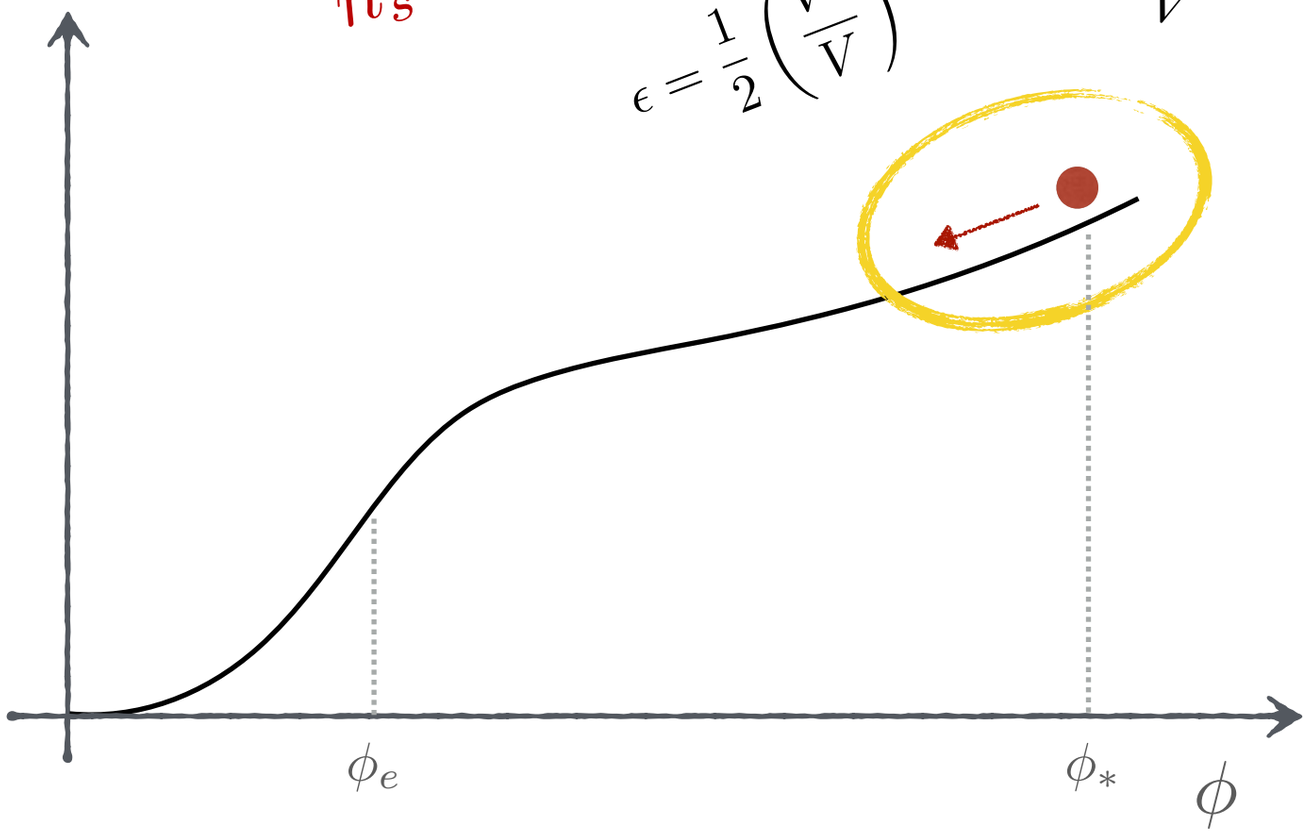
Slow-roll Inflation



Slow-roll Inflation

$$n_s = 1 + 2\eta - 6\epsilon \qquad r = 16\epsilon$$

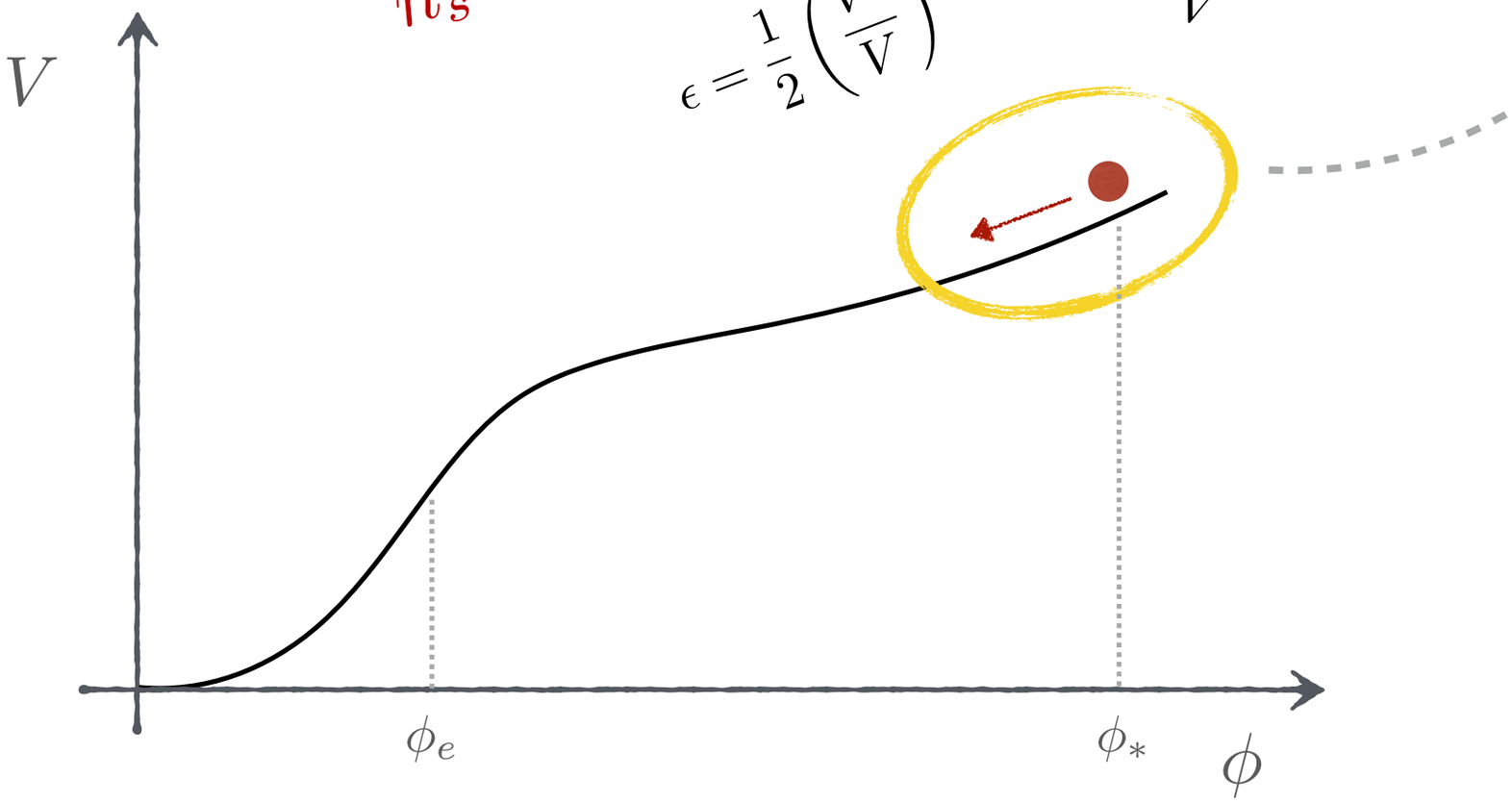
$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{V''}{V}$$

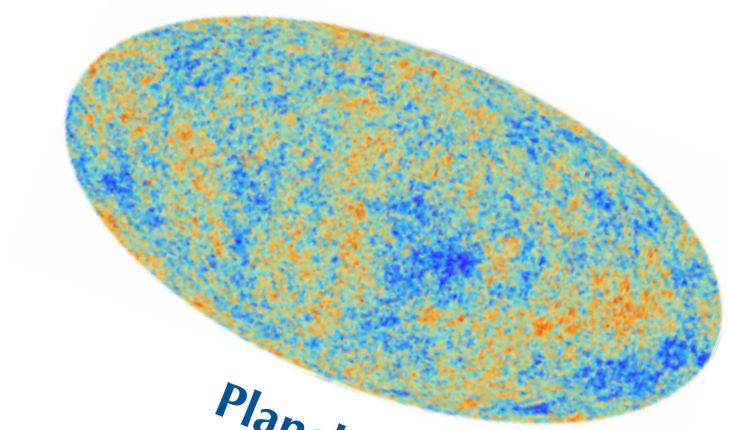


Slow-roll Inflation

$$n_s = 1 + 2\eta - 6\epsilon \qquad r = 16\epsilon$$

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{V''}{V}$$

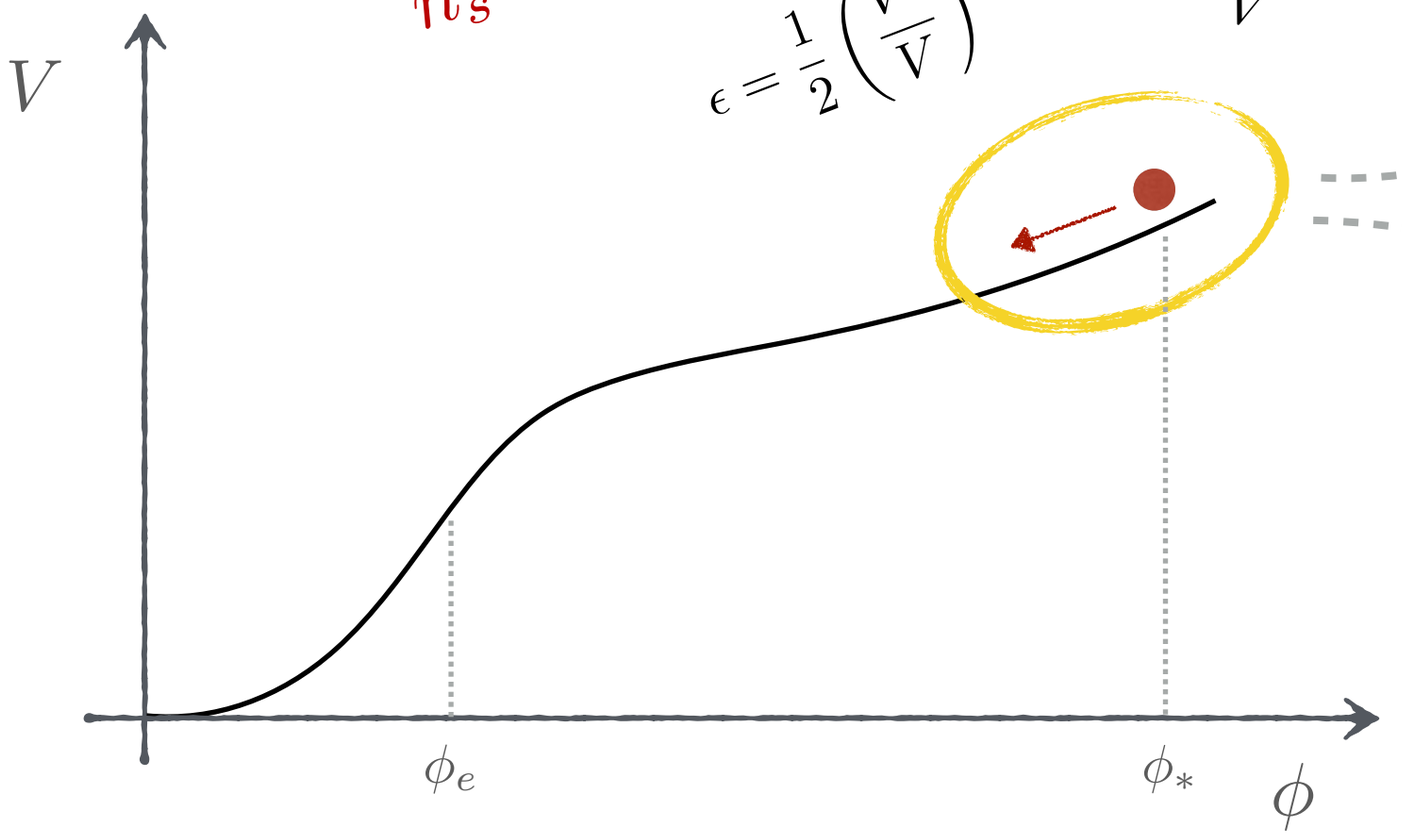


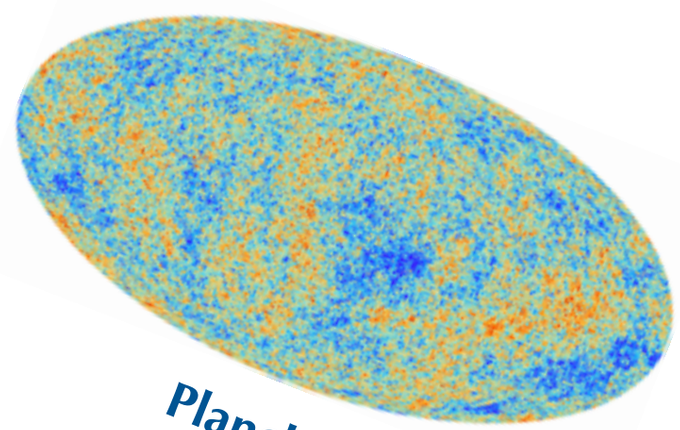


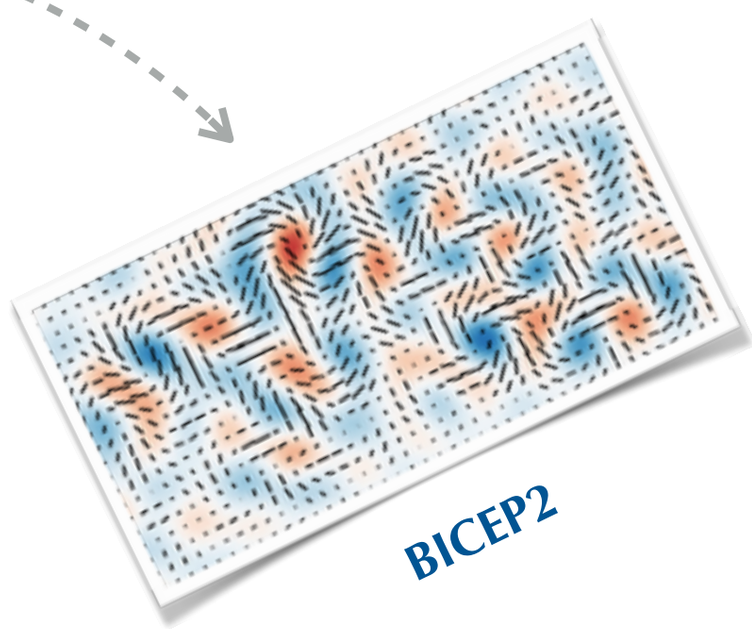
Slow-roll Inflation

$$n_s = 1 + 2\eta - 6\epsilon \qquad r = 16\epsilon$$

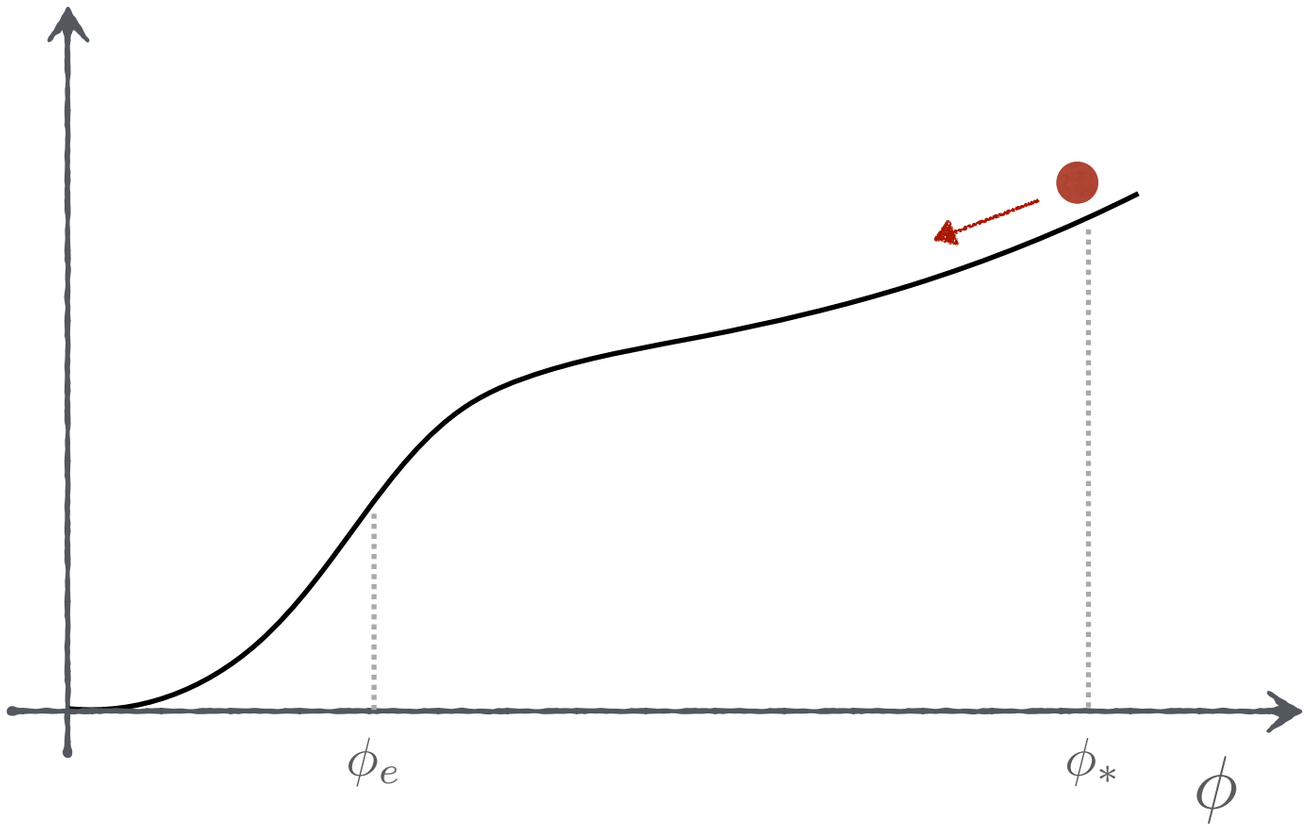
$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{V''}{V}$$



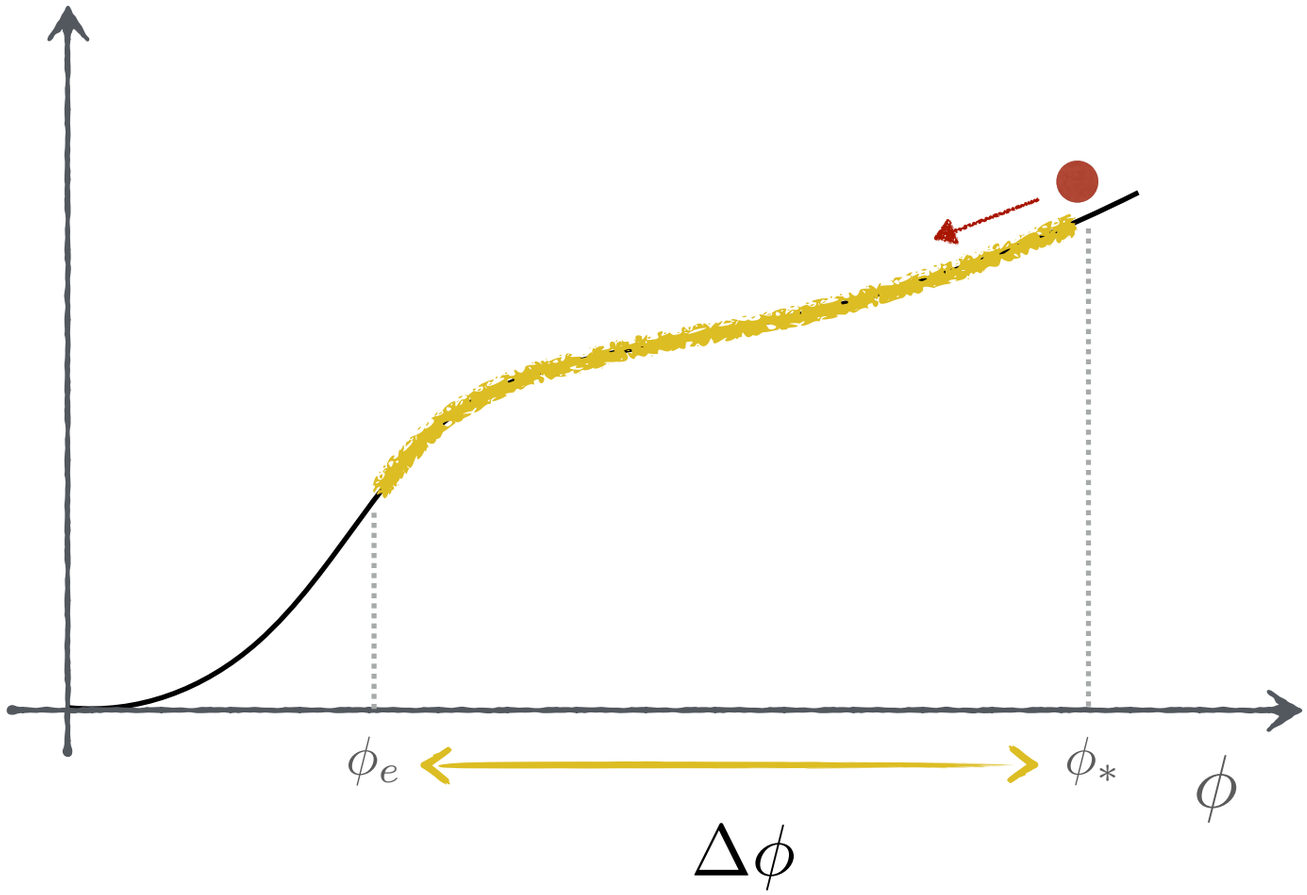

Planck


BICEP2

Slow-roll Inflation



Slow-roll Inflation



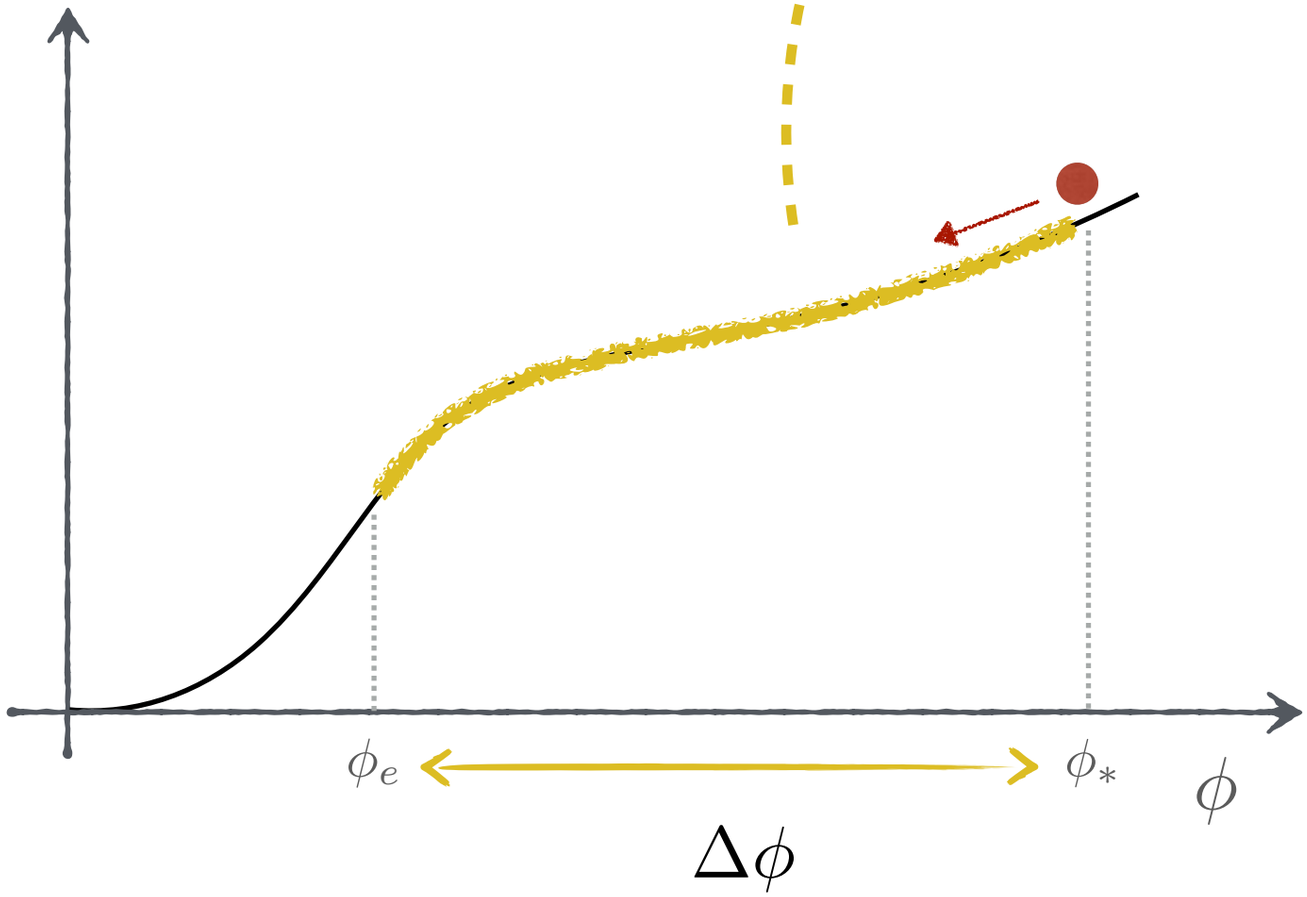
Slow-roll Inflation

$M_P = 1$

LARGE field models $\Delta\phi > 1$

&

SMALL field models $\Delta\phi < 1$



Slow-roll Inflation

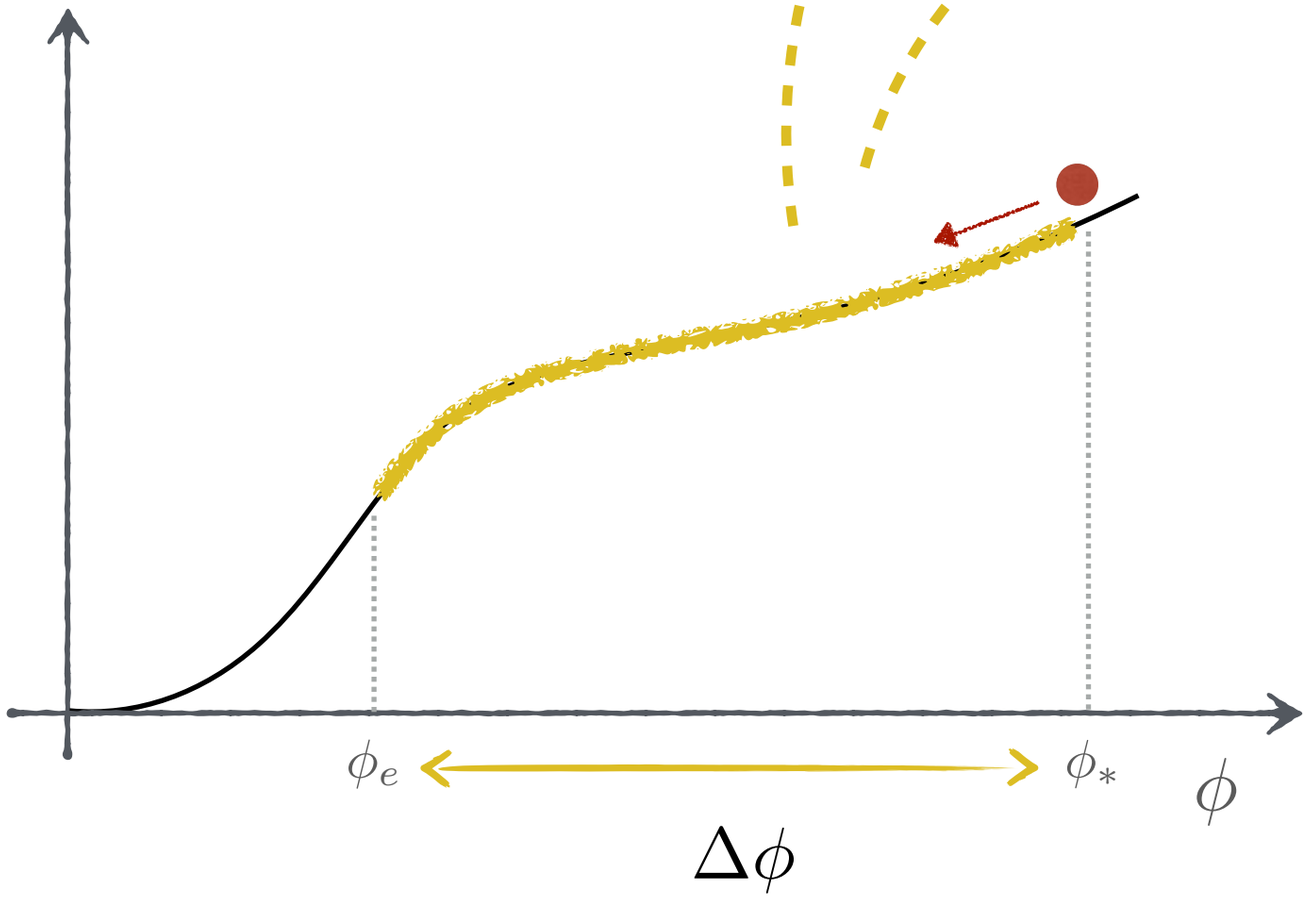
$M_P = 1$

LARGE field models $\Delta\phi > 1$

&

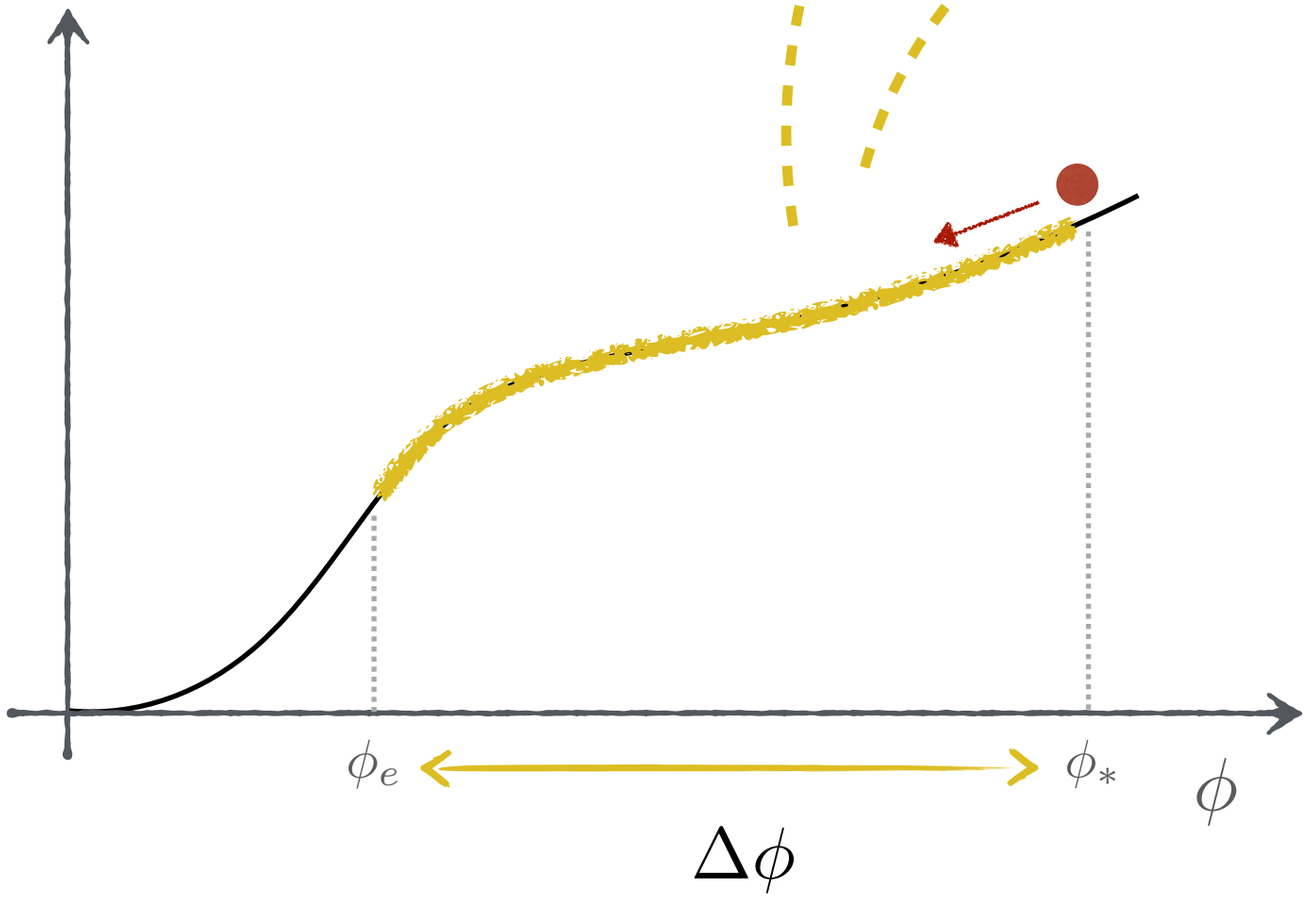
SMALL field models $\Delta\phi < 1$

Crucial Question
in EFT of inflation



Slow-roll Inflation

$M_P = 1$



LARGE field models $\Delta\phi > 1$

&

SMALL field models $\Delta\phi < 1$

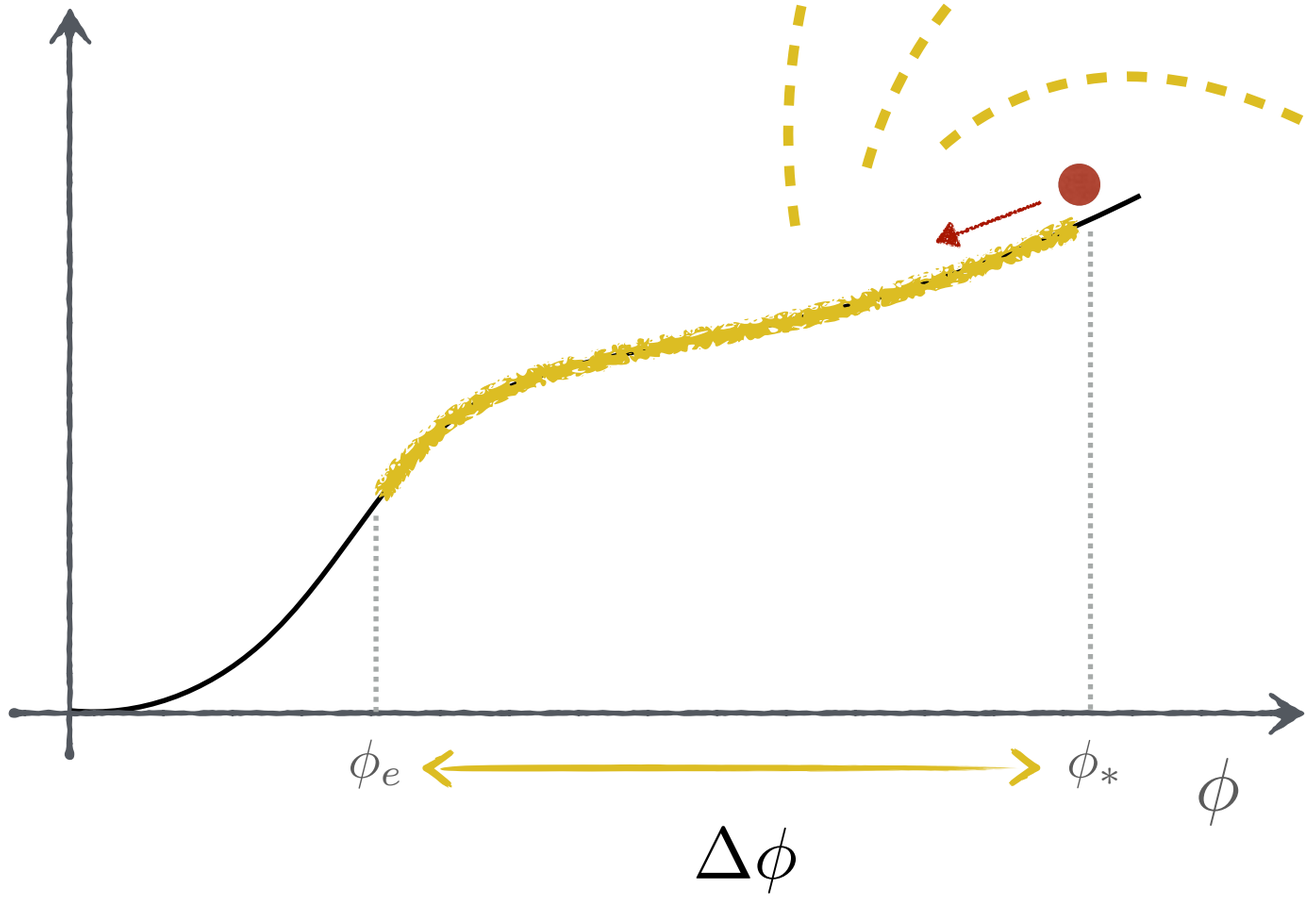
Crucial Question

in EFT of inflation

[see **Westphal** & **Shiu** 's talks on Friday]

Slow-roll Inflation

$M_P = 1$



LARGE field models $\Delta\phi > 1$

&

SMALL field models $\Delta\phi < 1$

Crucial Question

in EFT of inflation

[see **Westphal** & **Shiu** 's talks on Friday]

$\Delta\phi \longleftrightarrow (n_s, r)$?

The Lyth bound(s)

The Lyth bound(s)

$$\frac{d\phi}{dN} = \sqrt{\frac{r(N)}{8}}$$

The Lyth bound(s)

Number of e-folds

useful to quantify the **amount** of inflation

$$N \equiv \int_{a_*}^{a_e} d\ln a$$

$$\frac{d\phi}{dN} = \sqrt{\frac{r(N)}{8}}$$

The Lyth bound(s)

Number of e-folds

useful to quantify the **amount** of inflation

$$N \equiv \int_{a_*}^{a_e} d\ln a$$

$$\frac{d\phi}{dN} = \sqrt{\frac{r(N)}{8}}$$

$$r(N) = 16\epsilon(N)$$

The Lyth bound(s)

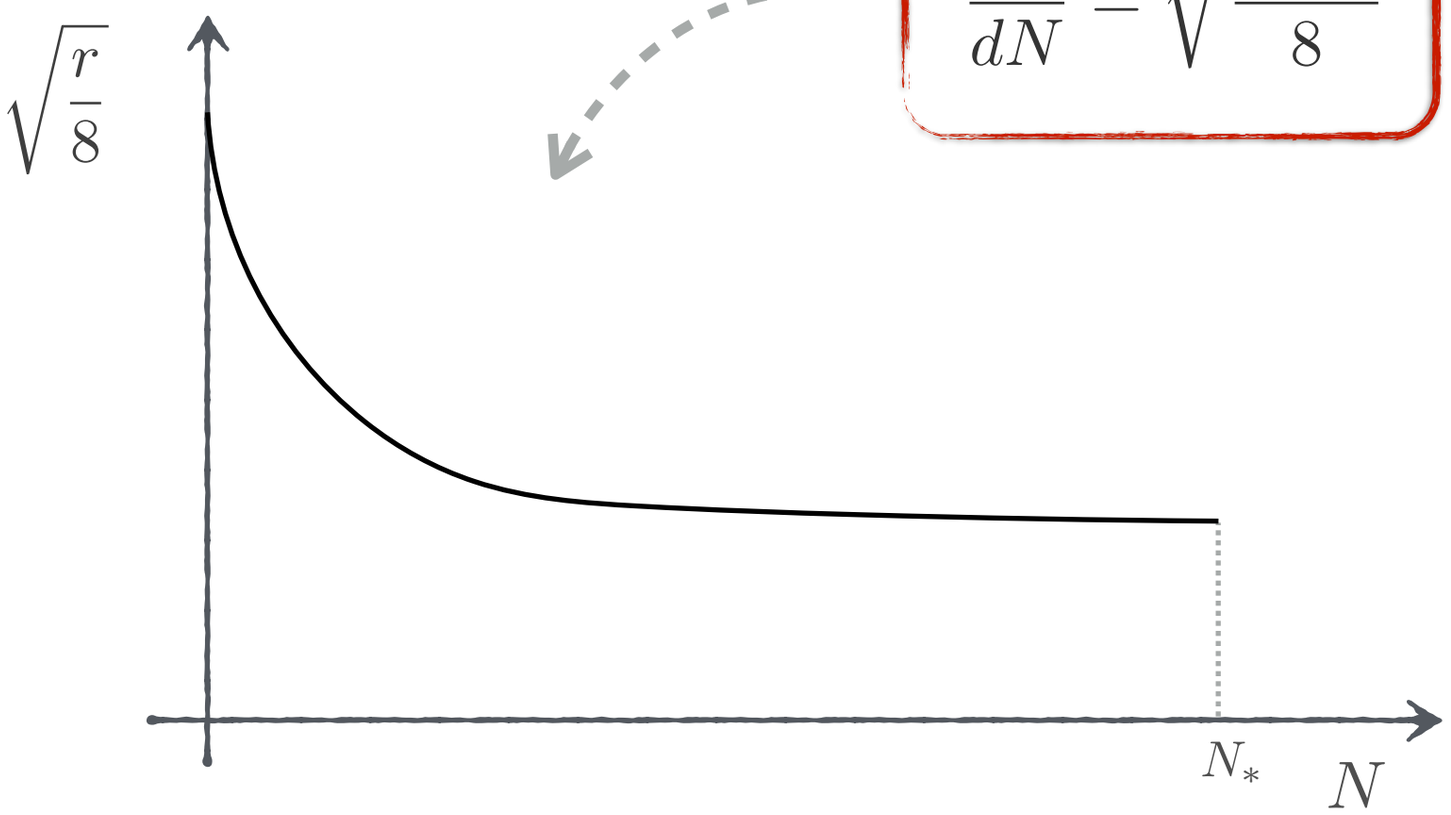
Number of e-folds

useful to quantify the **amount** of inflation

$$N \equiv \int_{a_*}^{a_e} d\ln a$$

$$\frac{d\phi}{dN} = \sqrt{\frac{r(N)}{8}}$$

$$r(N) = 16\epsilon(N)$$



The Lyth bound(s)

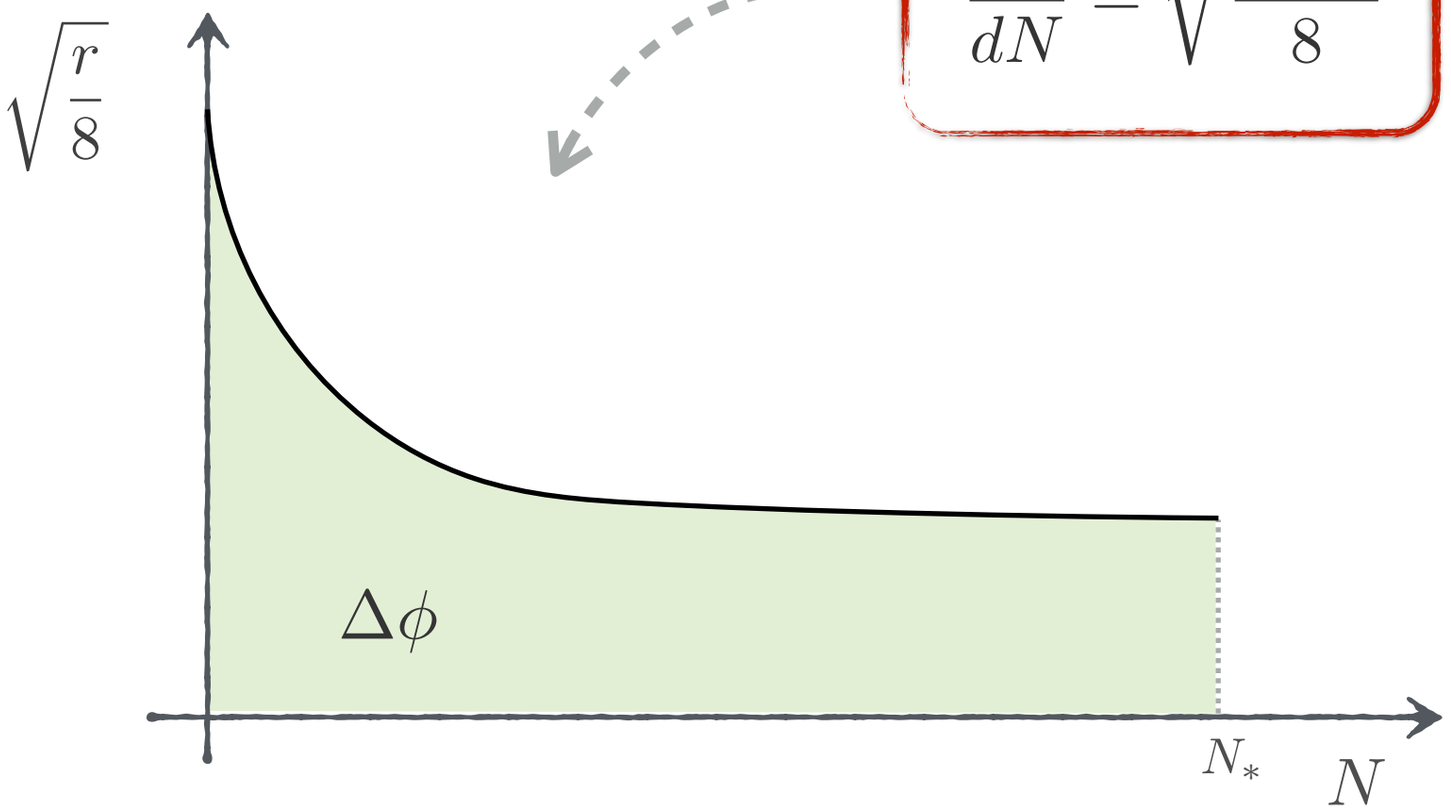
Number of e-folds

useful to quantify the **amount** of inflation

$$N \equiv \int_{a_*}^{a_e} d\ln a$$

$$\frac{d\phi}{dN} = \sqrt{\frac{r(N)}{8}}$$

$$r(N) = 16\epsilon(N)$$



The Lyth bound(s)

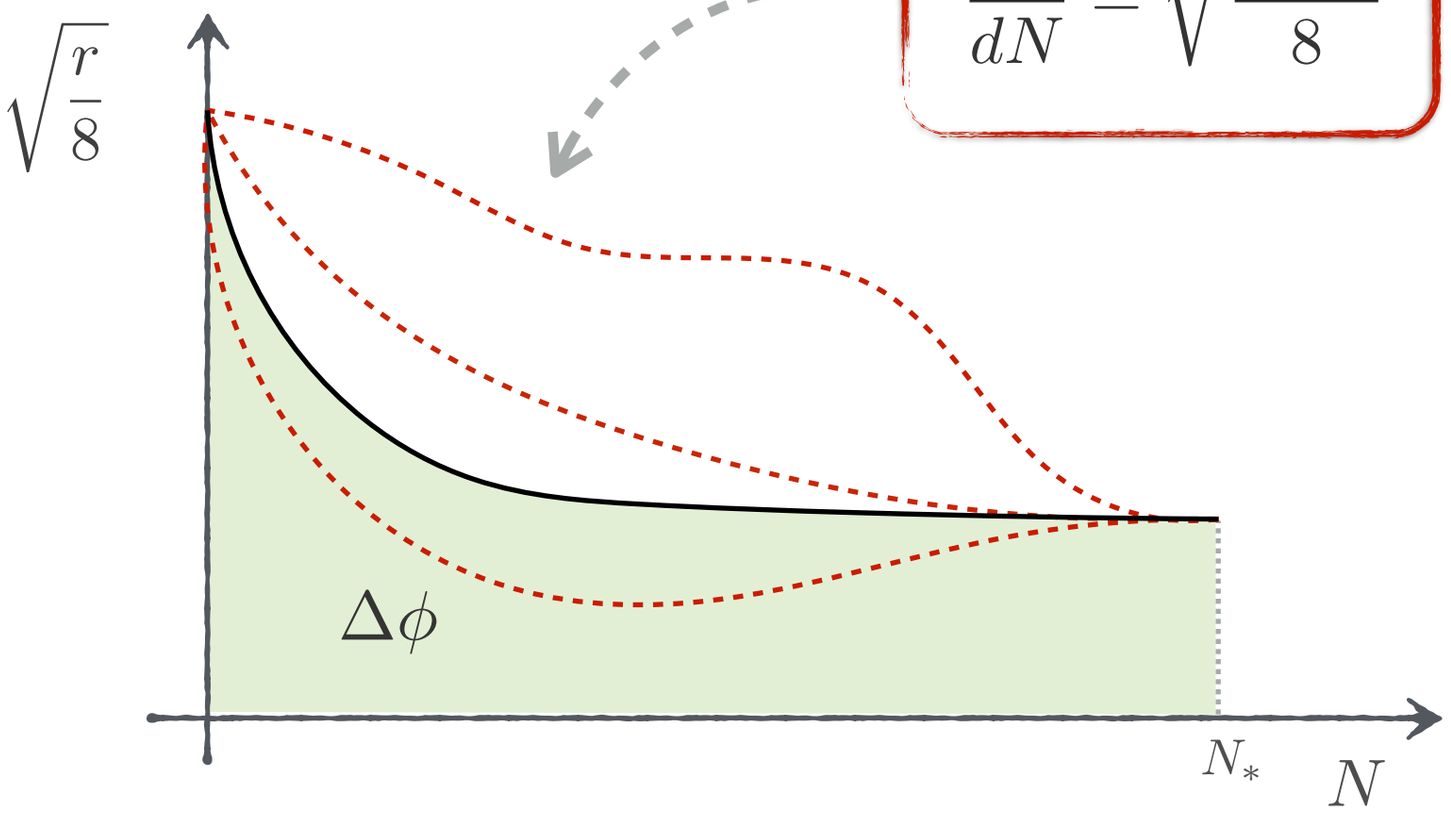
Number of e-folds

useful to quantify the **amount** of inflation

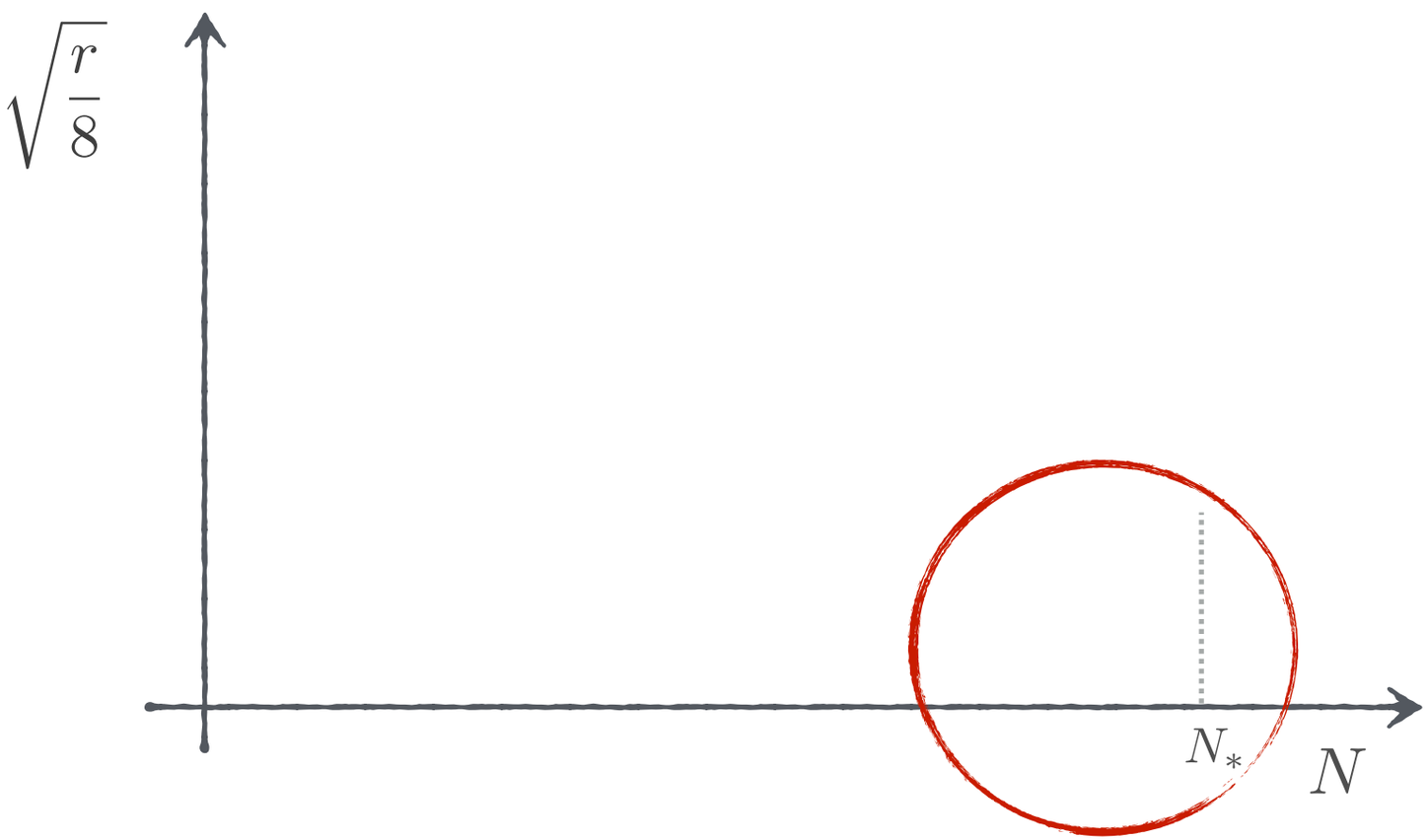
$$N \equiv \int_{a_*}^{a_e} d\ln a$$

$$\frac{d\phi}{dN} = \sqrt{\frac{r(N)}{8}}$$

$$r(N) = 16\epsilon(N)$$



The Lyth bound(s)



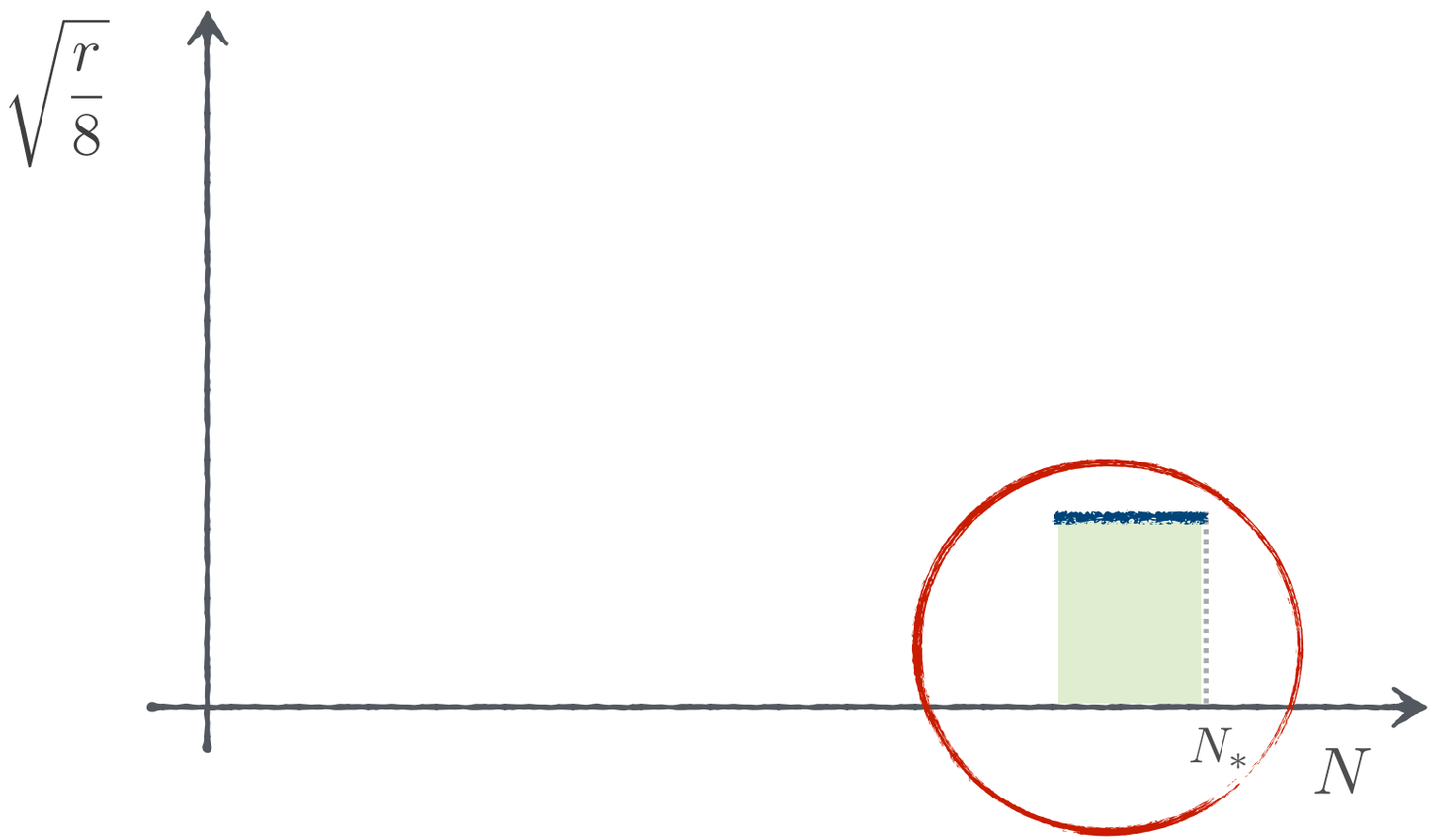
$\Delta N \simeq 8$

CMB window

The Lyth bound(s)

- **Constant tensor-to-scalar ratio** during the first **8 e-folds** of inflation [Lyth 1996]

$$\Delta\phi \sim \sqrt{\frac{r}{0.12}}$$



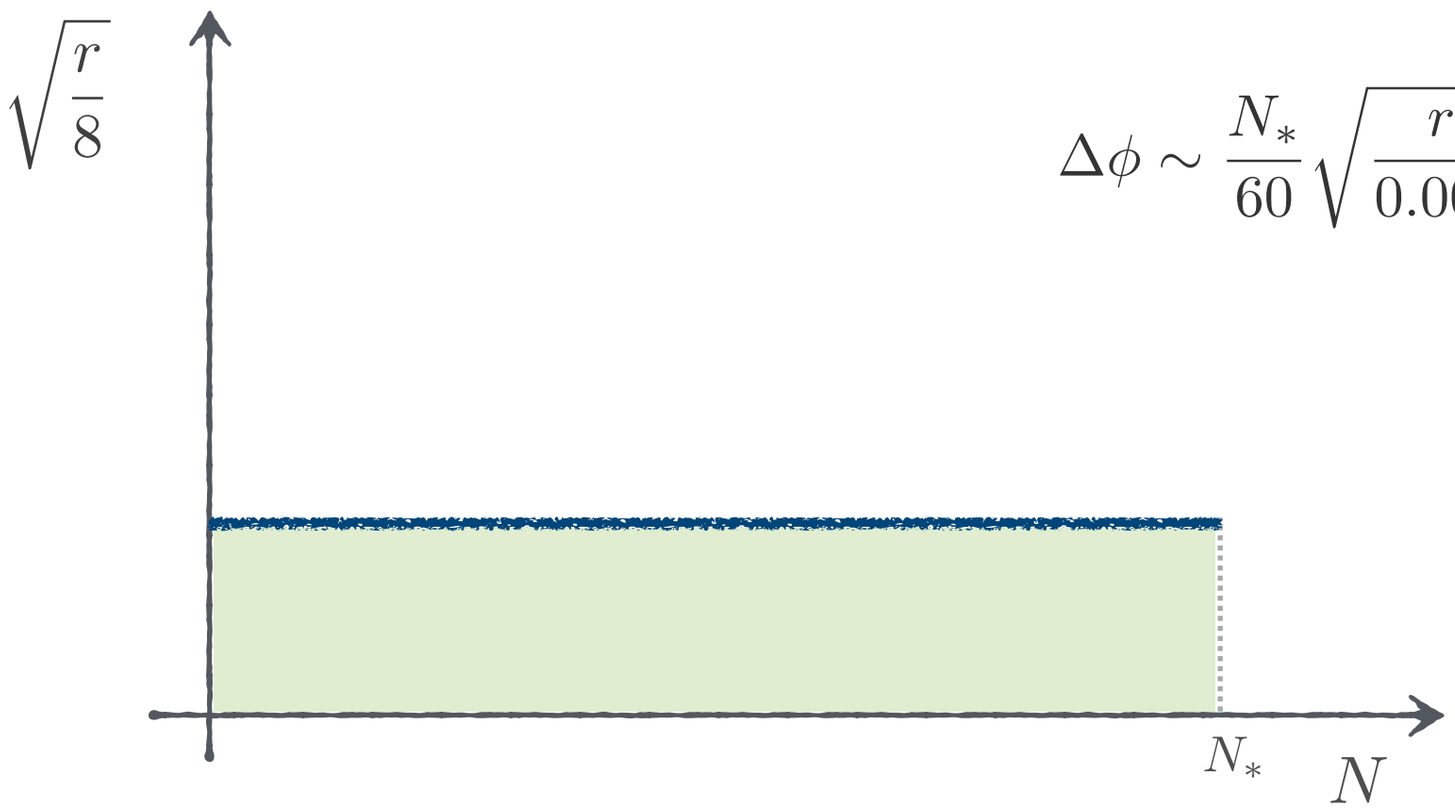
The Lyth bound(s)

- **Constant tensor-to-scalar ratio** during the first **8 e-folds** of inflation [Lyth 1996]

$$\Delta\phi \sim \sqrt{\frac{r}{0.12}}$$

- **Constant tensor-to-scalar ratio** during the whole **60 e-folds** of inflation [Boubekeur & Lyth 2005]

$$\Delta\phi \sim \frac{N_*}{60}\sqrt{\frac{r}{0.002}}$$



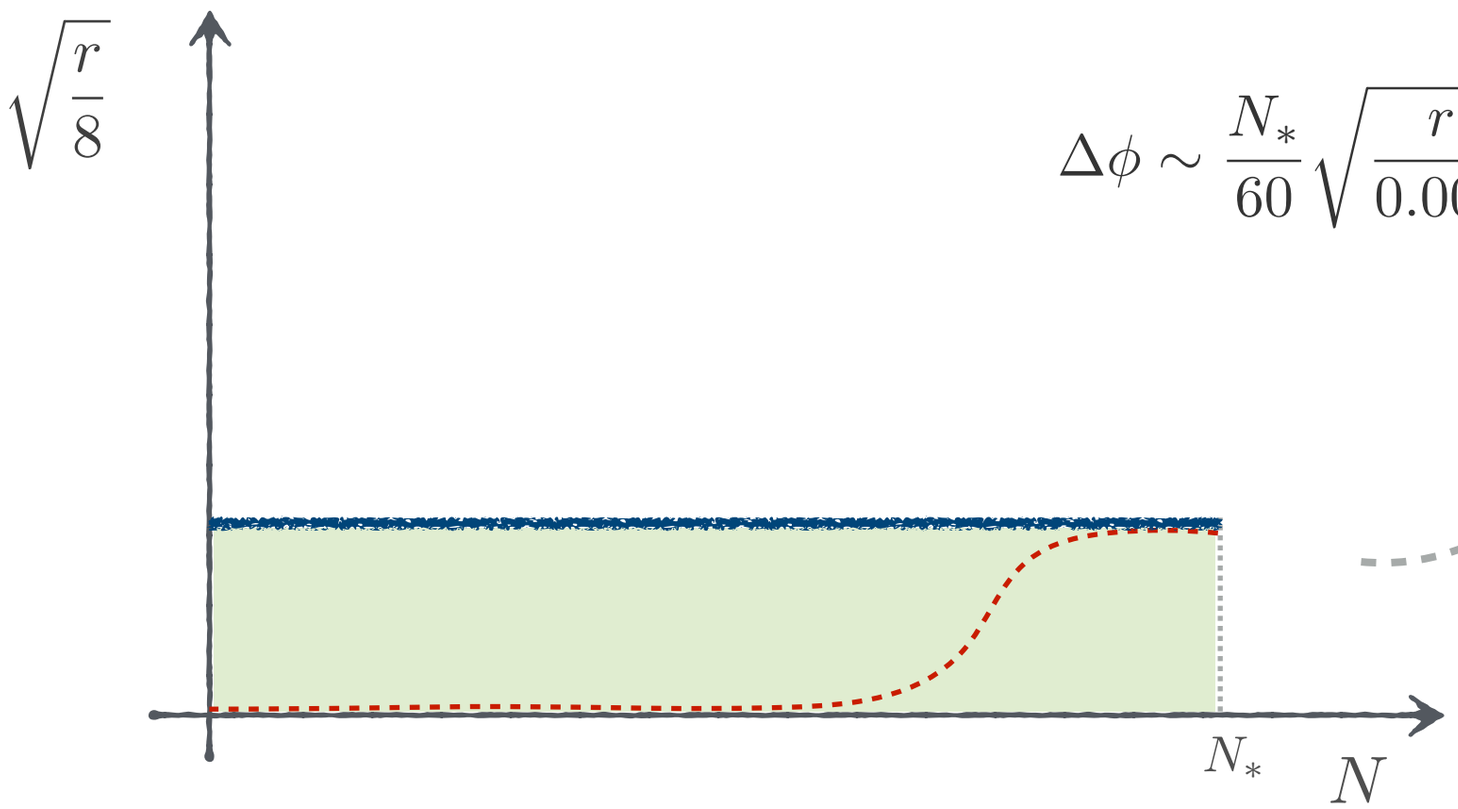
The Lyth bound(s)

- **Constant tensor-to-scalar ratio** during the first **8 e-folds** of inflation [Lyth 1996]

$$\Delta\phi \sim \sqrt{\frac{r}{0.12}}$$

- **Constant tensor-to-scalar ratio** during the whole **60 e-folds** of inflation [Boubekeur & Lyth 2005]

$$\Delta\phi \sim \frac{N_*}{60}\sqrt{\frac{r}{0.002}}$$



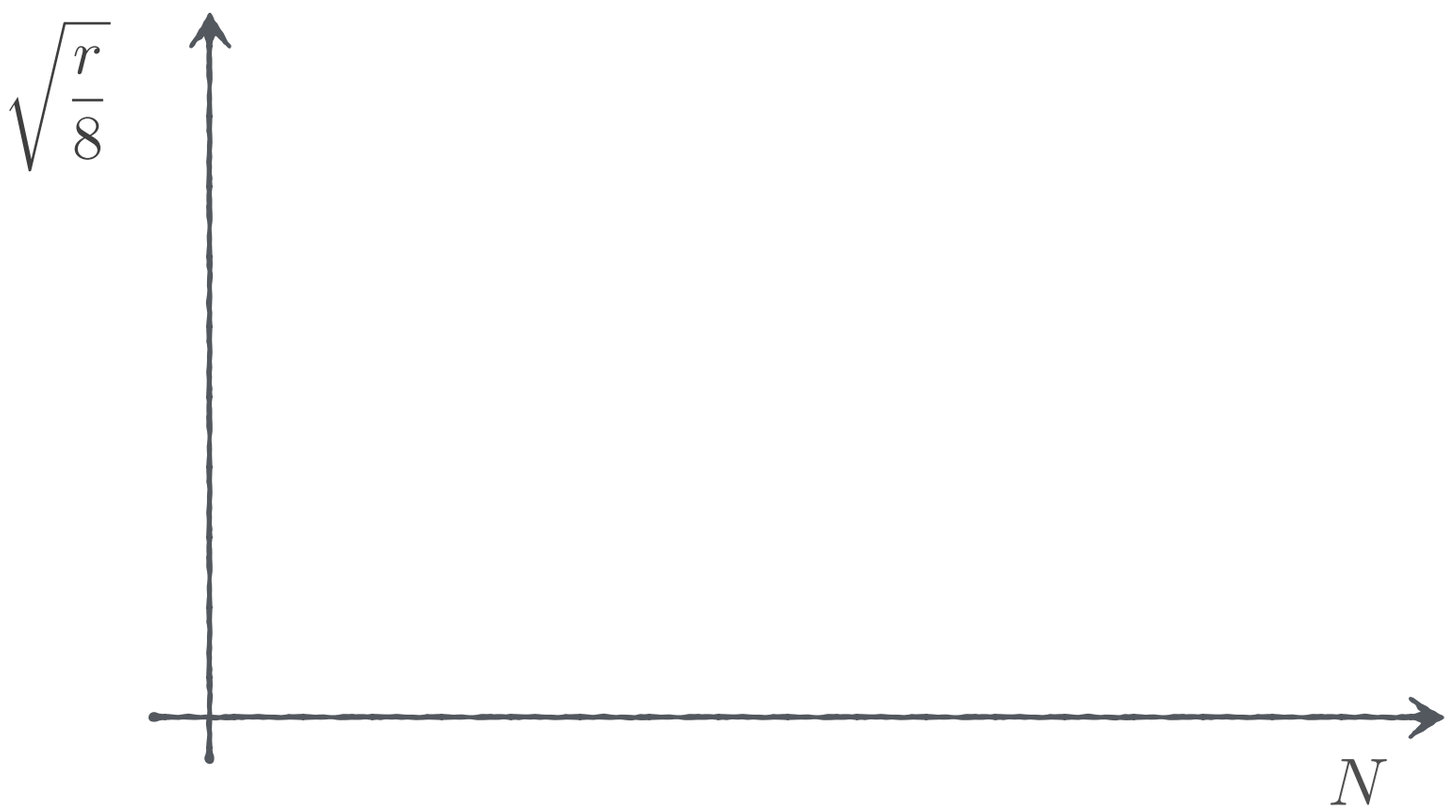
avoiding the bound
&
minimum estimate

[Hotchkiss, Mazumdar, & Nadathur, 2011]

[S. Antusch and D. Nolde, 2014]

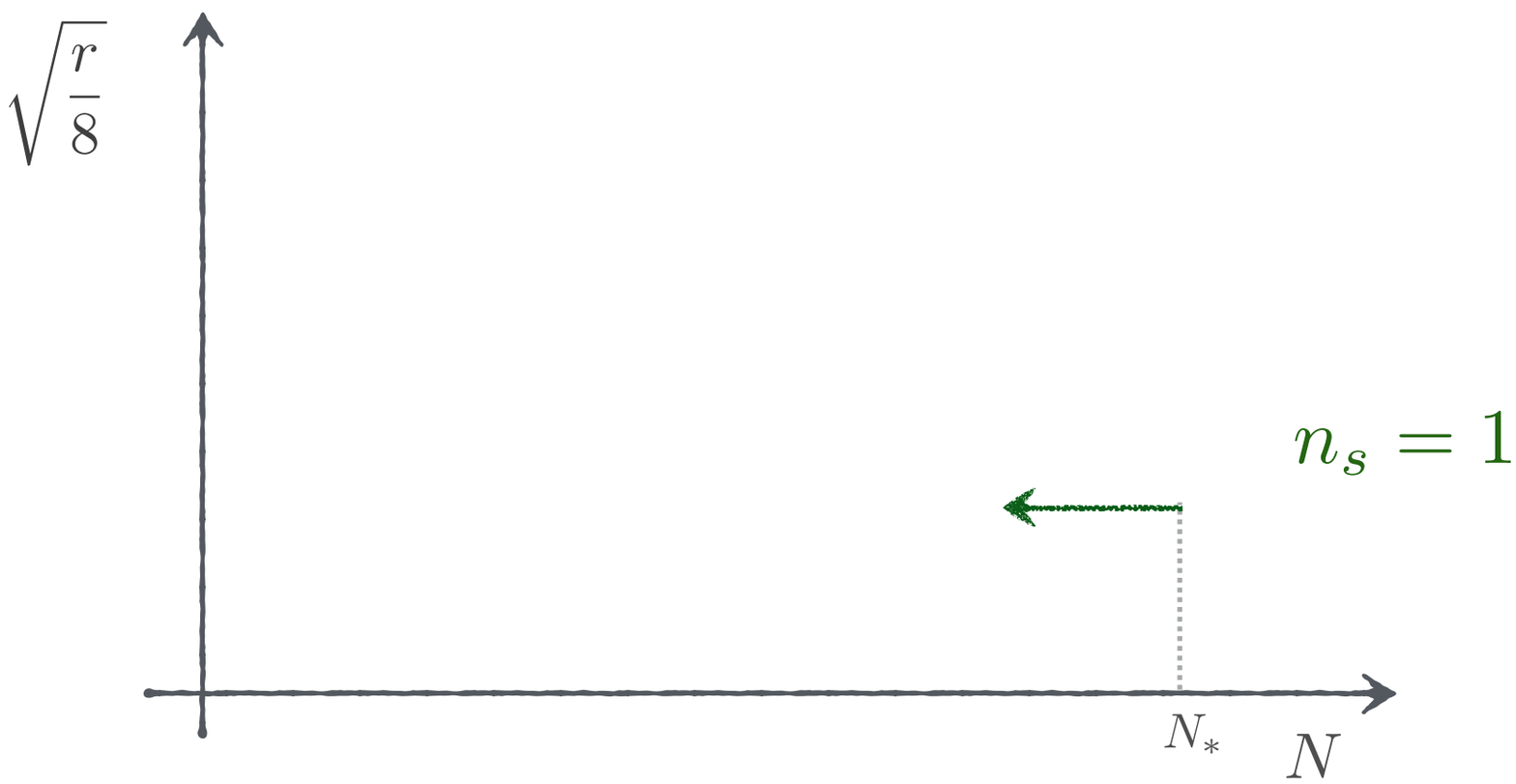
The Lyth bound(s)

when including info on the spectral index n_s



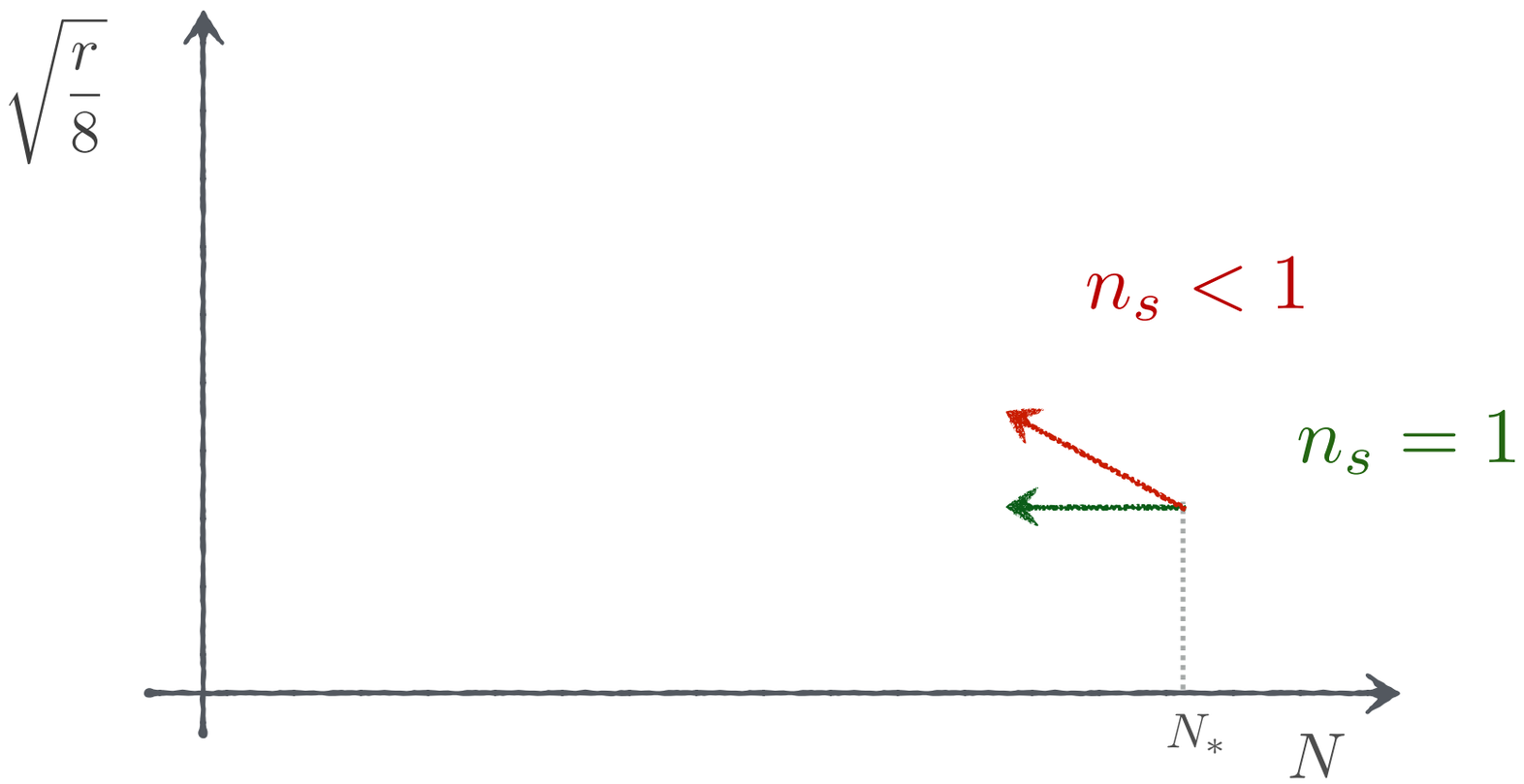
The Lyth bound(s)

when including info on the spectral index n_s



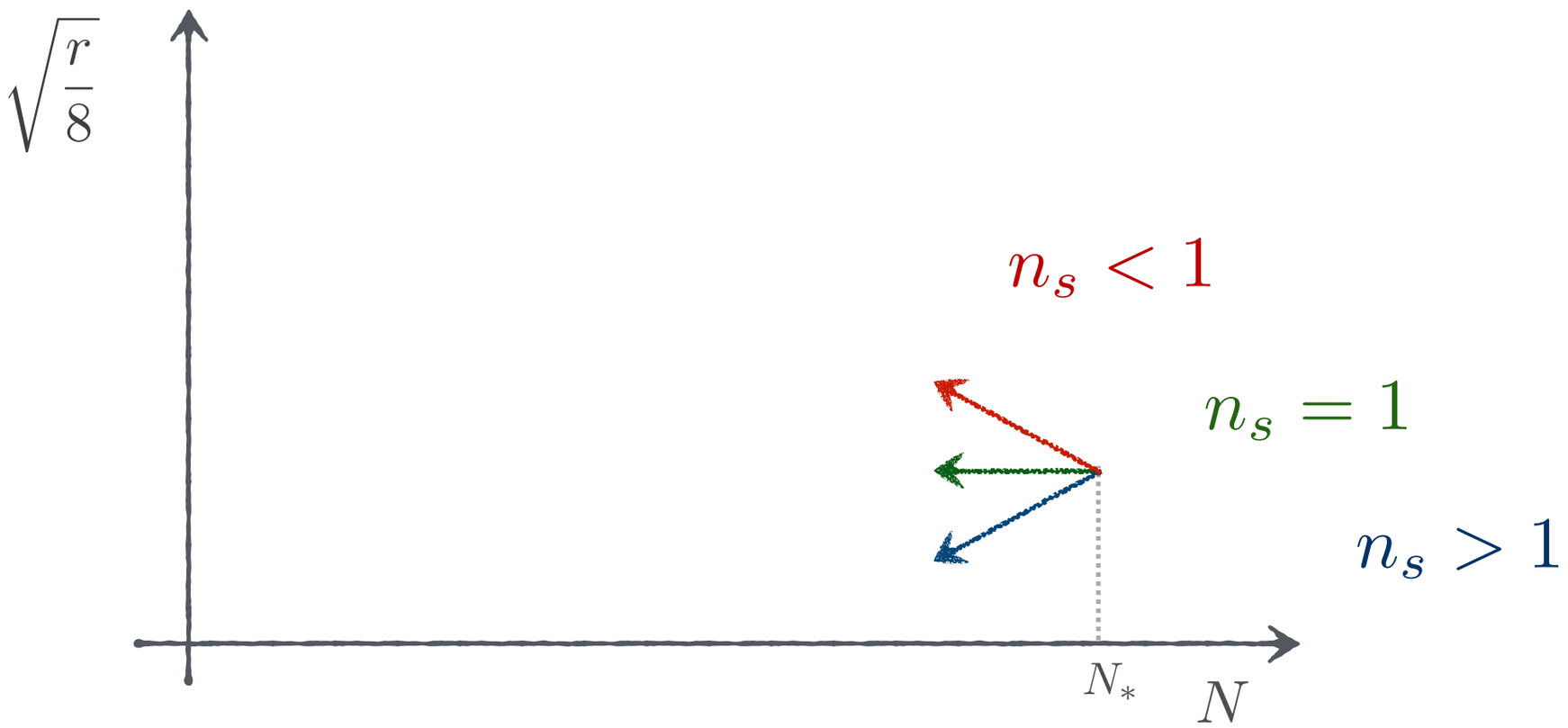
The Lyth bound(s)

when including info on the spectral index n_s



The Lyth bound(s)

when including info on the spectral index n_s

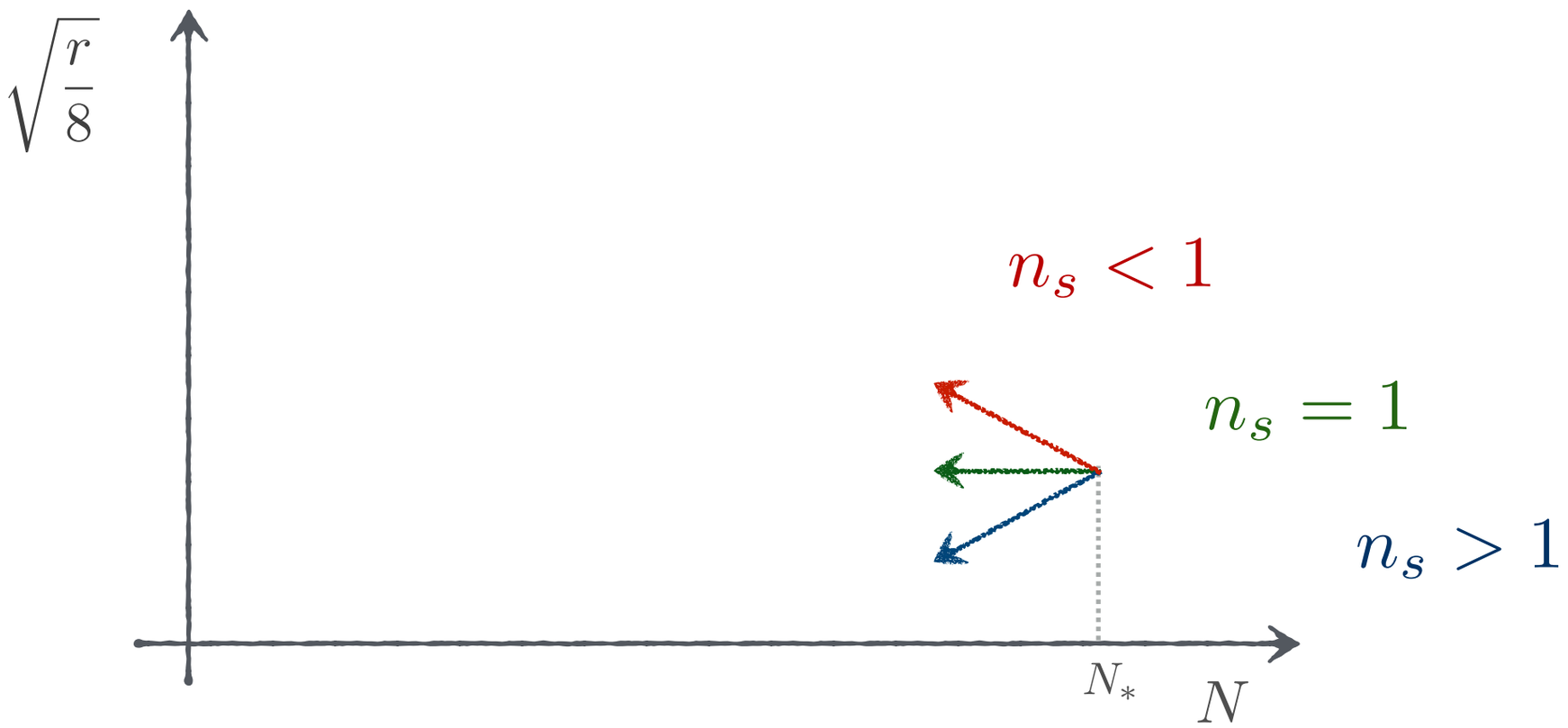


The Lyth bound(s)

when including info on the spectral index n_s

Planck's constraint on n_s

$$n_s = 0.9603 \pm 0.0073$$



The Lyth bound(s)

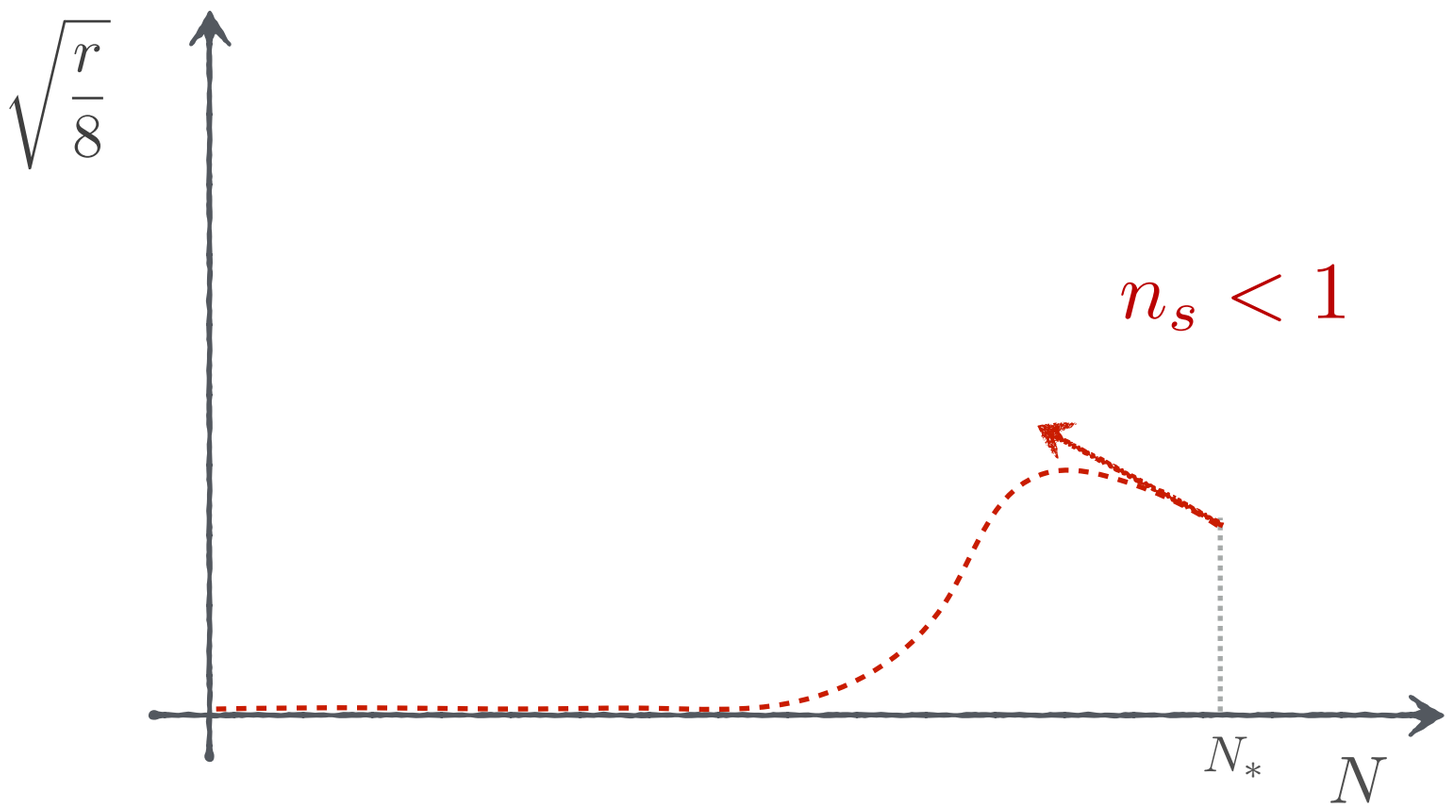
when including info on the spectral index n_s

Planck's constraint on n_s

$$n_s = 0.9603 \pm 0.0073$$

The Lyth bound(s)

when including info on the spectral index n_s



Planck's constraint on n_s

$$n_s = 0.9603 \pm 0.0073$$

$r \sim 0.1$

minimum estimate for $\Delta\phi$

[German, 2014]

[Bramante, Downes, Lehman, and Martin, 2014]

The Lyth bound(s)

when including info on the spectral index n_s

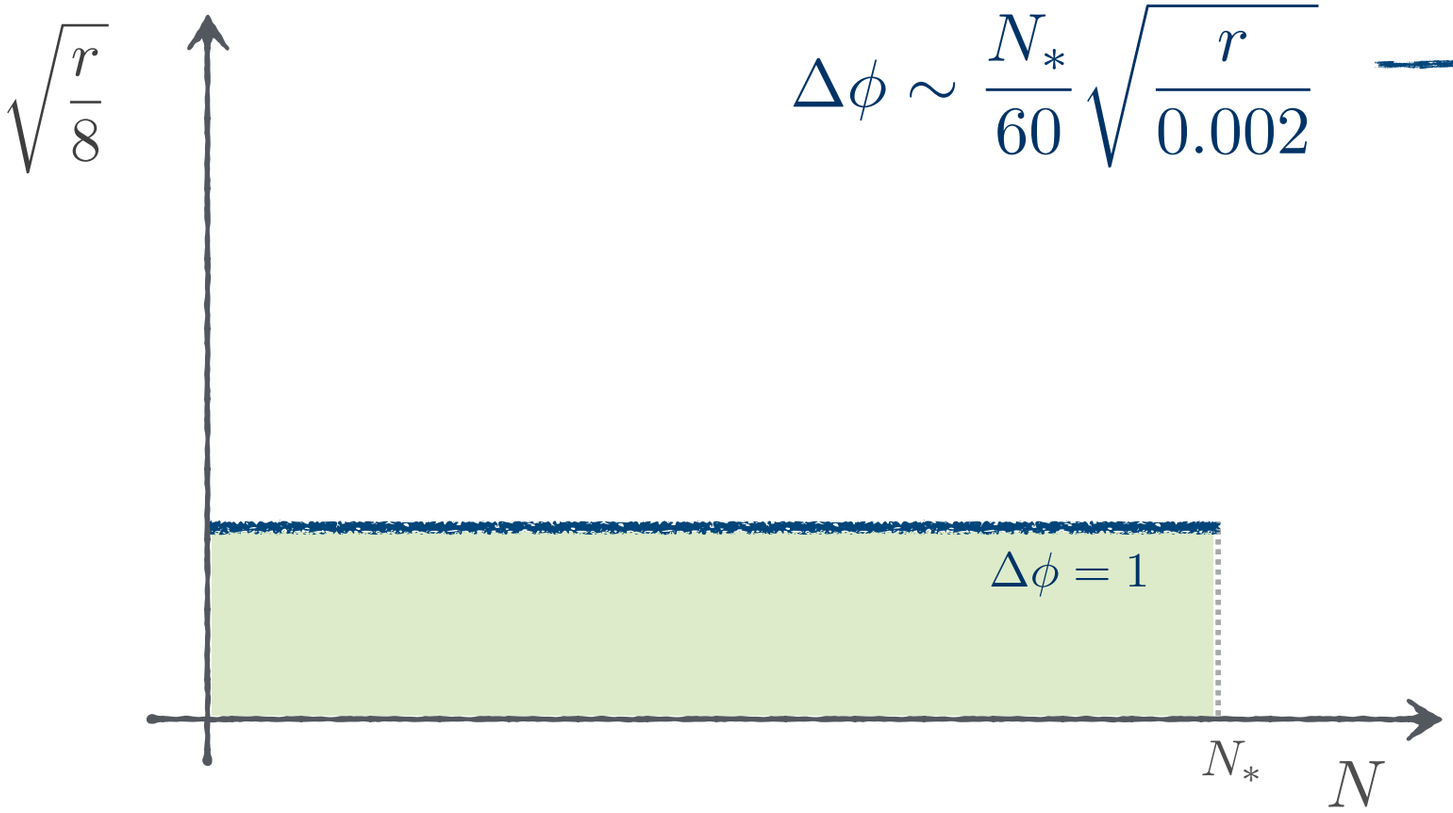
Planck's constraint on n_s

$$n_s = 0.9603 \pm 0.0073$$

$$\Delta\phi \sim \frac{N_*}{60}\sqrt{\frac{r}{0.002}} \longrightarrow$$

sub-Planckian field range for

$$r \lesssim 2 \cdot 10^{-3}$$



The Lyth bound(s)

when including info on the spectral index n_s

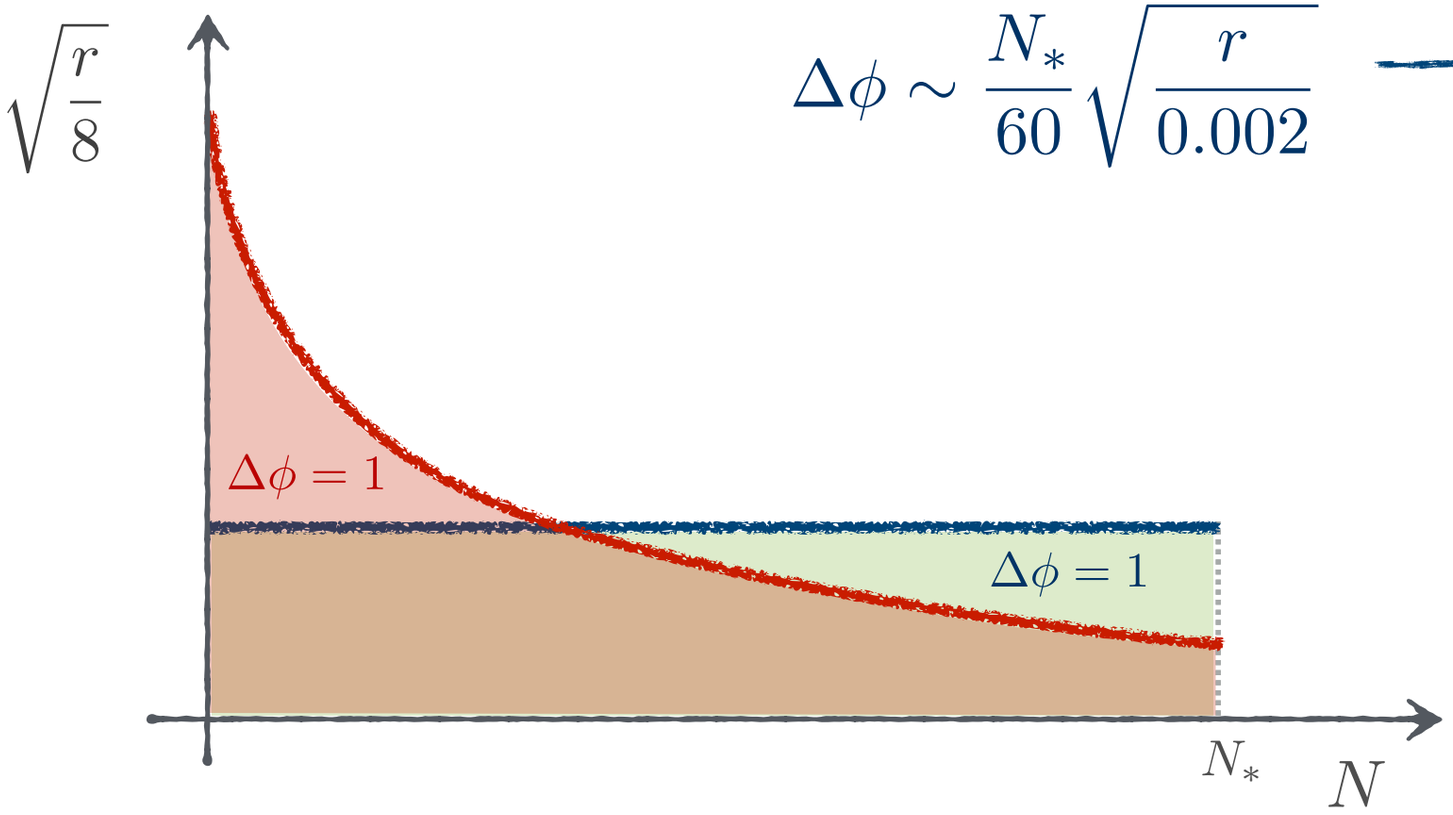
Planck's constraint on n_s

$$n_s = 0.9603 \pm 0.0073$$

$$\Delta\phi \sim \frac{N_*}{60}\sqrt{\frac{r}{0.002}} \longrightarrow$$

sub-Planckian field range for

$$r \lesssim 2 \cdot 10^{-3}$$



The Lyth bound(s)

when including info on the spectral index n_s

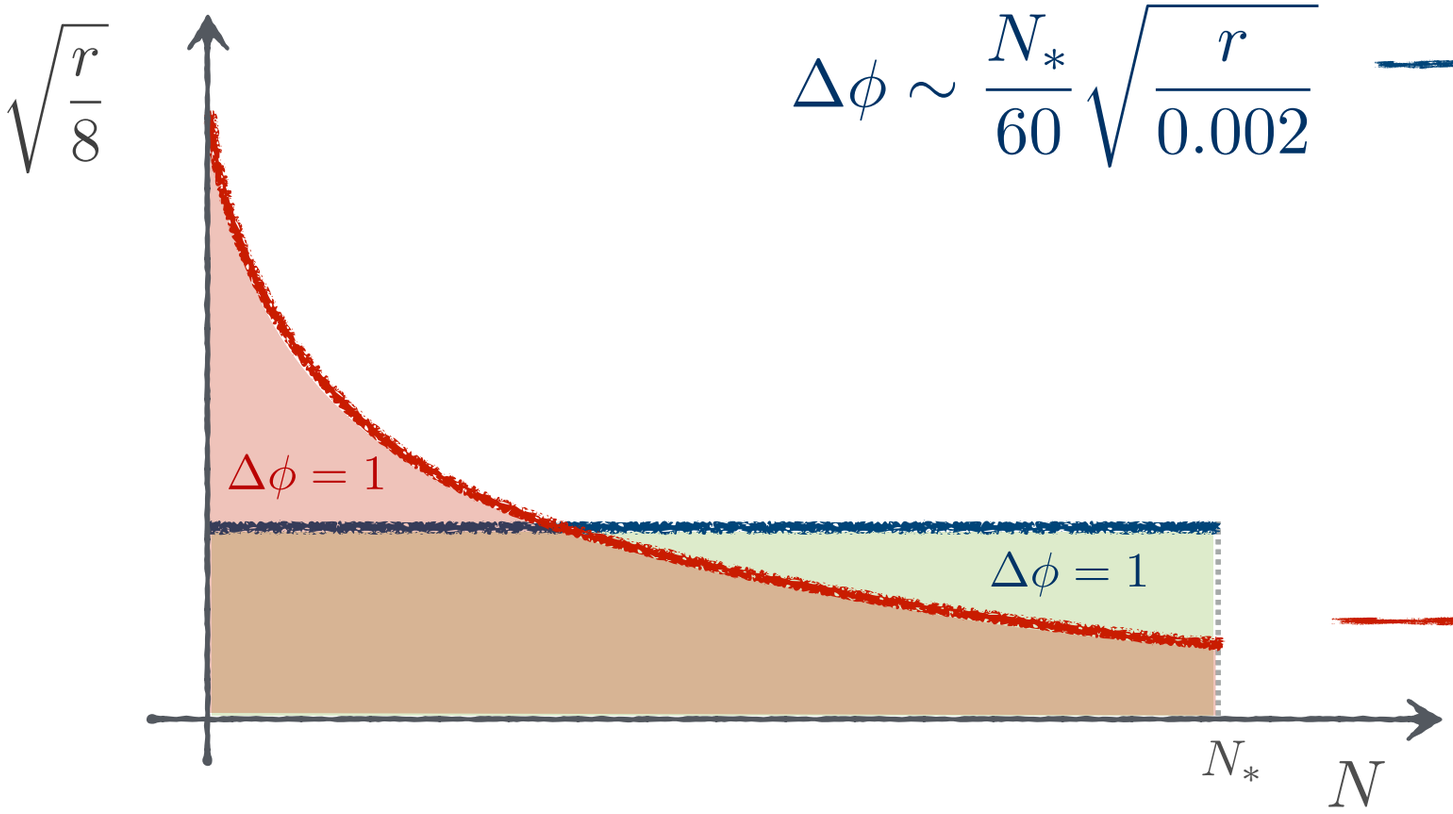
Planck's constraint on n_s

$$n_s = 0.9603 \pm 0.0073$$

$$\Delta\phi \sim \frac{N_*}{60}\sqrt{\frac{r}{0.002}}$$

→ sub-Planckian field range for

$$r \lesssim 2 \cdot 10^{-3}$$



when taking into account the value of the spectral index

sub-Planckian field range for lower values of r

The Lyth bound(s)

when including info on the spectral index n_s

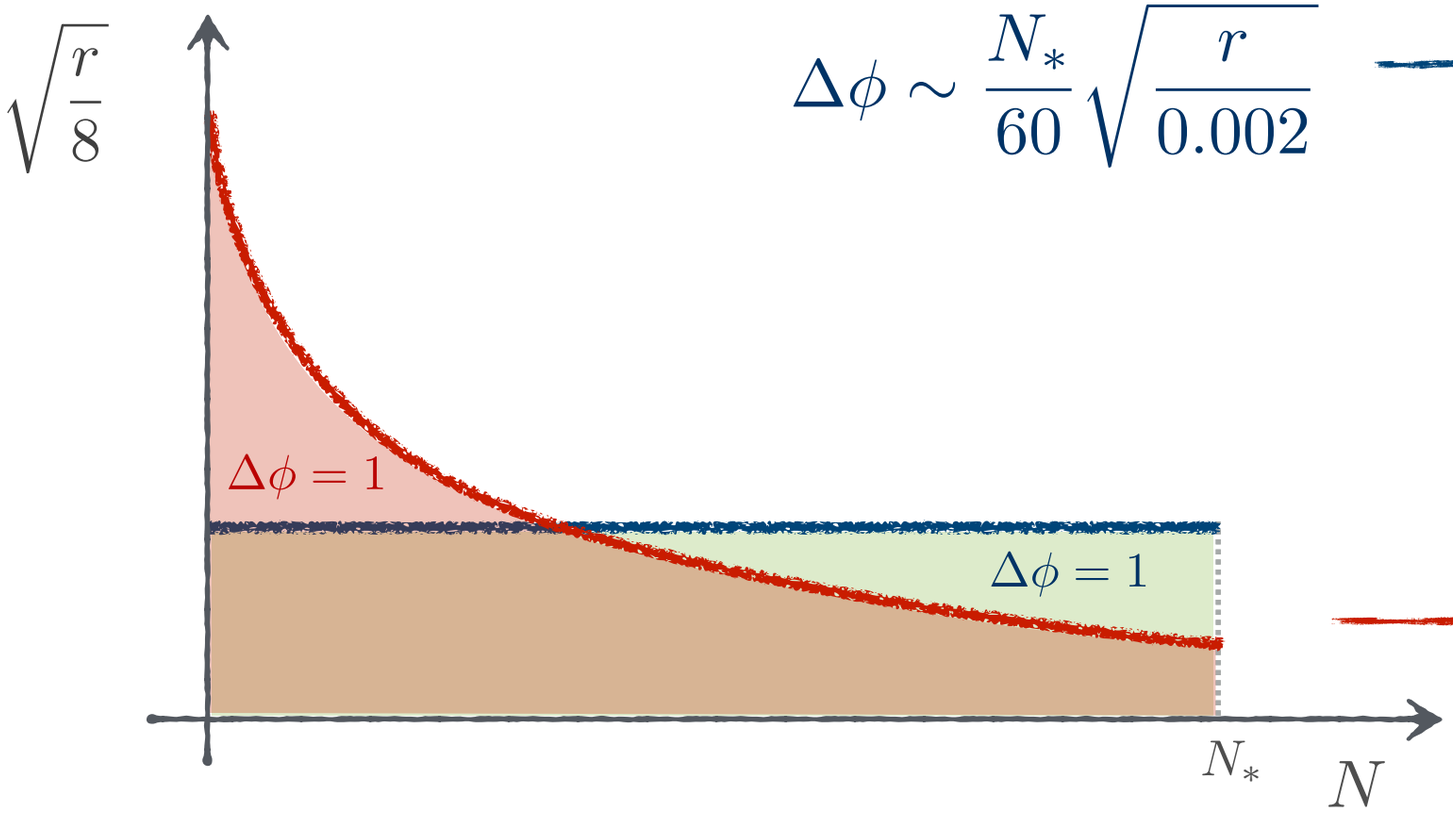
Planck's constraint on n_s

$$n_s = 0.9603 \pm 0.0073$$

$$\Delta\phi \sim \frac{N_*}{60}\sqrt{\frac{r}{0.002}}$$

sub-Planckian field range for

$$r \lesssim 2 \cdot 10^{-3}$$



when taking into account the value of the spectral index

sub-Planckian field range for lower values of r

$$r \lesssim 2 \cdot 10^{-5}$$

The Lyth bound(s)

when including info on the spectral index n_s

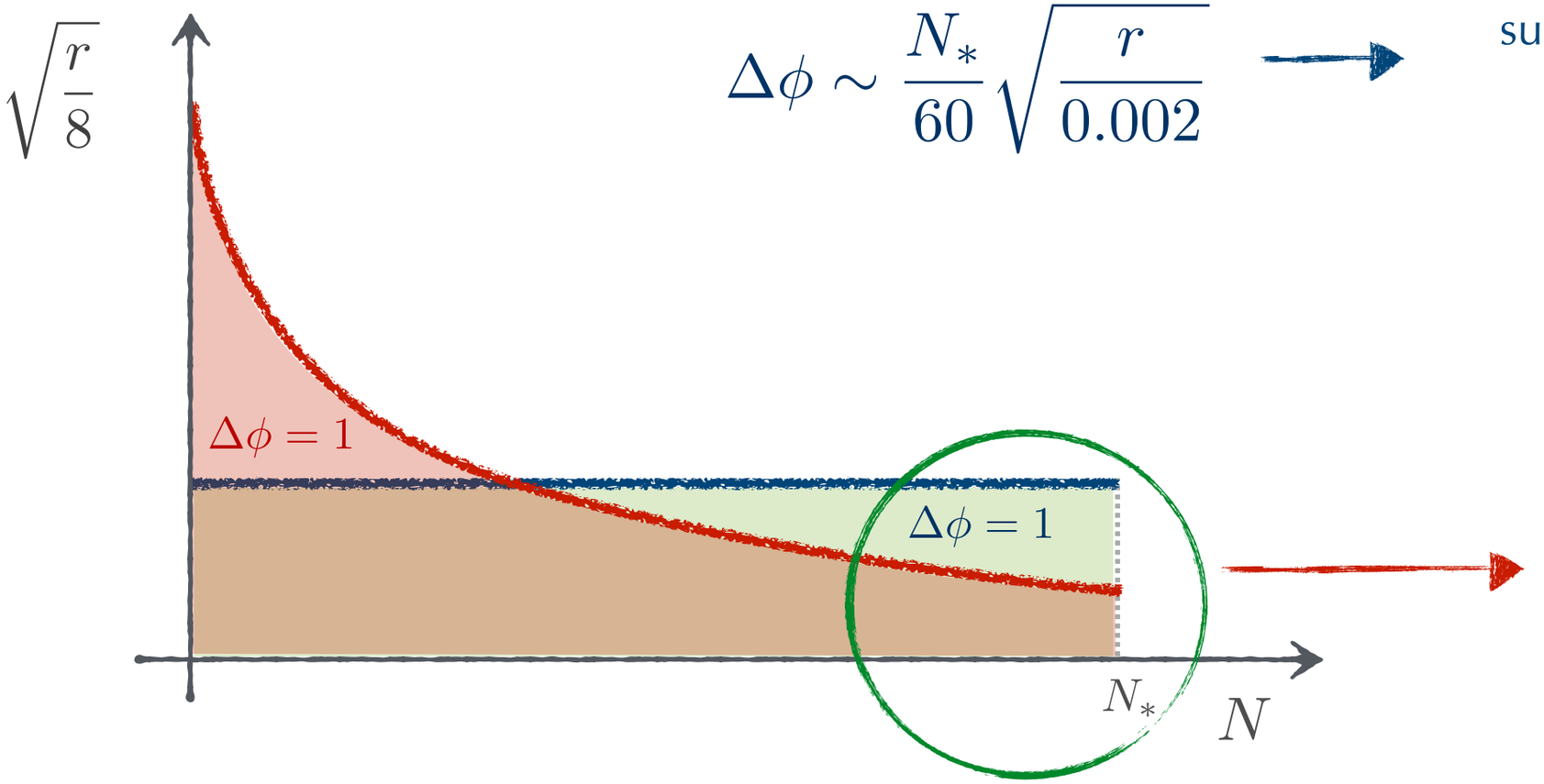
Planck's constraint on n_s

$$n_s = 0.9603 \pm 0.0073$$

$$\Delta\phi \sim \frac{N_*}{60}\sqrt{\frac{r}{0.002}}$$

sub-Planckian field range for

$$r \lesssim 2 \cdot 10^{-3}$$



when taking into account the value of the spectral index

sub-Planckian field range for lower values of r

$$r \lesssim 2 \cdot 10^{-5}$$

The Lyth bound(s)

when including info on the spectral index n_s

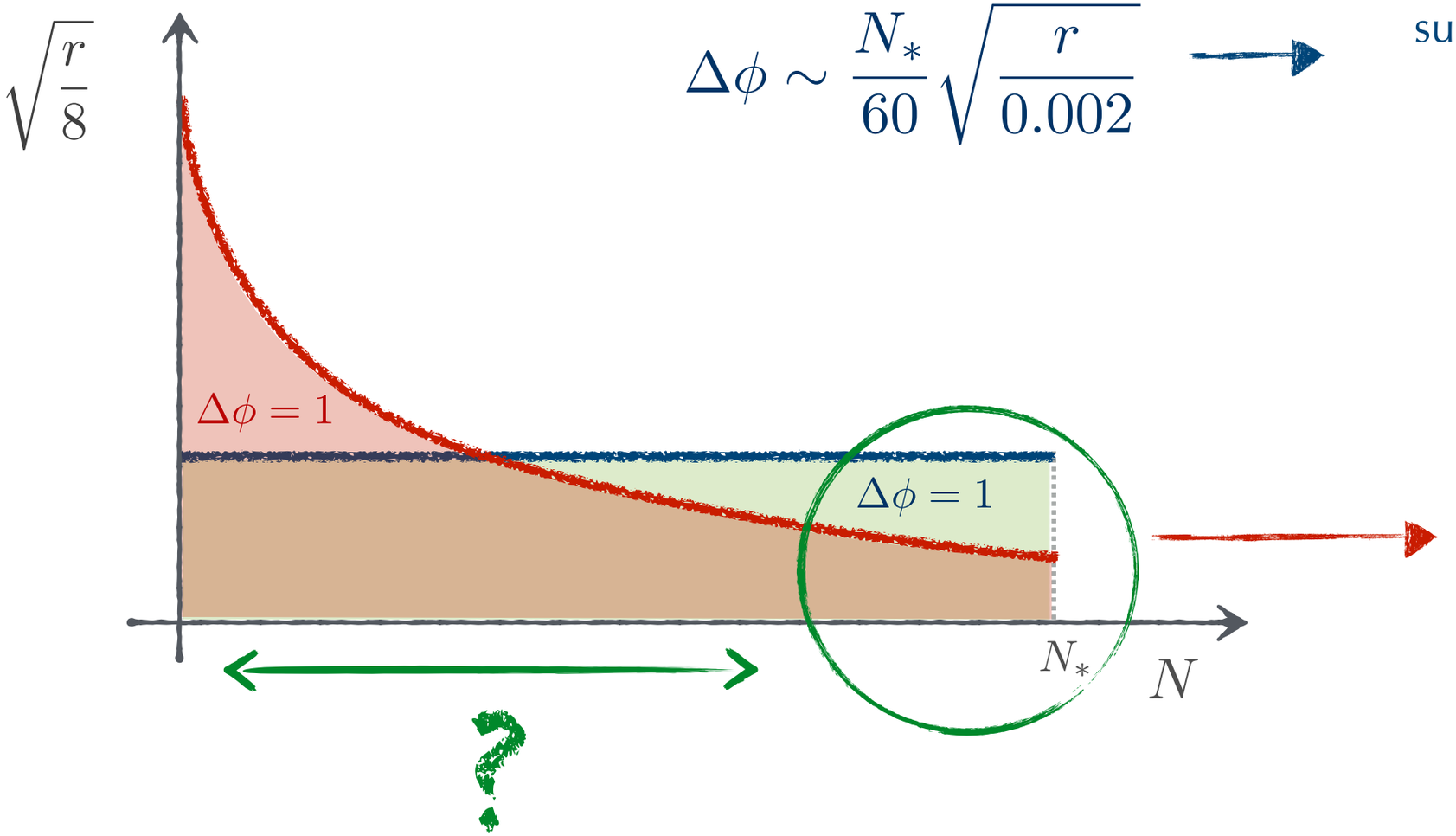
Planck's constraint on n_s

$$n_s = 0.9603 \pm 0.0073$$

$$\Delta\phi \sim \frac{N_*}{60}\sqrt{\frac{r}{0.002}}$$

sub-Planckian field range for

$$r \lesssim 2 \cdot 10^{-3}$$



when taking into account the value of the spectral index

sub-Planckian field range for lower values of r

$$r \lesssim 2 \cdot 10^{-5}$$

Universality

Universality at large N

percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

ϕ_e

Universality at large N

percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$\downarrow$$

$$n_s \propto \frac{1}{N}$$

ϕ_e

Universality at large N

percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} \quad + \quad \textbf{sub-leading corrections}$$

Universality at large N

percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

irrelevant for observations

$$\epsilon = \frac{\beta}{N^p} \quad + \quad \textbf{sub-leading corrections}$$

Universality at large N

percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} \quad + \quad \textbf{sub-leading corrections}$$

irrelevant for observations

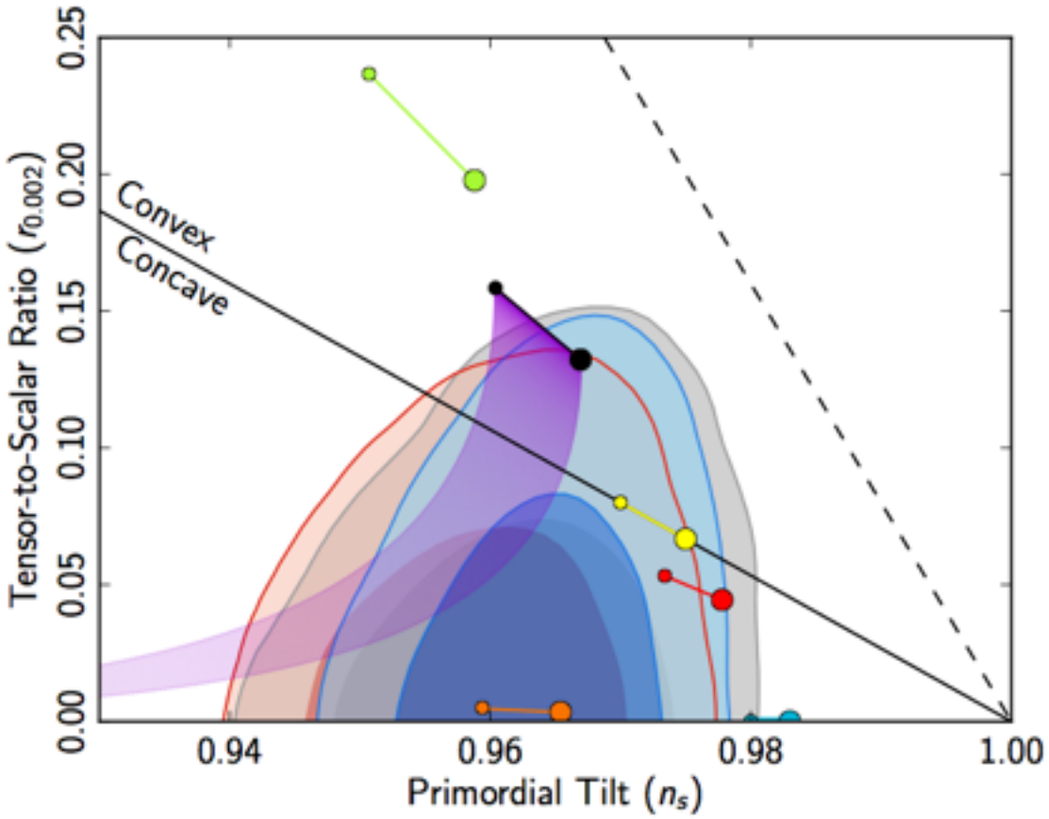
universality classes of inflation

in terms of n_s & r

[Mukhanov 2013]

[Roest 2013, Garcia-Bellido & Roest 2014]

Universality at large N



percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

irrelevant for observations

$$\epsilon = \frac{\beta}{N^p} \quad + \text{ sub-leading corrections}$$

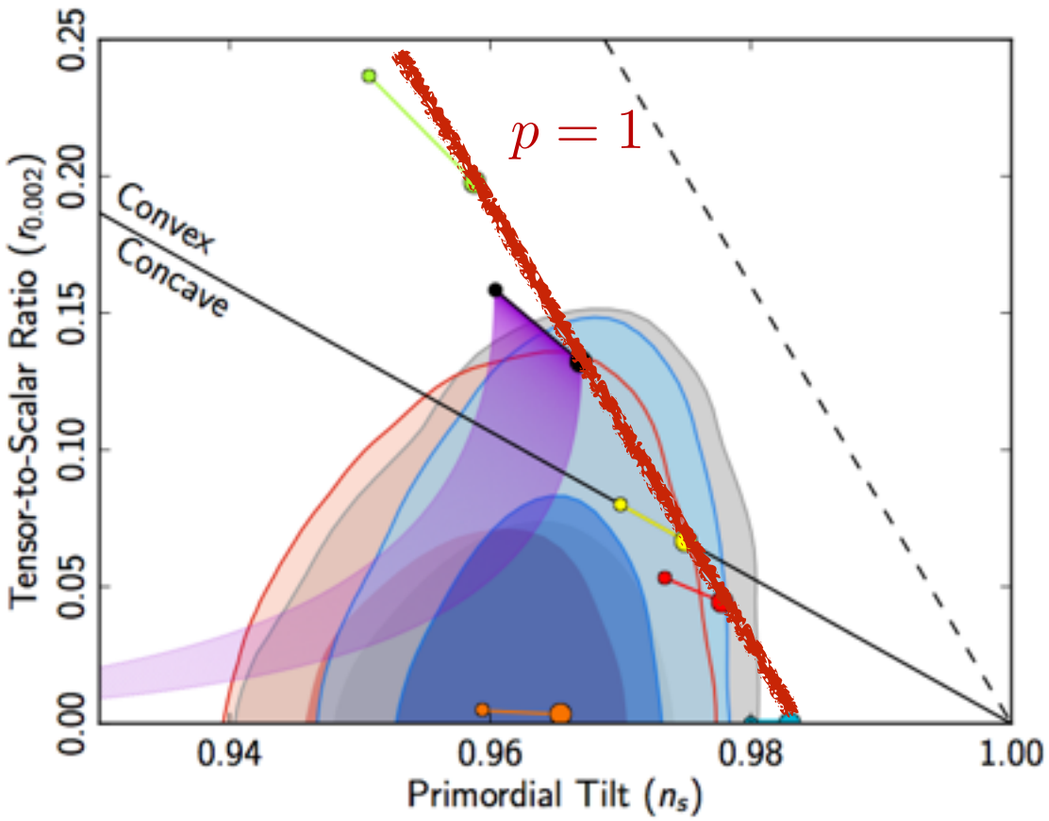
universality classes of inflation

in terms of n_s & r

[Mukhanov 2013]

[Roest 2013, Garcia-Bellido & Roest 2014]

Universality at large N



percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} + \text{sub-leading corrections}$$

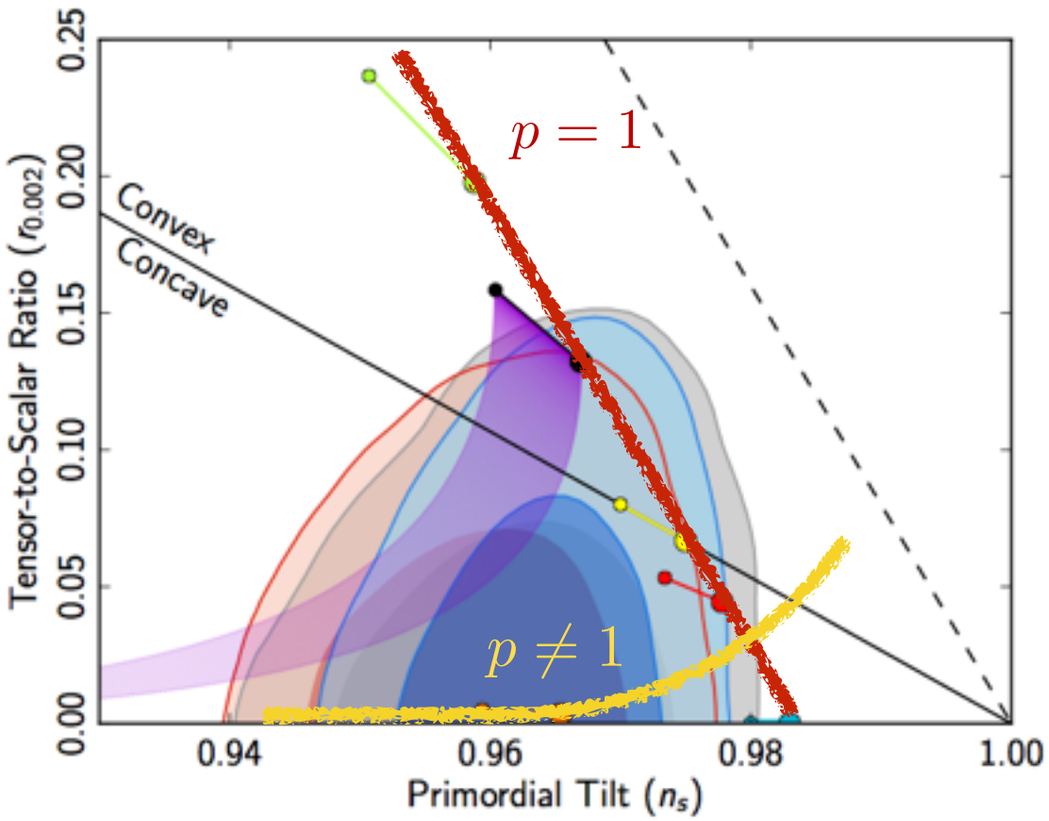
irrelevant for observations

universality classes of inflation
in terms of n_s & r

[Mukhanov 2013]

[Roest 2013, Garcia-Bellido & Roest 2014]

Universality at large N



percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} + \text{sub-leading corrections}$$

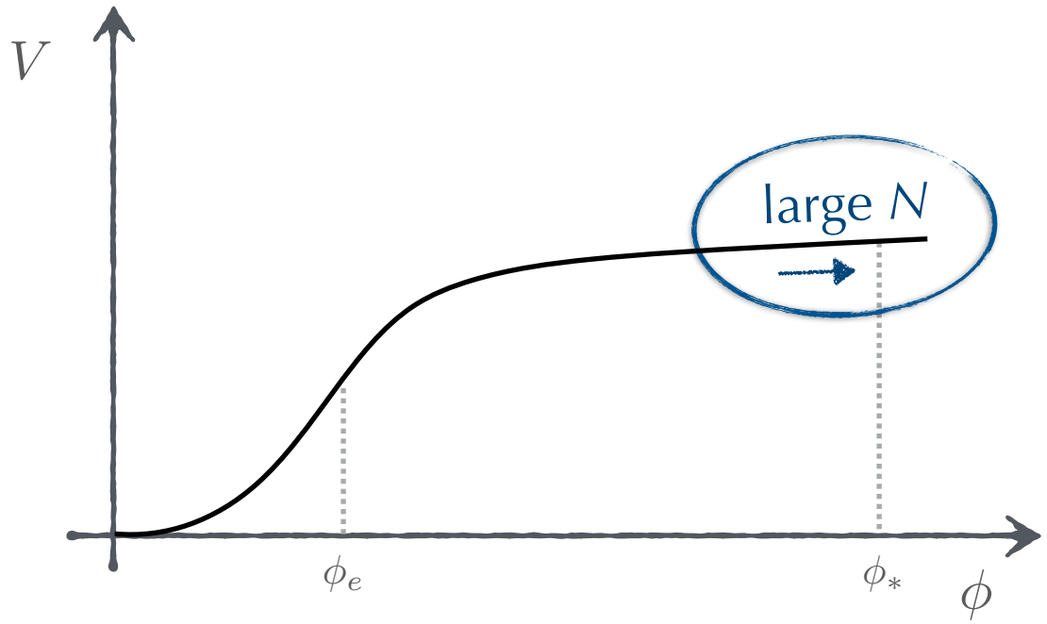
irrelevant for observations

universality classes of inflation
in terms of n_s & r

[Mukhanov 2013]

[Roest 2013, Garcia-Bellido & Roest 2014]

Universality at large N



percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} + \textbf{sub-leading corrections}$$

irrelevant for observations

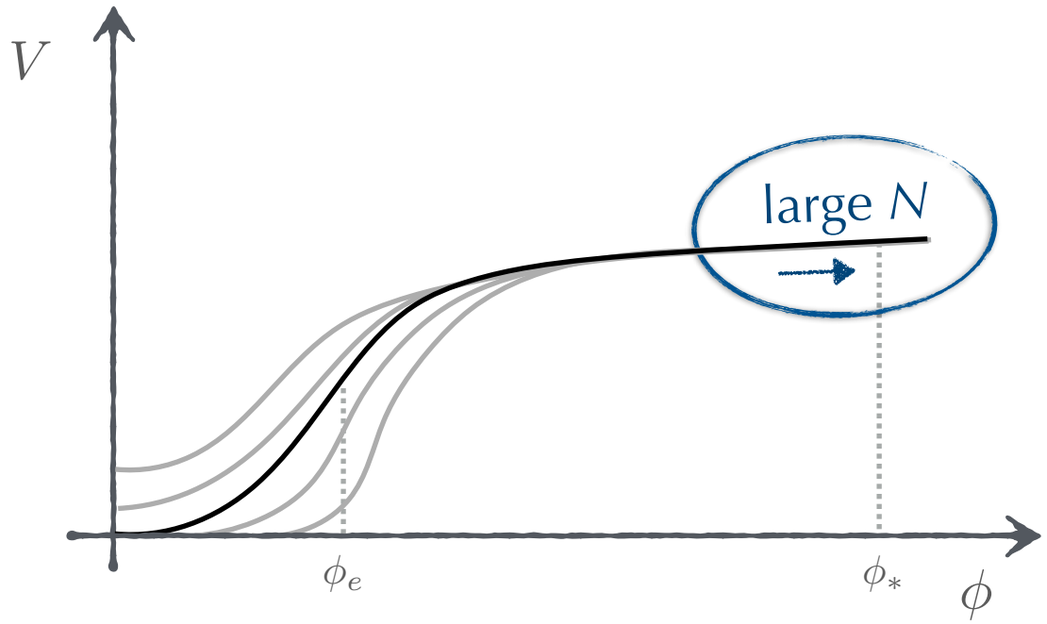
universality classes of inflation

in terms of n_s & r

[Mukhanov 2013]

[Roest 2013, Garcia-Bellido & Roest 2014]

Universality at large N



percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

irrelevant for observations

$$\epsilon = \frac{\beta}{N^p} + \text{sub-leading corrections}$$

universality classes of inflation

in terms of n_s & r

[Mukhanov 2013]

[Roest 2013, Garcia-Bellido & Roest 2014]

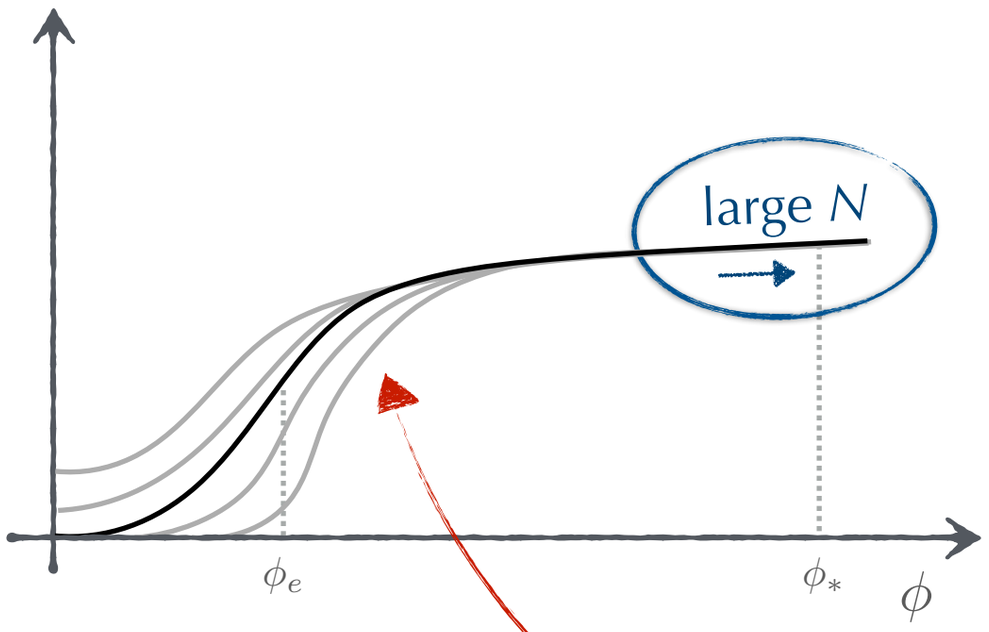
Universality at large N

percentage level deviation from scale invariance

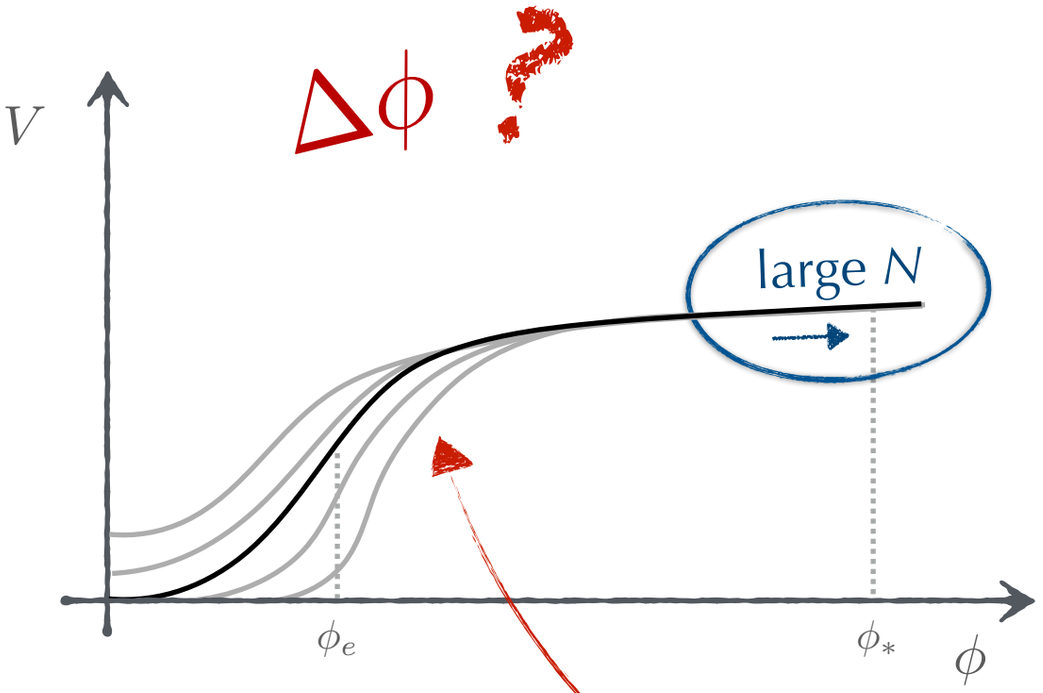
$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} \quad + \text{ sub-leading corrections}$$



Universality at large N



percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} \quad + \text{ sub-leading corrections}$$

Universality at large N

percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} \quad + \quad \text{sub-leading corrections}$$

for sub-Planckian field ranges

[Garcia-Bellido, Roest, Scalisi & Zavala 2014]

Universality at large N

percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} + \text{sub-leading corrections}$$

for sub-Planckian field ranges

Universality of the field range in the sub-Planckian regime

$$(n_s, r) \longleftrightarrow \Delta\phi$$

[Garcia-Bellido, Roest, Scalisi & Zavala 2014]

Universality at large N

percentage level deviation from scale invariance

$$n_s = 0.9603 \pm 0.0073$$

$$n_s \propto \frac{1}{N}$$

$$\epsilon = \frac{\beta}{N^p} + \text{sub-leading corrections}$$

for sub-Planckian field ranges

Universality of the field range in the sub-Planckian regime

$$(n_s, r) \longleftrightarrow \Delta\phi$$

please, see [arXiv: 1408.6839] for detailed derivation

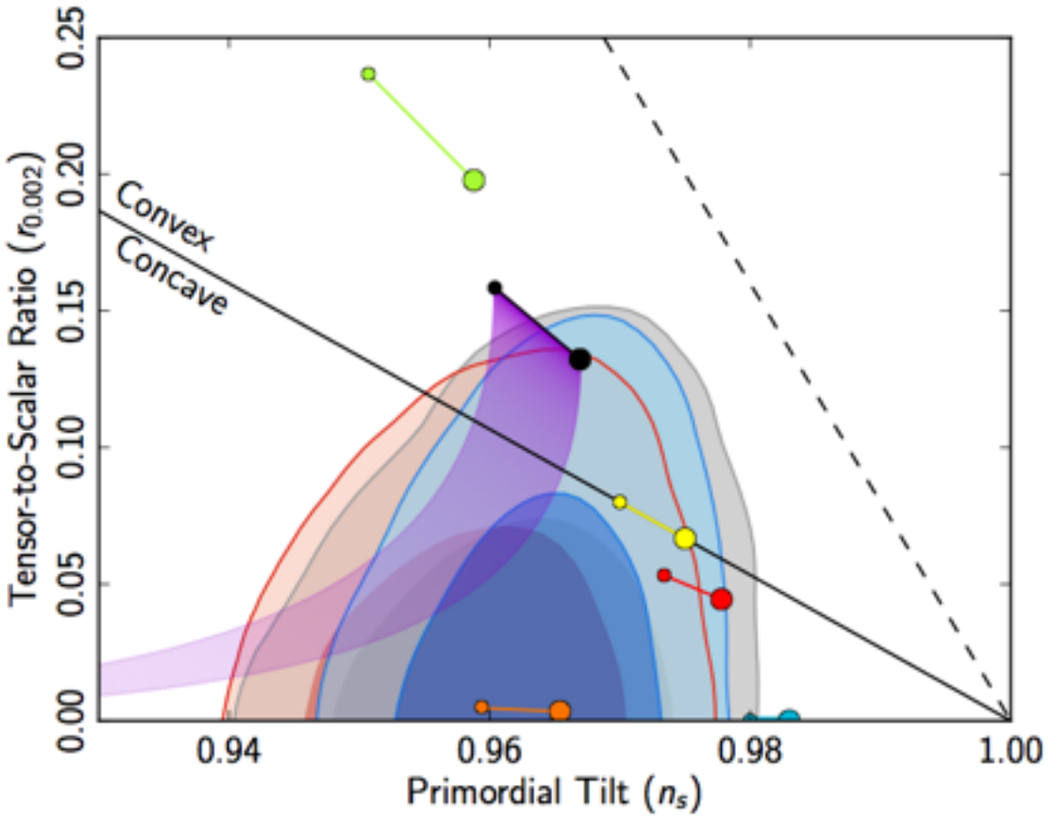
[Garcia-Bellido, Roest, Scalisi & Zavala 2014]

Universality at large N

$$\Delta\phi = \phi(N) - \phi_e$$

Universality at large N

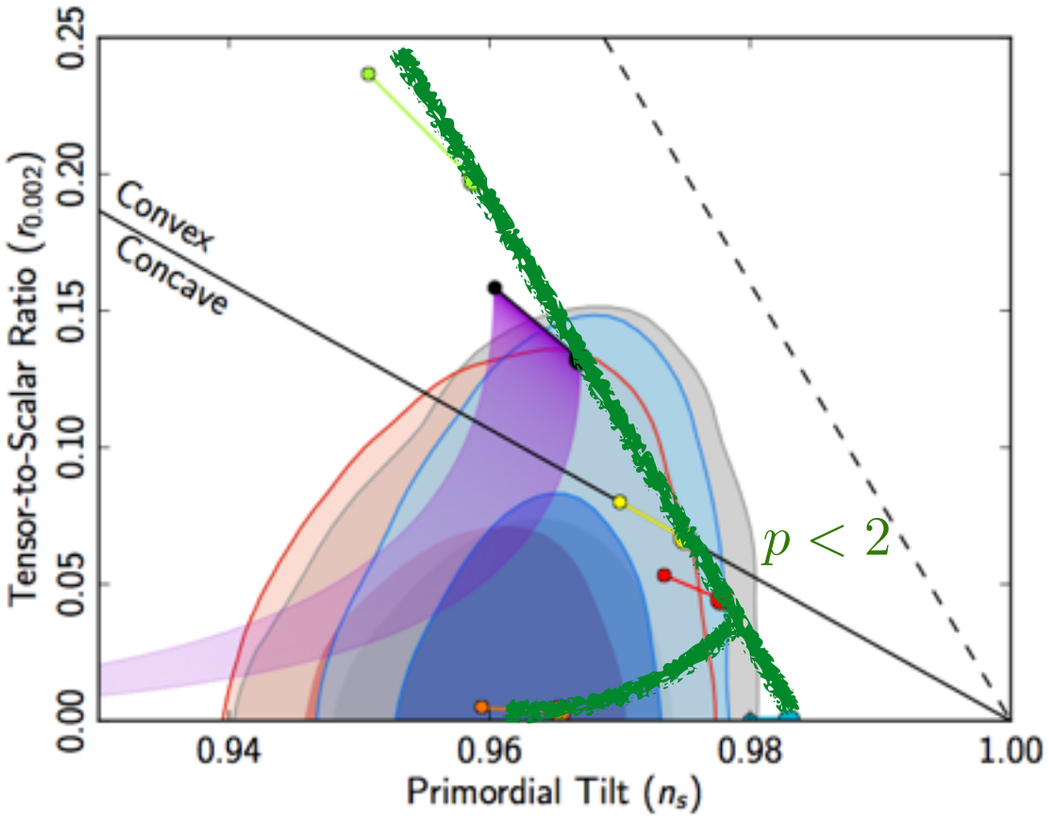
$$\Delta\phi = \phi(N) - \phi_e$$



Universality at large N

$$\Delta\phi = \phi(N) - \phi_e$$

$$p < 2 \qquad \phi(N) \sim N^{\#}$$

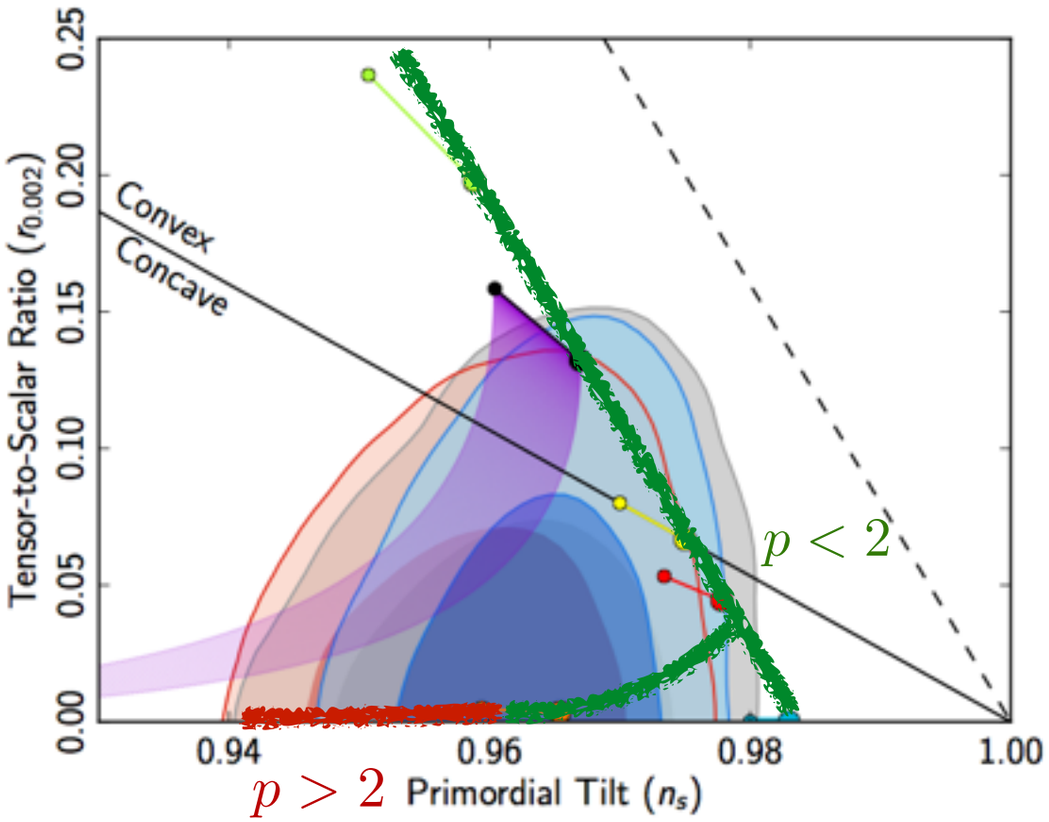


Universality at large N

$$\Delta\phi = \phi(N) - \phi_e$$

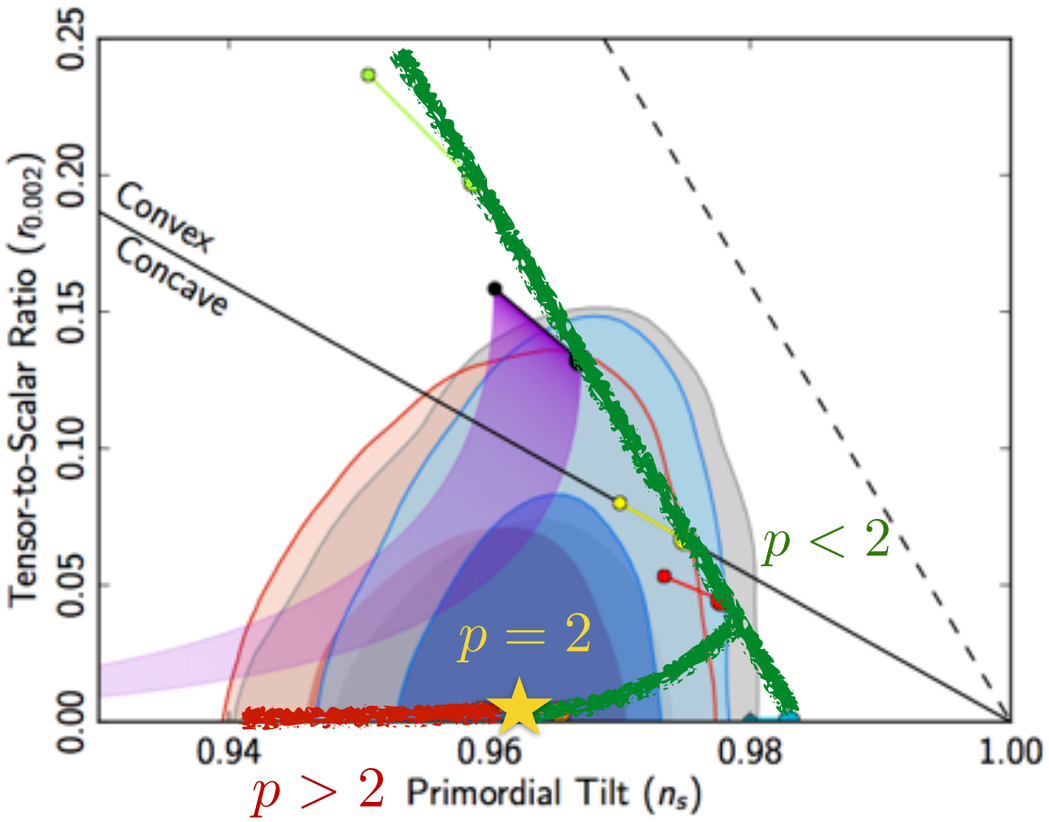
$p < 2$ $\qquad \phi(N) \sim N^{\#}$

$p > 2$ $\qquad \phi(N) \sim \frac{1}{N^{\#}}$



Universality at large N

$$\Delta\phi = \phi(N) - \phi_e$$



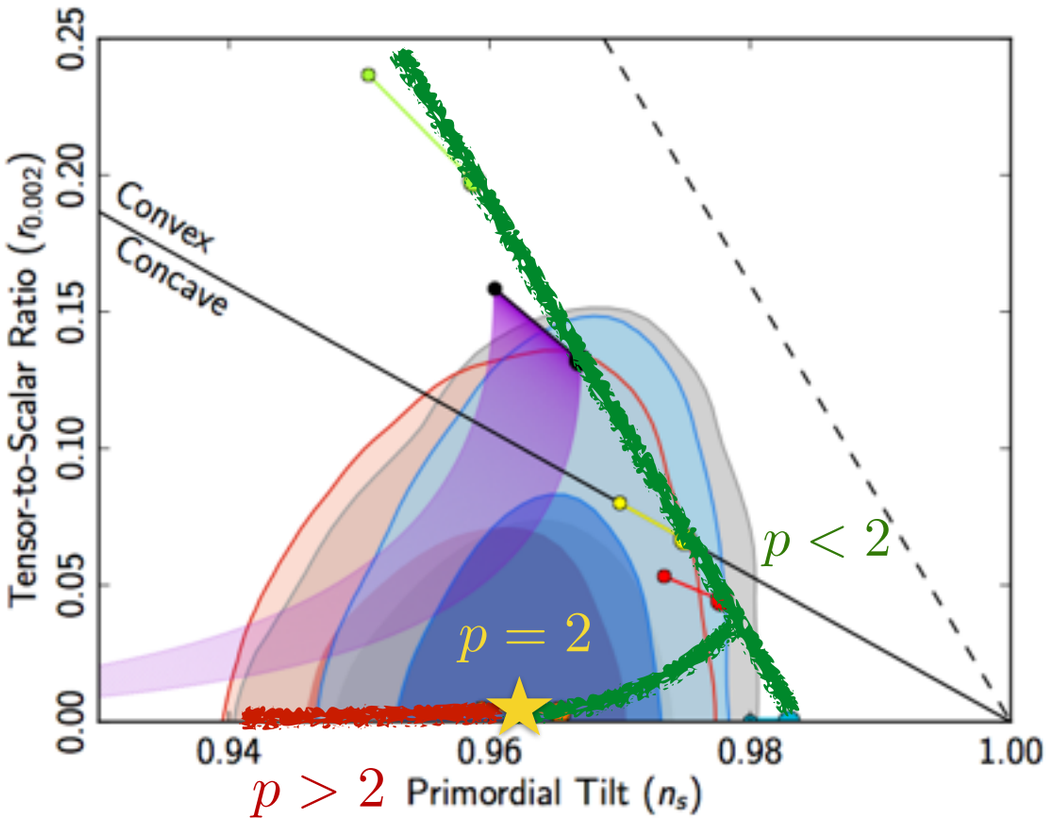
$$p < 2 \qquad \phi(N) \sim N^{\#}$$

$$p > 2 \qquad \phi(N) \sim \frac{1}{N^{\#}}$$

$$p = 2 \qquad \phi(N) \sim \ln(N)$$

Universality at large N

$$\Delta\phi = \phi(N) - \phi_e$$



$$p < 2 \qquad \phi(N) \sim N^{\#}$$

$$p > 2 \qquad \phi(N) \sim \frac{1}{N^{\#}}$$

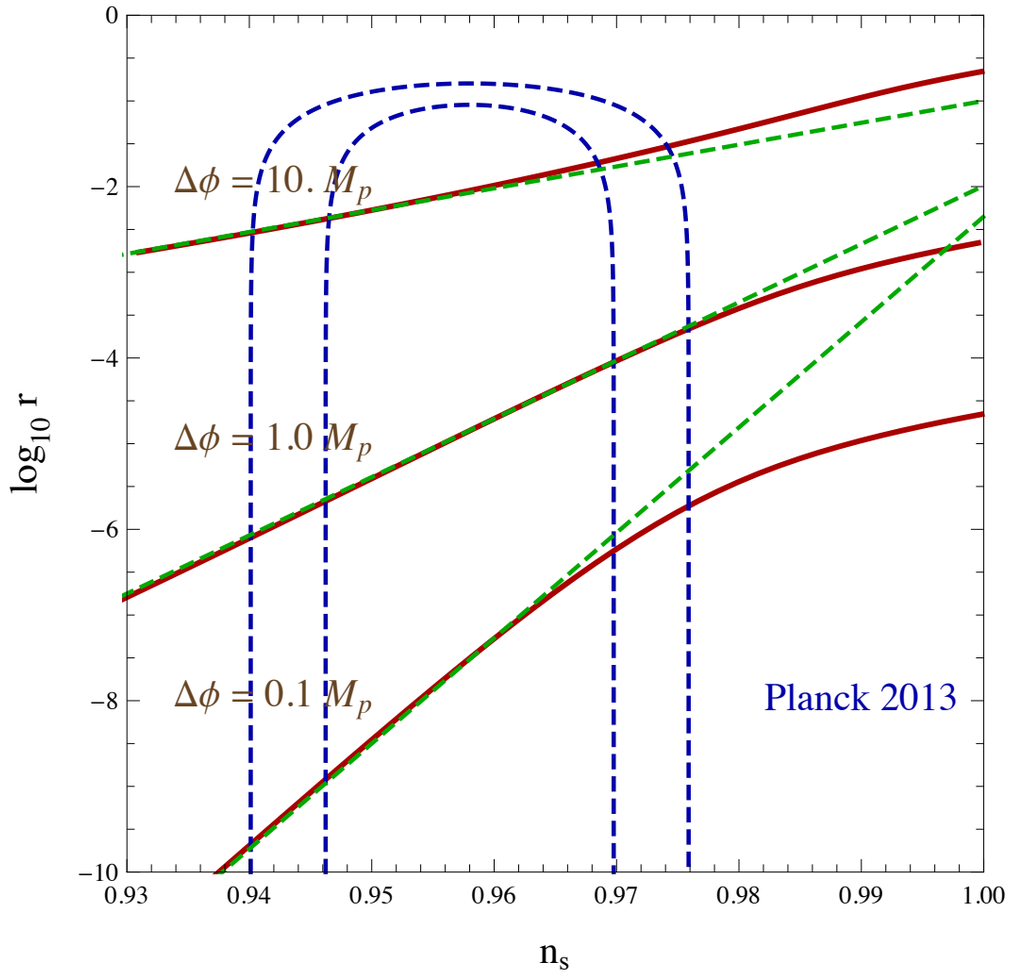
$$p = 2 \qquad \phi(N) \sim \ln(N)$$

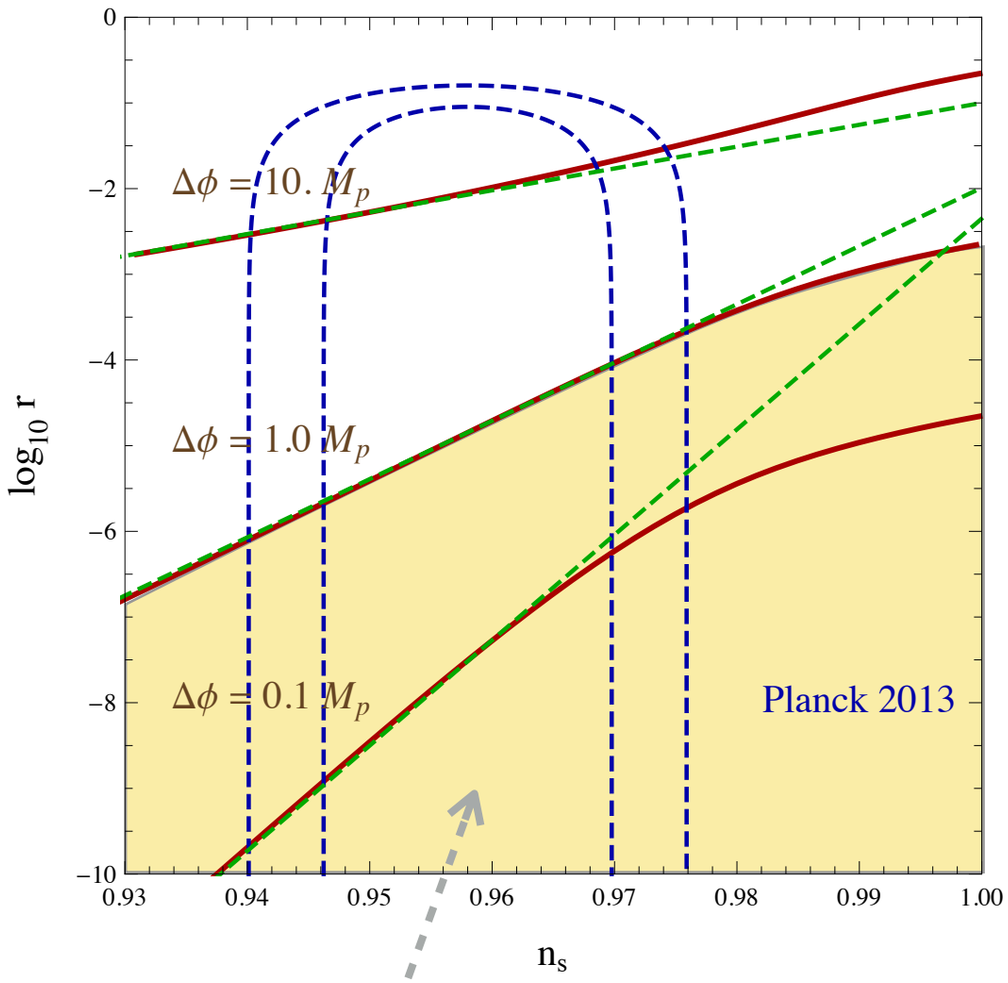
Special point

Inflationary attractors
[Kallosh, Linde & Roest 2013-2014]

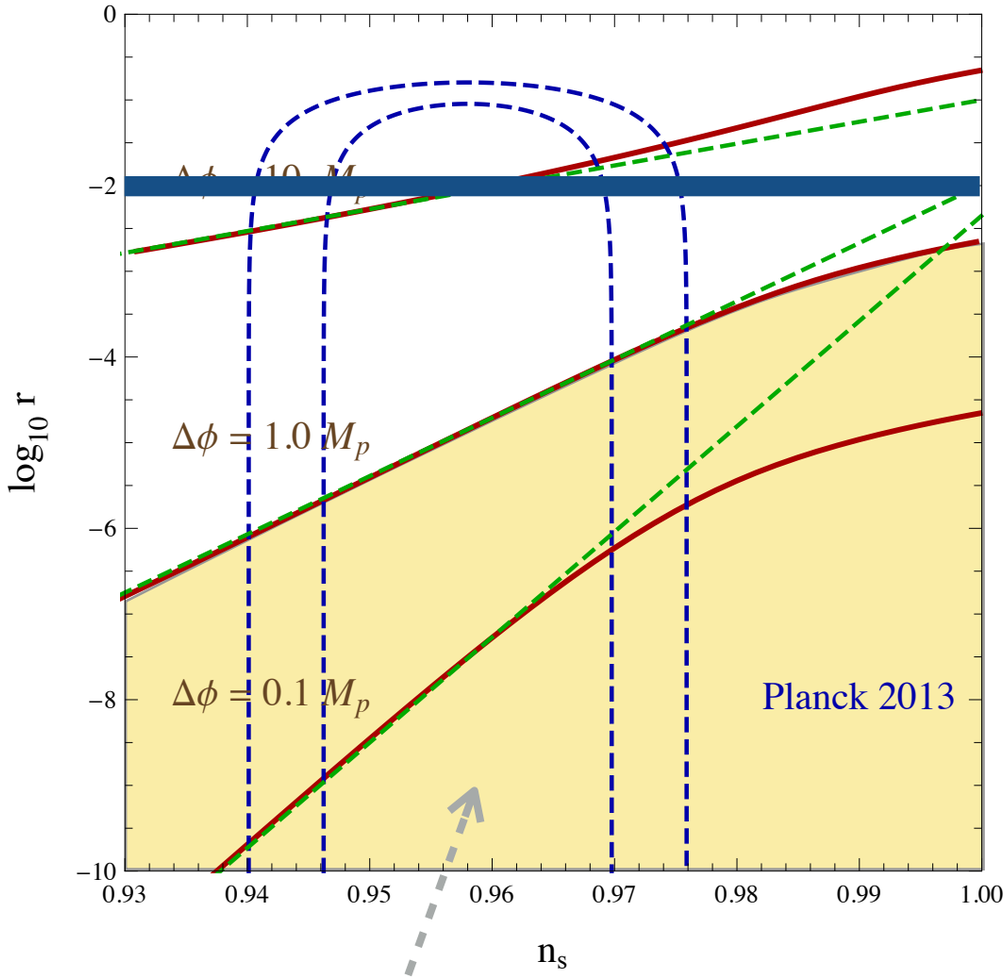
non-compact symmetry breaking
[Burgess, Cicoli, Quevedo & Williams]

A stronger bound


0
−2
−4
−6
−8
−10
0.93
0.94
0.95
0.96
0.97
0.98
0.99
1.00
$\log_{10} r$
n_s
$\Delta\phi = 10.\ M_p$
$\Delta\phi = 1.0\ M_p$
$\Delta\phi = 0.1\ M_p$
Planck 2013



sub-Planckian field ranges

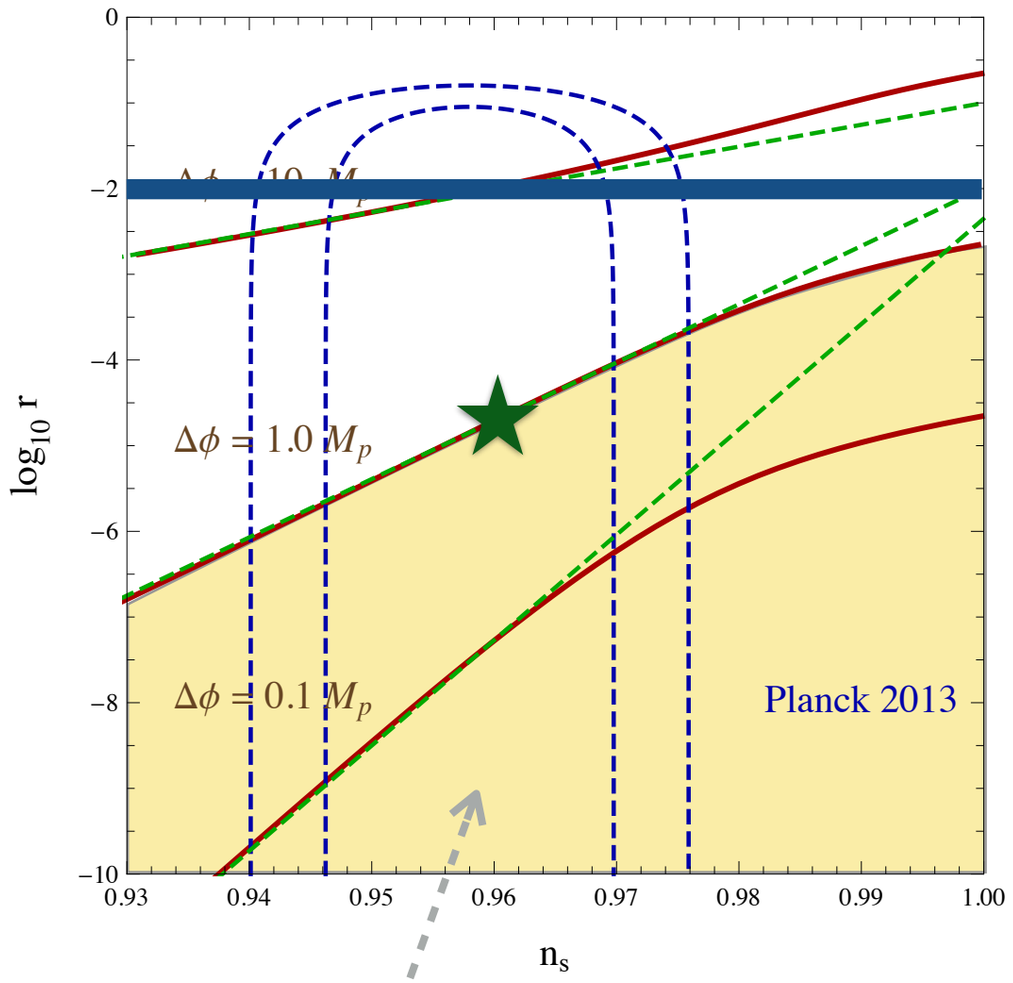


sub-Planckian field ranges

no info on spectral index

sub-Planckian field ranges for

$$r \lesssim 2 \cdot 10^{-3}$$



sub-Planckian field ranges

no info on spectral index

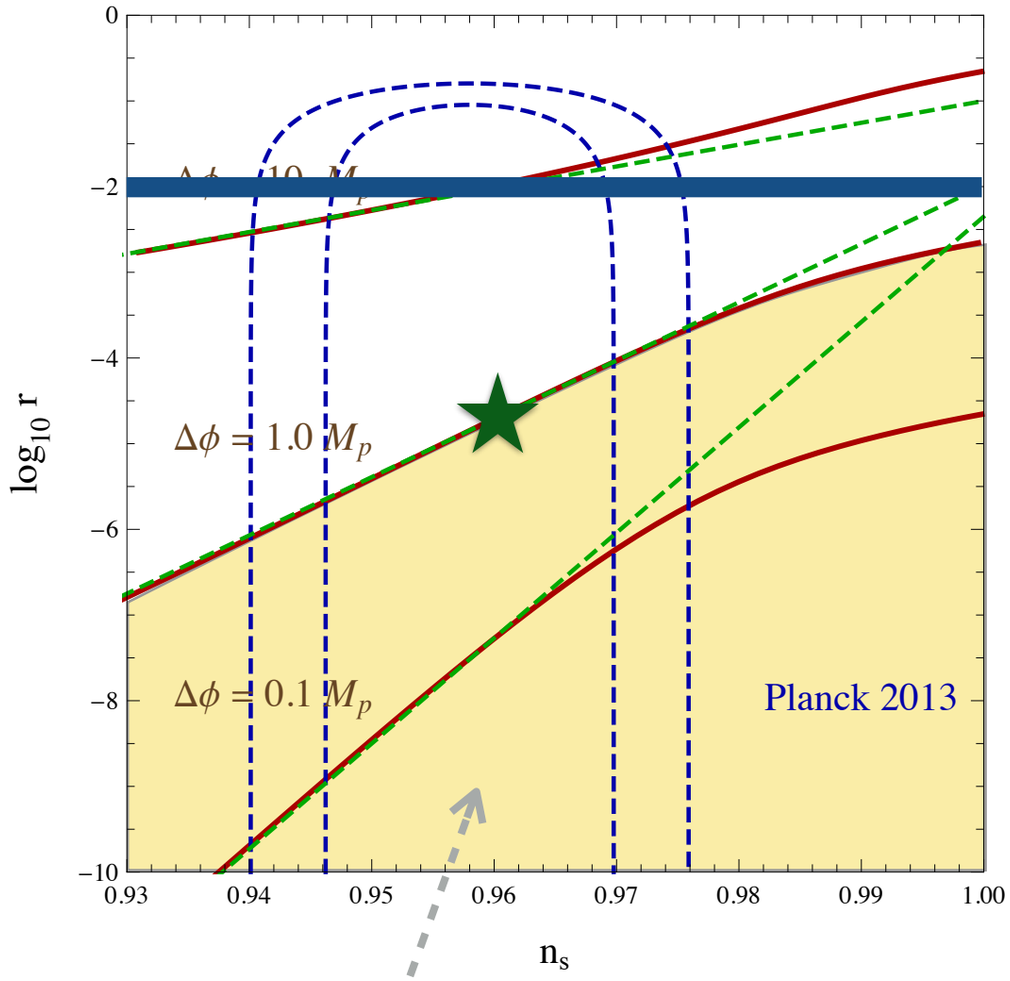
sub-Planckian field ranges for

$$r \lesssim 2 \cdot 10^{-3}$$

when considering the central value of the spectral index from Planck

sub-Planckian field ranges for

$$r \lesssim 2 \cdot 10^{-5}$$



sub-Planckian field ranges

no info on spectral index

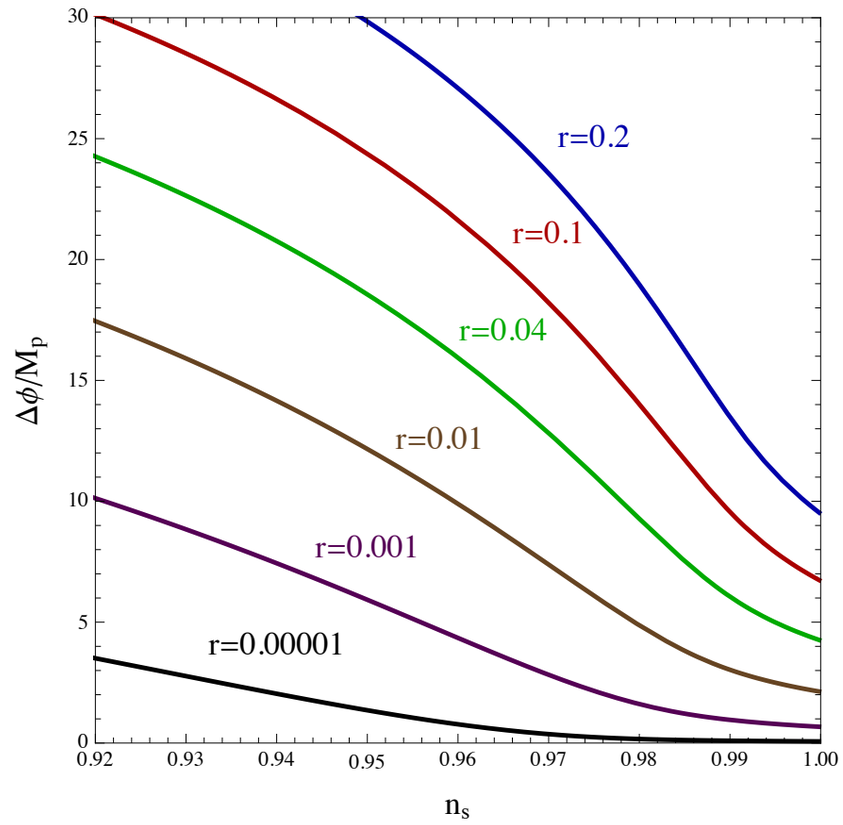
sub-Planckian field ranges for

$$r \lesssim 2 \cdot 10^{-3}$$

when considering the central value of the spectral index from Planck

sub-Planckian field ranges for

$$r \lesssim 2 \cdot 10^{-5}$$



Conclusions

- We provide strong evidence for **universality of the field range** in the **sub-Planckian regime**

$$(n_s, r) \quad \Longleftrightarrow \quad \Delta\phi$$

- We provide strong evidence for **universality of the field range** in the **sub-Planckian regime**

$$(n_s, r) \quad \Longleftrightarrow \quad \Delta\phi$$

- **We lower the usual Lyth bound by 2 orders of magnitude** by including information on the spectral index

sub-Planckian field range for $r \lesssim 2 \cdot 10^{-5}$

- We provide strong evidence for **universality of the field range** in the **sub-Planckian regime**

$$(n_s, r) \quad \Longleftrightarrow \quad \Delta\phi$$

- **We lower the usual Lyth bound by 2 orders of magnitude** by including information on the spectral index

sub-Planckian field range for $r \lesssim 2 \cdot 10^{-5}$

"observables gravitational waves would point towards super-Planckian field models"

thank you!

Extra Slides

observational predictions

$$n_s = \begin{cases} 1 - \frac{2\beta+1}{N}\,, & p = 1\,, \\ 1 - \frac{p}{N}\,, & p > 1\,, \end{cases}$$

$$r = \frac{16\beta}{N^p}$$

[Mukhanov 2013]

[Roest 2013, Garcia-Bellido & Roest 2014]

benchmarks potentials

$$V(\phi) = \begin{cases} V_0\,\phi^n\,, & p = 1\,, \\ V_0\left[1 - \exp\left(-\phi/\mu\right)\right]\,, & p = 2\,, \\ V_0\left[1 - \left(\phi/\mu\right)^n\right]\,, & p > 1\,, p \neq 2\,. \end{cases}$$

Chaotic monomial inflation

Starobinsky-like inflation

(Inverse-) Hilltop inflation

$p \neq 2$

$$\Delta\phi = \frac{2\sqrt{2\beta}}{2-p} N^{1-\frac{p}{2}} - \phi_e$$

$p = 2$

$$\Delta\phi = \sqrt{2\beta} \ln N - \phi_e$$

Universality at small μ

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu} \right)^n + \sum_{q=n\pm1}^{\pm\infty} c_q \left(\frac{\phi}{\mu} \right)^q \right]$$

n_s & r

generically insensitive to higher order terms

$$\Delta\phi = \left[\frac{2-p}{\sqrt{2}(1-p)} \right]^{1-\frac{2}{p}} \mu^{2-\frac{2}{p}} - 2^{2-p} \frac{(p-2)^{p-2}}{(p-1)^{\frac{p}{2}-1}} \mu^{p-1} N^{1-\frac{p}{2}}$$

e.g.

$$p = 4$$

$$\phi_e = \sqrt{\frac{\sqrt{2}}{3}}\, \mu^{3/2} + \frac{2\sqrt{2}}{9}\, c_4 \mu^2 + \frac{5(4\, c_4^2 + 3\, c_5)}{27 \cdot 2^{1/4}\sqrt{3}} \mu^{5/2}$$

$$\log_{10} r = -1.0 + 25.5\,(n_s - 1)\,, \qquad \Delta\phi = 10\,,$$
$$\log_{10} r = -2.0 + 68.0\,(n_s - 1)\,, \qquad \Delta\phi = 1.0\,,$$
$$\log_{10} r = -2.35 + 123\,(n_s - 1)\,, \qquad \Delta\phi = 0.1\,.$$